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ABSTRACT

This document summarizes and reviews mathematics education research reported in Australasia, or conducted by Australasians and reported elsewhere, during 1992-1995. Part I, "Contexts for Mathematics Education," contains four papers: "Social and Cultural Contexts in Mathematics Education" (R. Zevenbergen, B. Atweh, C. Kanes, & T. Cooper), "Research into Ethnomathematics in Australasia" (R. Peard), "Gender and Mathematics" (M. Barnes & M. Horne), and "Politics of Mathematics Education: Australasian Perspectives" (K. Clements & J. Thomas). Part II "Issues in Learning and Instruction," contains five papers: "Constructivist Research in Teaching and Learning Mathematics" (J. Malone & D. Ireland), "Mathematical Cognition and Instruction" (M. Chinnappan & L. English), "Advanced Mathematical Thinking" (P. White & J. Pegg), "Research in Mathematical Problem Solving" (S. Nisbet, I. Putt, & M. Taplin), and "Researching Language Factors in Mathematics Education: The Australasian Contribution" (N. Ellerton & K. Clements). Two papers are contained in Part III, "Teachers and Learners": "Research into the Professional Development of Mathematics Teachers" (C. Kanes & S. Nisbet), and "Understanding the Adult Learning of Mathematics" (G. FitzSimons). Part IV, "Specific Curriculum Areas," contains four papers: "Research in Early Arithmetical Learning and Teaching" (B. Wright, J. Mulligan, R. Stewart, & J. Bobis); "Space, Geometry, Measurement, and Visualisation" (K. Owens, M. Mitchelmore, L. Outhred, & J. Pegg); "Research into the Learning and Teaching of Stochastics" (J. Truran & K. Truran); and "Research into the Teaching and Learning of Algebra" (M. MacGregor & C. Quinlan. Includes a list of contributors and sponsors. (MKR)

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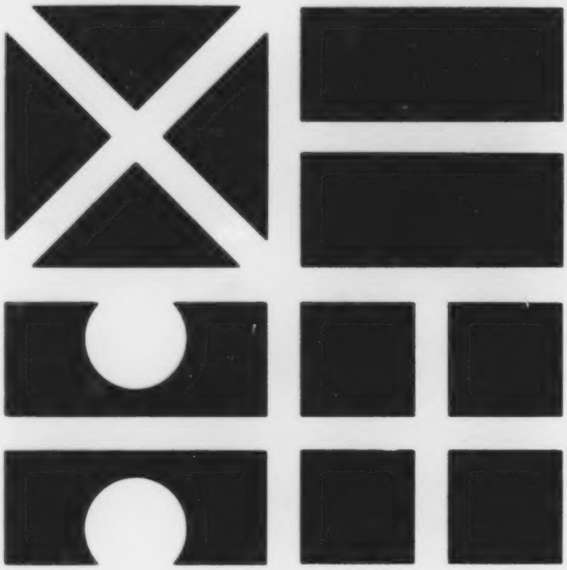

Edited by Bill Atweh, Kay Owens
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The Mathematics Education Research Group of Australasia was established as a forum for mathematics educators to disseminate research information on the teaching and learning of mathematics.

The purposes of the Group include:

- to encourage and promote quality research in all areas related to the teaching and learning of mathematics at all levels particularly in Australasia;
- to provide permanent means for sharing of research results and concerns among all members through regular publications and conferences;
- to seek means of implementing research findings at all decision levels to the teaching of mathematics and to the preparation of teachers of mathematics; and
- to maintain liaison with other organisations with similar interests in mathematics education or educational research.

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Research in Mathematics Education in Australasia
1992 - 1995

Edited by
Peter Sullivan, Kay Owens, and Bill Atweh

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Disclaimers

The views expressed in the chapters of this review are those of the authors of the chapter and are not necessarily those of MERGA or the editors.

While authors have attempted to review all research in the field, this book cannot claim to be comprehensive. Authors and editors apologise for research which may have been difficult to source or which may have been overlooked.

Mathematics Education Research in Australasia: The Context of This Review

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This review supplements two previous reviews (Blane & Leder, 1988; Atweh & Watson, 1992) published by the Mathematics Education Research Group of Australasia (MERGA). It summarises and reviews mathematics education research reported in Australasia, or conducted by Australasians and reported elsewhere. The term Australasia is used by different authors in different ways depending on the availability of the literature. In general, the term is used to include Australians, New Zealanders, those from Pacific Islands, Melanesia, and far East Asian countries. It is noted that research published in the latter three regions is often difficult to source; little research is reviewed from these areas.

The primary purpose of the review is to indicate the scope and focus of mathematics education research conducted in this part of the world and to identify sources for researchers who may wish to examine particular aspects of the research in more detail.

This chapter summarises the objectives of MERGA and its major activities and indicates some political processes in which MERGA participates. It also presents an indicative summary of the content of each of the chapters, each of which reviews a distinct aspect of research in mathematics education in Australasia.

MERGA

MERGA is a professional association for those interested in mathematics education research. It has nearly 300 members predominantly from Australia and New Zealand, but with members from Papua New Guinea, Malaysia, Singapore, and Korea, as well as from the United States of America, the United Kingdom, Canada, Brazil, and Sweden.

The constitution lists the purposes of the MERGA as:

1. to encourage and promote quality research in all areas related to the teaching and learning of mathematics at all levels particularly in Australasia;

2. to provide permanent means for sharing of research results and concerns among all members through regular publications and conferences;

3. to seek means of implementing research findings at all decision levels to the teaching of mathematics and to the preparation of teachers of mathematics; and

4. to maintain liaison with other organisations with similar interests in mathematics education or educational research.

The main activity of MERGA is the annual conference, the venue of which has been in most states of Australia, and which is to be held in New Zealand in 1997. Over 200 members have attended the recent conferences and over 100 papers have been presented in each of the four years covered by this review.

The following extract from the guidelines for presentation of papers illustrates the focus of the conferences:

> MERGA is concerned with research in mathematics education. MERGA acknowledges a broad range of approaches to research, including quantitative approaches such as experimental and quasi-experimental designs and surveys, qualitative studies such as case studies or ethnographies, participatory research projects, post-critical research, as well as appropriate combinations of these. The MERGA conference offers an ideal forum for the presentation and publication of papers which report research, which are critical reviews of research, or which are significant reviews of a body of literature. The MERGA conference is also suitable for reports of components of larger studies, theoretical papers, confirmatory studies which attempt to replicate projects conducted elsewhere, research in progress, and research which uses innovative methodologies or which applies conventional methodologies in new ways.

Papers for publication in the conference books are critiqued by two reviewers selected from a panel of senior researchers approved by the conference committee. The refereeing process provides a mechanism for peer review and critique and so contributes to the overall quality of mathematics education research in Australasia.

MERGA publishes a scholarly journal, the *Mathematics Education Research Journal*, and much of the research reviewed in this book is from either the conference publications or the journal. MERGA also commissions occasional additional publications such as this volume, and has recently published a collection of chapters on early number learning (Mulligan & Mitchelmore, 1996), and commissioned another book on research and supervising research.

The association also maintains a directory of members including information on research specialities and contacts. MERGA organises two awards for research, the Practical Implications Award, sponsored jointly with the National Key Centre for Teaching and Research in School Science and Mathematics, and the Early Career Award which is judged on paper presentations at the conference as well as on the quality of the papers.

Currently MERGA does not have a formal policy on mathematics education research. At the conference in 1996, the association will adopt such a policy. Elements of the draft of this policy address the purposes of research in mathematics education, the components which should be present in such research, and the role of the association in supporting research.

Support for Mathematics Education Research

The research reported in this review has generally not been funded at levels common in some other countries. Even within Australasia, in comparison to other disciplines, research in education generally, and mathematics education in particular, is underfunded.

Each university in Australia and New Zealand has a system of internal research grants which have supported mathematics education researchers. There is a competitive system of external grants funded by the Australian Research Council, a number of which have been won by mathematics education researchers. Nevertheless, there are very real barriers which confront all education researchers in seeking competitive funding.

As explained by Clements and Sullivan (1991), among the difficulties experienced by Australasian researchers are that some senior academics, not in the field of education, believe that methods used by education researchers are less rigorous than those used by scientists who engage in more empirically based research, and that the data obtained are often difficult to analyse and interpret. There is a need for education researchers to explain to the academic community generally that sites for educational research are complex with many direct and subtle influences operating. Clements and Sullivan argued that standard experimental designs are often unsuited to educational contexts, and contrived environments in which attempts are used to control variables are unlikely to yield data that can be meaningfully related to the complexities of normal educational settings. Often the most appropriate data are obtained through ethnographic methods, case studies, and participatory research. The dilemma is that decisions on funding are often made by researchers from disciplines in which such methods are not valued.

A cause for some concern, and one which MERGA seeks to address, is that there has been quite limited commissioned research or research funded by consumers, such as schools or school systems. At times major curriculum documents, and well funded programs, are developed without drawing on findings of research and without seeking input from experienced researchers. One recent positive trend is a wealth of case studies of teacher professional development and action research projects which have been produced as part of the National Professional Development Program. There has been in recent years greater collaboration between researchers and systems and it is hoped that this trend will continue.

Paradoxically, one of the strengths of mathematics education research in Australasia is that much of it has not been specifically funded. While this may at times result in smaller samples, and studies which are limited in scope, it also attests to the commitment of the researchers and assures their independence from the goals of funding bodies.

Participation in Political Processes

Partly in response to the low level of direct financial support for mathematics education research, and partly because some educational developments were undertaken in isolation from research evidence and without input from researchers, MERGA has sought to participate more actively in political processes over recent years. Particular government initiatives in which MERGA has been interested have included a review of the discipline of teacher education in mathematics and science, and also the development of curriculum documents intended to guide teaching of mathematics in Australian schools towards greater emphasis on competencies and specific outcomes (see Ellerton & Clements, 1994, for a critique of this development). The latter development also occurred in New Zealand. MERGA and similar organisations have contributed to the debate surrounding these initiatives and subsequent developments. The chapter by Clements and Thomas not only reviews these processes but includes a further contribution to these debates.

As one of its main avenues of political activity, MERGA participates in the Australian Mathematical Sciences Council (AMSC) which is a member of a political lobby group, the Federation of Sciences and Technological Societies. The AMSC includes MERGA, the Mathematics Education Lecturers' Association, the Australian Mathematics Society, and the Statistical Society of Australia. The Australian Society of Operations Research is now an observer, and the Australian Association of Mathematics Teachers recently relinquished its membership.

The challenge for professional associations interested in mathematics education is to find ways to collaborate with each other and to eschew short term opportunities in favour of building strong relationships between associations, so that united they form a significant political entity with which governments will seek to work in developing initiatives which are owned and driven by the profession.

Components of the Review

This review of research has 15 chapters. Authors have sought to provide a comprehensive guide to research in the respective fields although there may be instances in which relevant research has been overlooked or has been difficult to source. Authors consulted a variety of databases such as ERIC, AUSTROM, PsychAbstracts, hard copies of journals, books, conference proceedings, collected papers, and some authors were ablt to made enquiries from faculties and libraries in all universities in Australia and New Zealand.

The first part of the review contains four chapters on contexts for educational research.

The chapter by Robyn Zevenbergen, Bill Atweh, Clive Kanes, and Tom Cooper commences by a theoretical discussion on the wide spread of perspectives adopted by different researchers investigating the social context of mathematics education: frameworks drawn from discourse theory, critical theory, sociology, non-cognitivist learning theory, the philosophy of language, and poststructuralism are represented. The chapter identifies the broader cultural context and the micro-culture of the classroom as two main sections to review the research. Research on cultural aspects of mathematics education, in particular crosscultural and indigenous studies, takes a leading role in the chapter. With possible overlap with the chapter on language and mathematics, this chapter concentrates on the social aspects of language research. Likewise, research on the classroom context considers aspects such as teacher-student and student-student interactions in the classroom, group learning, teacher factor and student perception.

Peard provides an overview of work in Australasia which is classified within the area of ethnomathematics. Because of the newness of this field, Peard pays considerable attention to varying definitions of ethnomathematics and himself provides a broader definition than commonly is used. The review summarises the significant work which has been done addressing ethnomathematics as it appears in non-industrial cultures, including a number of significant studies which have contributed to the literature worldwide. Peard also takes an interesting perspective in that he examines aspects of mathematics from within industrial cultures under the ambit of the term ethnomathematics. These include aspects of the mathematics of finance and the mathematics of gambling, which become relevant for particular sub-cultural groups within industrialised countries and which can inform mathematics teaching and learning in those countries. The chapter also examines the influence of culture on mathematics education and the way that these two are linked. The chapter is an inclusive summary of the work in the field and is a useful starting point for researchers seeking to identify Australasian work on the topic of ethnomathematics.

Gender issues have been a concern for Australian educators for some time and there has been considerable research as well as policy documentation in this area.

The chapter by Barnes and Horne presents a review of key studies which have either researched gender issues or which have examined other issues from a gender aware perspective. The list of references is comprehensive and the research diverse but the chapter gains coherence by presenting a theoretical perspective on research on gender. The chapter summarises succinctly different types of research in the area and gives examples of each. Much research has focussed on the lack of participation by women in mathematics and reasons for the lack of participation. The effects of curriculum documents, assessment procedures, classroom organisation, discourse in the classroom, impact of technology, beliefs and attitudes, and social contexts have all been studied.

As mentioned above, MERGA as an organisation has become more aware of the political dimension of research, education, and mathematics. This has resulted in a range of activities by members interfacing with governments and government policies. The chapter by Clements and Thomas presents a perspective on some of the key issues which have arisen over recent years. They explain the way mathematics has been used politically and the way that it has been used by policy makers. They are critical of a perceived tendency to import models and curricula from elsewhere and argue that the resultant trend to national curriculum and system wide testing can have detrimental effects both on educational programs and opportunities for learning by all students. Clements and Thomas argue that policy makers have based curricula on premises which have not been researched and for which no supporting evidence exists. They call for more profession-driven approaches in the future and cite the AMSC as a model for cooperation between the professions.

The second part of the review contains five chapter which address issues in learning and instruction.

In their chapter reviewing research conducted from a constructivist perspective in the Australasian region John Malone and Dennis Ireland date the commencement of interest in this area to the mid 1980s. They assert the Australiasian research has adopted a mix-and-match approach to the definitions of constructivism. They divide the research into four areas: constructivist models for teaching and learning mathematics, teacher beliefs and change, constructivist approaches to assessment, critical responses to the constructivist canon, and future research trends. In their conclusions the authors note the predominance of primary school research in this area hypothesising that the secondary school teaching and learning tends to be subject matter orientated are less open for change.

The chapter on cognition and instruction focusses on a view of cognition that recognises the importance of schemata and cognitive load. Chinnappan and English summarise studies which have linked the theoretical to the experimental results on a range of instructional areas, particularly at the micro well-controlled level. There are some studies considering longer periods of instruction and others analysing instruction on problem solving. A significant part of the chapter shows the relationship between mathematical knowledge construction and mathematical understanding from the cognitive viewpoint.

The chapter on advanced mathematical thinking, by White and Pegg, considers the importance of beliefs about mathematics held by tertiary students as they undertake their studies and the effects that these beliefs may have on their mathematical thinking and learning. In addition, it explores the earlier conceptualising of students about algebra, calculus, functions, and proof. The effects of schooling and technology on learning and conceptualising are addressed. Some studies are interpreted in terms of theoretical positions such as van Hieles' theory and the SOLO taxonomy, giving a further interpretation of these viewpoints and the meaning of the term abstraction.

The chapter by Nisbet, Putt, and Taplin provides a comprehensive overview of research into problem solving in the teaching and learning of mathematics over the period. Problem solving is one of the more productive and popular topics among Australasian researchers and this is reflected in the high profile given to problem solving in curriculum documents and in school assessments in the region. There has been considerable research attention to general cognitive processes which contribute to problem solving including the distinction between the use of school taught or intuitive methods, and also to strategies for representing problems, with an emphasis on visualisation. There is also a large body of research summarised on metacognition over a range of levels and topic areas, which in all cases is reported as enhancing problem solving success rates. The chapter also identifies studies which address gender issues, the needs of special groups, the use of technology, the impact of attitudes and beliefs, and research on particular mathematical contexts. The underlying trend is that researchers are seeking to identify processes which are used by individuals in solving problems, and there is an explicit focus on learning and classroom application in the research.

In the comprehensive chapter on language factors in mathematics education, Ellerton and Clements highlight one of the major areas of strength in research in Australasia. They identify research on the interaction of other cultures with western mathematics, which complements earlier work on ethnomathematics, including studies with Aboriginal cultures, the major work on counting systems in Papua New Guinea, and the projects that draw on the best of Asian education. Ellerton and Clements report on studies of classroom discourse and of modes of communication, including assessment. They summarise the extensive body of research on errors, on bilingualism, on open-ended questions, and on cooperative group learning. They note the significant implications of this research for teacher education.

The third part of the review contains reviews of research on teachers and adult learners.

Many researchers in mathematics education in Australasia have addressed issues related to teacher education and professional development. The chapter by Kanes and Nisbet reviews such research. The chapter locates this evolving area of research within the context of curriculum changes during the past decade. They present research reported during the last four years in four areas according to the primary focus of each study: technical requirements for teaching, professional needs of teachers, teaching context as a site for curriculum change, and theoretical studies on the concept of teacher development. In the conclusion of their chapter, the authors employ three types of curriculum interests articulated by Habermas, technical, practical, and emancipatory to reflect critically on the status of knowledge in this area and recommend new research from new perspectives.

An increasing area of research in mathematics education relates to students returning to higher education after a short or long interruption. The chapter by Gail FitzSimons reviews such literature. The chapter commences with a discussion of theoretical aspects of issues relating to adult education and definitions of numeracy. Several areas of research have been identified that relate to adult learners: bridging programs, Indigenous students, language and multicultural issues, calculators and computer. the affective domain, and workplace education. The chapter concludes that research in this area is in its infancy and identifies some areas where more systematic research in needed.

The fourth part of the review addresses specific curriculum areas.

The chapter on early arithmetical learning, by Wright, Mulligan, Stewart, and Bobis, addresses a diverse range of important topics and issues. These include classroom-based studies examining student collaboration, the use of games, and the use of an extending curriculum; studies of intervention programs and recovery programs for at-risk students; longitudinal studies of strategy and knowledge development; and studies of the role of calculators.

Perhaps the most coherent body of research on a particular area was reviewed by Owens, Mitchelmore, Outhred, and Pegg. They summarised research on space, geometry, measurement, and visualisation. An important component of the chapter is research reviewed on problem solving and spatial tasks. The chapter also addresses the broader aspects of visualisation, including studies which address visual imagery, those which examine the drawing of diagrams and learning style, and the use of concrete representations. Some studies on affective and social factors, including gender and cultural factors, are also reviewed. There is a comprehensive review of studies which address spatial conceptual development, including particular topics in geometry and the use of levels of understanding to describe spatial development. A similar analysis is provided for research on measurement. There is a clear focus throughout on issues related to learning. This comprehensive body of research must surely have a major impact on what is known about spatial thinking and conceptual development in space and measurement, and future teaching of these strands.

Given its current prominence in school curriculum documents, it is amazing that there has been quite little fundamental research on probability and statistics. The chapter by Truran and Truran provides a comprehensive overview of the research which has been conducted in Australasia. They refer to research which addresses the learning of probability and statistics from the perspective of cognitive psychology and the issue of assessment instruments. They review research on curriculum pedagogy, and research related to affective and ethnographic issues, including some work on the history of probability and statistics and the use of technology. As Truran and Truran explain, there is now a need for those involved in the formation of curricula, and the preparation of support materials for teachers, to acknowledge findings from research and what we are beginning to know about the way that students and teachers approach probability and statistical concepts so that there can be a better match of curriculum to students' classroom needs and abilities.

In the review of research on algebra, MacGregor and Quinlan cover two main areas. The first section covers studies that have considered difficulties and approaches taken by students when learning algebra. These difficulties include the problems arising when symbols are considered as standing for objects rather than variable numbers, the interpretation of English sentences into algebraic statements, the complexity of operations required by some problems, the differences between computer and textbook notation for functions, differences in concepts of a function, and the cognitive factors related to processing in algebra. The chapter then highlights the change in research on algebra. More recently, research has been addressing the issue of teaching in order to improve the effectiveness of school and early university algebra experiences. Some of the different approaches that have been evaluated are the use of concrete materials, patterning, "backtracking," and technology.

Conclusion

As in many other fields, the amount of research being conducted and reported in education is greater now than at previous times. One of the tasks of researchers, new and experienced, is to identify previous research on which to build. This review aims to facilitate the task of identifying research which has been conducted in Australasia to make it accessible to others.

There are other purposes for this review. One is to celebrate the achievements of the association itself. Given that one of the more important functions of a research organisation is to stimulate research, this publication attests to the significant success of MERGA.

Another purpose of the review is to signal to other researchers in the field of education, to other researchers in other fields, and to the education community generally, that there is a healthy and active research community in mathematics education, and that MERGA as an organisation, and members as individuals, are most willing to participate with others in using this research in the design and delivery of educational programs.

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PART 1

CONTEXTS FOR MATHEMATICS EDUCATION

Social and Cultural Contexts in Mathematics Education

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Introduction

For the purposes of this chapter, we understand the term *context* in its sociological meaning. Within this construction, we see those factors external to the individual as foundational to this chapter. This means that many psychological factors such as affect, attributions, internal learning variables, and factors such as gender are effectively excluded from this chapter. It is inevitable that many of the aspects of research addressed within this chapter will be dealt with in others. Similarly, there are many areas of research which could well have been covered within this chapter, but which we have deliberately omitted or discussed in brevity to avoid overlap with other chapters. These deliberate limitations which we have imposed on the writing of this chapter may have excluded some works which other researchers may have felt appropriate

In structuring the chapter, we commence with a discussion on the theoretical perspectives which researchers have employed in the conceptualisation of their work. These are varied and provide a wide scope for the theorising of context. We then consider the macro conceptualisations of context. These include the social and cultural contexts of mathematics education along with the role that language plays in these areas. From this wide lens on the research within this area, we move to a more focussed analysis to consider the micro contexts of mathematics education. Within this conceptualisation we have included studies of schools, classrooms, social interactions, and the role of teachers within these situations.

Theoretical Perspectives on the Social Context of Mathematics Education

Research into the significance of social context for mathematics education has made use of a variety of theoretical approaches reflecting aspects of recent developments within the arenas of cultural studies, philosophy, sociology, and history. A common goal of these works has been to examine critically a range of presuppositions underscoring aspects of contemporary theory and practice in mathematics education. For instance, Klein (1993, 1994) investigated social constructivism in the context of teacher education, and raised important questions concerning the adequacy of this paradigm with respect to the development of mathematics-teacher knowledge. Other authors have explored the adequacy of conventional assumptions relating to difference (such as social, cultural, and/or gender-related differences), the role of language in teaching and learning, and the nature of communication. These, and other similar contributions, will be discussed in greater detail below. Theoretical frameworks drawn from discourse theory, critical theory, sociology, non-cognitivist learning theory, the philosophy of language, and poststructuralism are represented in the approaches adopted by researchers. These are discussed below.

Frequent use is made of the notion of *discourse*. While this term has assumed various fairly loose meanings and multiple significances in recent times, it is usually taken to refer to a development of language which transcends purely communicational ends. An important theme in the style of discourse analysis developed by Michel Foucault, for instance, is to understand how knowledge and power are constituted simultaneously through the operations of language. When applied to topics within mathematics education, such analyses have been used to provide accounts of curriculum knowledge in mathematics alternative to those derived from more conventional paradigms. For instance, Zevenbergen (1992, 1993a, 1993b, 1994b, 1995) has argued that it is appropriate to interpret certain constructs (such as rationality, gender, child development, and pedagogy) as these apply to mathematics education not as *ontological realities*, but as socially constructed discursive phenomena located within historical domains. Moreover, these social constructions imply significant distortions when considered from the perspectives of a socially just society. Not only are dominant discourses culturally bounded and by implication exclusionary, but they reify constructs of difference such as gender, affective and motivational variation, and ability, and in so doing abstract the constructs from social and political contexts germane to their validity. Zevenbergen argued that the discourse of cognitive psychology provides an instance of this phenomenon. Likewise, Klein (1993, 1994) argued that constructivism as a model for student-teacher education is deficient in that it evades consideration of the historical and social situatedness of the nature of knowing and learner subjectivity. Klein argued that adequate notions of culture and power are largely absent from the constructivist apparatus, and that these are more appropriately highlighted through analyses of the discursive practices of mathematics education. Arguing in parallel with Zevenbergen, Klein warned that the dominant position of cognitive psychology, albeit in its latest mode of constructivism, is inconsistent with the espoused aims of curriculum reform and the objectives of social justice in mathematics education.

Another theoretical framework adopted by researchers in the area of social context is broadly consistent with critical social theory, as associated with the work of Habermas. An important feature of this theory is the importance of illuminating the contradictory tendencies embedded within oppressive social organisations and systems of thought. Social injustice, it is argued, can be combatted once the social construction of thought is recognised, contradictory tendencies unmasked, and action taken to reconstitute social relations in the light of purportedly *enlightened analyses*. Taylor and Campbell-Williams (1993), for instance, argued that mathematics teaching as it is usually practised holds on to certain invisible frames of reference which act as constraining influences on the mathematical activities of the teacher and students. These authors asserted that Platonism and formalism, two traditional models for the philosophy of mathematics, afford examples of such myths. Their analysis made use of Grundy's adaptation of Habermas' theory of human knowledge-constitutive interests which analyses the curriculum with respect to three possible kinds of interest: technical, practical, and emancipatory. The first concerns the instrumental means of action required for a predesignated end, and is predominantly product oriented. In contrast, the second is concerned with the intersubjective process of interpretation, personal judgement, and reflective deliberation, rather than with the objective products of learning. The third is concerned with notions of power and the way ideas can dominate the perceptions and actions of other less powerful members of society. It might be said, for instance, that the work of Klein and Zevenbergen, reviewed above, was motivated by the intention to advance just such emancipatory interests in the curriculum. In contrast to these authors, Taylor and Campbell-Williams have chosen to work within the social constructivist paradigm, albeit enriched by Habermas' notion of an *ideal speech situation*. Roughly speaking, such a speech situation requires that communication be conducted without constraints, disclose the true intentions of the speakers, and be free of coercion. Taylor and Campbell-Williams contended that a critical classroom discourse, constructed on the bases of these premises, and directed toward unmasking hitherto invisible constraining frames of reference which underpin the traditional culture of the classroom, can serve as a starting point for advancing emancipatory interests within the curriculum. Neyland (1994) adopted a broadly similar perspective in his analysis of the potential for social constructivism to advance emancipatory rather than purely instrumental interests within the curriculum.

Other theoretical perspectives adopted by researchers have drawn on avenues deriving from non-cognitive theories of learning as found, for instance, in: (a) the situated studies of Lave and Rogoff, (b) the sociological investigations of Bourdieu, (c) the linguistic philosophy of Wittgenstein, and (d) the critiques of *meaning* as found in the works of Kristeva and Derrida.

Grier (1994), for instance, reported on her work-in-progress concerned with adult learners' formal and social ways of knowing and applying mathematics. This study was concerned with how alternative kinds of knowing are interrelated by learners in the development of their mathematical knowledge. Lave's notion of learners participating in a community of practice, would appear to be highly relevant to these investigations.

Kanes (1992) argued that Wittgensteinian style analyses provide another framework for investigating the role of language in pedagogic interactions. Whereas discourse theory attempts to forge links between language, epistemology, and the various logics of personal and social domination, Wittgenstein's approach concentrates on the operationality of language. Kanes argued that Wittgenstein's concepts of *language-game* and *grammar* allow researchers to focus on the *roughness* of actual communicative discourse. *Slippage* within the discourse is found to be both an actual and a necessary component of a successful linguistic interaction. One implication of this view is that there is no necessary connection between *clarity of thinking* and *clarity of expression*. This problematises the view that mathematical ideas are, in principle, available for transparent communication. This conclusion, though not the argument, is, of course, shared with those arguing from a social constructivist perspective.

In the context of contributing to a critical appraisal of the social constructivist paradigm, Kanes (1993) developed four arguments, each quite distinct from those of Klein, Zevenbergen, Taylor, and Campbell-Williams, as reviewed above. The first argument is methodological. Following Bourdieu, he argued that social constructivism does not and cannot justify the interpretative protocols used to posit defining relations between teachers and students. Social constructivist theory lacks the capacity for a reflexive self examination, and this limits the interpretative validity of its' own claims. Second, Kanes argued that the theory is incoherent regarding the role of representation in the learning process. Third, despite claims to the contrary, Kanes argued that the theory preserves the *Cartesian duality*, albeit in the form of a dichotomy between the mind of the learner and the social domain, and seems to imply that each learner develops a private language, wholly separate from the social domain. But this view leads to the philosophical problems identified by Wittgenstein in his later philosophy of language. Fourth, Kanes claimed social constructivism makes use of structural rather than *poststructural* concepts of meaning and its production. One consequence of this is that social constructivism is inconsistent: at the level of knowledge formation it is radically anti-essentialist, whereas at the level of meaning formation it relies heavily on essentialist assumptions. In the light of these arguments, Kanes hinted at the emergence of a non-cognitivist postconstructivist theory of learning.

In summary, the theoretical expositions which have been discussed in the preceding sections raised a number of considerations. They offered alternative approaches to the understandings in contemporary mathematics education. For us, they suggested that new ways of thinking and conceptualising research in the field will offer new understandings of teaching and learning whereby the social context is integral to the teaching/learning process.

In the remainder of this chapter we draw on research which supports the notion that the social and cultural contexts of mathematics education are vital considerations to development of more complete understandings of teaching, learning, and schooling in contemporary society. In reviewing this research we identified four key categories which we have used to structure the remainder of the chapter. These were:

1. the cultural context which was taken to be the wider conceptualisation of culture and its impact on the field;
2. the background of the students which was taken as the socially constructed individual and the impact that their entering culture, complete with language, has on the learning of mathematics;
3. classroom as a microcosm of the wider social and cultural milieu; and
4. the unique situation of Aboriginal mathematics education.

Culture and Context

At the 1993 conference of the Mathematics Education Research Group in Australasia (MERGA) two of the keynote speakers, Barton and Bishop, addressed the cultural context of mathematics and mathematics education. Barton (1993) raised the question of justification of many different mathematical knowledges as developed in different cultures. He commenced by sketching the development of views and philosophies on the nature of mathematics as developed in our western culture during the past 2000 years. He argued that the predominant concern of mathematicians since Plato and up to the early part of this century has been the establishment of the reality of mathematical objects and the truths of mathematical statements. He showed that in the second half of this century there has been an increasing trend to look at the social-convention nature of mathematics. Such a movement, he argued, commenced with the writing of the French philosopher Bachelard who suggested a philosophy of science based on the historical development of the concept of rationality. Thus subjectivity was introduced to mathematical objects and truth. Referring to the writings of Kuhn, Popper, Lakatos, and Wittgenstein, Barton concluded that there is a "shift from objective concerns to subjective ones" (p.1). According to the author, however, these philosophies fail to explain the development of different kinds of mathematics in different cultures. "These theories either lead to a concept of a one true (ideal) mathematics, or they lead to a concept of a developing subject forever guided along a path towards a rational ideal of mathematics" (p. 2).

Barton presented a view of mathematics as a lens through which people see the world. In other words, mathematics is to be seen as a process rather than consisting of objects and statements. This view, Barton argued, brings the actors back into the development of mathematics. "The consequence of this view is that mathematics will be culturally determined to the extent to which a community of mathematicians share cultural characteristics (e.g., language, values, perceptions)" (p.2). This view of mathematics allows for the definition of what mathematics is since it starts by considering behaviours that have mathematical characteristics which in turn become the mathematics as defined by and for a particular culture. In a previous paper, Barton (1992) identified some of the theoretical justifications of ethnomathematics.

Bishop's (1993) keynote address considered the importance of conceptualising the social context within different research studies. He demonstrated how in the past ten years there has been a steady, but slow, increase in attention to social factors as explanations of difficulties that students face in their study of mathematics. Bishop then proposed a structural model consisting of "concentric circles of increasing contextual influence on the individual learner" (p. 8) consisting of individual, pedagogical, institutional, societal and cultural influences. Bishop then raised the challenge that

> our research practices are no longer adequate to meet the challenges of today. In particular, it is now clear that we have to begin to examine critically our research practices and our value laden constructs if we are to make more progress in developing our social and cultural knowledge (p.7).

Contextual factors, Bishop argued, enter research in two ways: context in research study and context of research study. Bishop concluded by compiling a list of research questions as examples of questions that may be raised related to the study of the social and cultural context, in particular in the study of cultural conflicts. The concept of cultural conflict was also the basis of an address that Bishop (1994) presented at the SEACME conference in Indonesia. In this paper Bishop argued that conflict between the culture of the learner and the dominant society can be experienced at the classroom level, at school level, and at societal level.

Often studies which are concerned with the macro context of mathematics education deal with relationships of power. The discussion of power most frequently is left implicit, but sometimes is made explicit. This was particularly the case with Begg (1995a). In contrast to the theoretical positions of Barton and Bishop where power is not addressed explicitly, Begg, in considering his role in numerous aid projects, raised questions about the relationships of power when international enterprises collaborate on mathematics education projects. He challenged mathematics educators to consider seriously whose interests are being served by many international projects, particularly when the relationships of power are skewed in favour of the colonising country. His concerns about cultural conflict were evident in his discussion of the language of instruction and in particular whether it is in the long-term interests of the country if the Indigenous language is not the language of instruction. He proposed that "when assisting a developing country improve its mathematics education it may be that these processes [problem solving, modelling, communicating, giving a human face to the subject] provide a better starting point than the traditional content topics" (p. 101).

Crosscultural Comparisons

Three studies reported in MERGA conferences dealt with comparison in achievement and curriculum between Australasian contexts and overseas countries. McIntosh, Bana, and Farrell (1995) argued that in spite of the emphasis on mental computation in many curricula around the world, it was not often assessed by standardised achievement tests and has not been addressed in other international studies. The study aimed at exploring the performance on students in three countries, Australia, Japan, and the United States of America. A group of 755 Japanese, 522 USA and 641 Australian students were selected from Grades 2/3, 4/5 and 7/6 to be of similar ages. Three tests were used: a Preference Survey to measure which calculations students prefer to do mentally, a Mental Computation Test and an Attitude Survey towards mental and written computations. While Japanese students were found to outperform other students on the Mental Computation Test, they also exhibited the highest variation. Further, this superiority tended to decline with age, with the Australian sample outperforming the Japanese sample at the upper grades. This finding lead the authors to conclude that early emphasis on mental computation is not necessarily beneficial to students.

Bell and Ok-Ki (1994) reported on a large sample study of performance on mathematical tasks selected according to the requirements of the New South Wales primary curriculum and grouped by their content of shape, measurement, number, and problem solving. The Korean sample of 1671 was drawn from unstreamed classes while the Australian sample of 1211 consisted of entries from Grade 6 in the Australian Primary School Mathematics Competition (August, 1990). While Australian females showed superior results to males, the reverse was true in the Korean population. This was attributed to the "strong gender differentiation in Korean society" (Bell & Ok-Ki, 1994, p. 93). The Korean sample was superior on number, computation, shape, relations, problem solving, and total test. The Australian sample was superior on measurement. The study failed to provide a discussion of the implications of their findings or a strong rationale for this type of research. Lastly, another comparative study, between curriculum in mathematics education in several countries, was reported by Begg (1995b). The study compared the curriculum in mathematics education in New Zealand with five countries in the Netherlands and Scandinavia which were judged to be similar to New Zealand in terms of size, population, and political environments.

Mathematics Education in Aboriginal Contexts

In writing this section, we recognised that there is great diversity between the cultures of Indigenous Australians. Within the confines of this review we were unable to do justice to this issue, but recognised that it represents a significant concern for the research community. Our purpose was to report on those studies which have been undertaken.

While few large scale funded studies in the period 1992-1995 considered questions related to mathematics education with Aboriginal groups, there was a general recognition within the mathematics education research community of the importance of this topic. This is demonstrated in the chosen theme of the 1995 annual MERGA conference in Darwin: *Galtha*, a Yolngu word chosen by the Indigenous elders for the curriculum in schools at Northwest Arnhemland. Similarly, the keynote address at the Sydney 1992 MERGA conference, presented by Harris, addressed the concept of space in Aboriginal culture and education. The keynote address was in three sections. The first section considered the spatial skills developed in traditional communities. Harris commenced her address by comparing the skill developed by Captain Cook in his voyage to Australia that depended on technology and mathematical computations, and the equally reliable navigation skills by traditional Aboriginal people in the difficult Australian terrain. She referred to evidence showing that in controlled studies, Aboriginal people were accurate within 2.8 degrees, on average, when asked to point to distant places where these places were sacred sites, while they were up to 67% inaccurate when asked to point to random places. Also, early settlers often marvelled at the ability of Aboriginal people to locate water holes in the hostile environment. In a second section of the talk, Harris identified components of a space curriculum developed in a central Australian community where the traditional classification of objects were used as a bridge between the way Aboriginal societies classified objects and the way Western geometry classified objects. A third section of the talk considered spatial representation in Aboriginal art.

Probably the most significant contribution to research on Aboriginal issues is the work of Howard. An ethnographic study (Howard, 1994c) involved 168 interviews based around three schools in Sydney and country New South Wales. The interviews were wide ranging and included some with Aboriginal and non-Aboriginal students, parents, teachers, and community groups. Interviews ranged between 15 to 60 minutes. A number of conference presentations resulted from this study. Howard (1994a, 1994b, 1995) considered methodological implications of working with Indigenous people in ethnographic framework. In a single case study, Howard (1994c) reported on the views on one Aboriginal female student. The case study highlighted the positive effect of the student developing a good relationship with a particular mathematics teacher, enhancing interest and confidence in mathematics that allowed her to achieve well in the subject. The student had internalised several myths about mathematics that dominated the culture of the school such as: (a) The way to learn mathematics is by listening carefully, and (b) failure in mathematics is the fault of the learner. In a paper co-presented at the World Indigenous People Conference (Frensh, Frensh, Matthews, Stephens, & Howard, 1994) by the people from the Gamilaroi and Wirajurri nations, the authors reported some views from Aboriginal students and teachers about school mathematics, mathematics and the community, language and mathematics and mathematics anxiety. At the outset of the paper, the authors presented an arguably controversial claim that "all students who come to school are very good at mathematics, that is their mathematics. The mathematics that they use everyday to survive in the playground, getting to and from school, in the games they play" (p. 10). School mathematics, highly valued in the classroom, is the mathematics of the dominant culture. It is often taught in ways that do not take into account the various learning styles of the students. It does not value the mathematics of Indigenous peoples, and is prescribed from outside the community in which the students live. Finally, Howard (1995b) presented a theoretical discussion of various issues related to mathematics education and Indigenous people. The paper called for a mathematics curriculum that is culturally sensitive, that acknowledges the varied types of mathematical knowledge used by different cultural groups, and that starts from this mathematics in establishing the mathematics that is needed by today's technological western society. The paper concluded by calling for closer co-operation between the mathematics research community and the Indigenous people themselves to investigate possible forms of such a curriculum. Howard recommended the establishment of a national Centre for Indigenous Learning and Mathematics.

The question of what pedagogy is applicable to Aboriginal students was considered in three other studies. McArthur (1995) reported on an action research project toward the achievement of a culturally inclusive curriculum in the junior secondary school. The project involved parents and students working in collaboration with the school teachers. Richards (1994) reported on a case study of a small group of urban Aboriginal students in a "transition" classroom. The study concluded that teachers who were sensitive to alternative behaviour enhanced the establishment of good relationships and brought a high level of involvement from the Aboriginal students. Knight, Hurley, and Flavel (1994) reported on a special program to assist students in Grades 8 to 10 at an urban high school. As students progressed in the program they were able to join the mainstream classes in mathematics. The program used appropriate teaching strategies for the students and provided an atmosphere where the students cultural heritage was treated with respect.

Finally, Kepert (1993) reflected on some of the lessons he learned in his work as a Primary Mathematics Project officer in the Northern Territory involving professional development of teachers of Aboriginal students. While this contribution did not represent research in the traditional sense, it presented useful insights on characteristics of programs that may be successful with Aboriginal students. Kepert argued for the importance of teachers' knowledge about how Aboriginal students learn in the traditional communities where the role of the elder/teacher is seen as a conveyor of knowledge gained by direct experience. The ability of the teacher to understand the culture of the students and their ability to motivate students by building on this knowledge was seen as essential for successful teaching and learning of mathematics. The author demonstrated several areas in western mathematics that depend on abstractions which have no equivalence in the world view of the Aboriginal communities. He rejected the solution that the students' problems can simply be achieved by deciding which language best fits the instructional tasks at hand; teachers should also take into account cultural differences. Kepert identified some aspects of mathematical knowledge that are strongly represented in the Aboriginal culture. These included a complex set of relations and spatial proficiency. These, he argued, should be the starting point for many school mathematics activities. Further, other western concepts such as fractions and percentages could be more meaningfully developed by using contexts from the traditional culture. The author concluded that

> some Aboriginal students have a cultural background that will not immediately allow them to access the mathematics syllabus taught in school. Through these activities I was trying to give their teachers an insight into the two cultural systems - one from mainstream Western culture and one from the Aboriginal

> child's world. These insights may lead to a more appropriate mathematics programs and particularly one that begins with an understanding of the students' world. (p.282)

Interactions at a Distance

Evaluating the effectiveness of a program to deliver mathematics teacher education units at a distance to Aboriginal and Torres Strait Island students living in remote communities, Putt, Henderson, and Stillman (1995) evaluated the units covering mathematics and mathematics education for primary schools. These units were delivered to the remote communities by a combination of texts, workbooks, readings, videos, and computer courseware. The material was developed to meet three contexts: (a) to be in harmony with the culture, knowledge, and preferred ways of learning of Aboriginals and Torres Strait Islanders; (b) to satisfy the requirements of university mathematics and mathematics education units for primary teacher education; and (c) to be capable of transmission through a computer environment. The computer environment was sufficiently flexible to allow text, visuals, animation, use of mouse ("click and drag"), and multiple-choice tests.

Putt, Henderson, and Stillman reported that Aboriginal and Torres Strait Island student responses to a survey indicated the students learnt best from the computer, preferred to work alone when studying material for the first time and when revising, and preferred "click and drag" and typing answers to true/false and yes/no tests. They argued that the students' reasons were related to the cultural learning methodology of the Aboriginal and Torres Strait Island students and reported that the students commented on their need for privacy, to avoid the shame of incorrect answers, to have control over the pacing of lessons, and to be able to test their own thinking. They also reported mixed success with attempts to have group mathematics problem-solving activities videotaped for return to the lecturers. The tapes were effective in allowing the lecturers to observe overt thinking actions, but taping appeared to confuse and stress the students. However, Putt, Henderson, and Stillman argued that students' thinking was still most challenged by having to prepare and implement lessons in actual classrooms. They indicated that further research was being completed to map how students actually negotiate the distance material and to investigate the role of the local tutor.

Social Background

The social background of the student has been identified as a factor in the mathematical meaning-making process. Few Australasian studies have directly addressed the social backgrounds of students in relation to their impact on learning mathematics. Two Australian doctoral theses have investigated this phenomenon. While these studies have been discussed in other sections of this chapter, they are noteworthy for their explicit recognition of the social background variable. Peard (1994) investigated the relationship between students background knowledge and experiences with gambling and their probabilistic understandings. Peard's work strongly resonates with the ethnomathematics literature and is dealt with in greater detail in that chapter. In contrast, Zevenbergen (1995) examined the practices and broader social contexts of mathematics classrooms in relation to the construction of social difference. The different approaches of these authors contributed to the growing recognition of the impact of social context in the construction of mathematical meaning.

Peard (1995) found that the students who were familiar with track betting were more familiar with probabilistic concepts than peers from non-gambling backgrounds. He also documented the more extensive vocabulary of students who had a familiarity with betting. He raised the concern that the curriculum needs to consider seriously the linkage between the personal non-school world of students with that of the formal school curriculum.

Using Sinclair and Coulthard's IRE (Initiation, Response, Evaluation) model of classroom interaction, Zevenbergen's (1995) analysis of classroom interaction demonstrated how the social background of students either hinders or enhances their potential to participate in classroom dialogue. The unspoken rules for participation in classroom interaction were learnt implicitly so that students who were familiar with such patterns of interaction had an advantage over those for whom such patterns were unfamiliar or incongruent with their out-of-school experiences. Using transcripts from Grades 1 and 5, Zevenbergen argued that the interactional styles of working-class students were incongruent with that of the formal classroom. Whether this was a display of resistance of working-class students, or a display of cultural incongruence was difficult to determine. However, the outcomes indicated that middle-class students, because of their familiarity with classroom interactional patterns are able to gain access to mathematical knowledge which was effectively denied to working-class students.

Language as a Social Factor

Most of the authors in this area acknowledged the work of Cummins and argued that there is a need to recognise the linguistic competence in the first language as a factor in the competence in the second language. Students whose literacy levels in both languages were low would be cognitively disadvantaged whereas those whose competency levels were high in both languages will have a cognitive advantage.

Language background of students has been the topic of attention of some studies in the recent years. Clarkson's (1992) study was concerned with issues of bilingualism and mathematical performance. His work was conducted in Papua New Guinea where he considered the interaction of language and mathematics. He controlled for variables including cognitive development and home background which included father's occupation, quality of housing, parental education, students' expectation of life after leaving school, and parental encouragement In one of the reports emanating from his study he concluded that "bilingualism should not be treated as a single unidimensional entity in research studies, and hence, care should be exercised if global statements are to be made concerning bilingual students" (Clarkson & Galbraith, 1992, p. 42).

Once the effects of covariance were removed, Clarkson found that bilingual students competent in both languages gained higher scores than other groups of students. However, these differences were not significant. Further, he argued that it is important to consider the distinct resourcing advantage of monolingual students attending International schools. When this variable was treated as a factor, then the performance of bilingual students could be considered significant (Clarkson, 1992).

Dealing with the issue related to overseas students studying in Australia, Thomas (1995a, 1995b) warned of the danger of stereotyping all Asian-Australians as being good at mathematics and science. She argued that like all social groups, these students also needed explicit instruction in the hidden curriculum of schools; that is, the values which are imparted from the home. She also commented that there is a sense of obligation that Australian universities do not accept overseas students unless the students are ready for studying in this country; when they are accepted, necessary support must be provided to support their understandings of the culture, the expectations of the university, and language demands placed upon them. Thomas also alerted us to the need for teachers to be able to recognise the difficulties of reading mathematics. She argued that secondary mathematics teachers do not have any training in the identification of reading difficulties faced by students.

Support for Thomas' and Clarkson's positions was found in the research conducted by Leder, Rowley, and Brew (1995) who studied the achievement of senior students' performance on the Victorian Certificate of Education (VCE) Common Assessment Tasks (CATs). CATs 1 and 2, which have a strong language component, were of particular interest. Overall scores did not vary greatly across the four forms of assessment which included an investigative report, a challenging problem, a multiple-choice facts and skills tasks, and a short answer analysis task. They commented that "One can only conclude that ESL [English as a Second Language] students perform better across the range of studies and CATs, and that no clear advantage or disadvantage can be attributed to the introduction of modes of assessment that CATs 1 and 2 represent" (p. 431). (See Barnes and Horne this volume for more details on VCE assessment, especially in relation to gender issues.) However, what was of importance was that the standard deviation was higher for ESL than non-ESL students indicating that while there were high achievement scores for the ESL students, there were also a substantial number of ESL students who did not perform well. This finding supported claims made by Thomas (1995a) and by Clarkson and Galbraith (1992) that it is not advisable to treat ESL students as a homogenous group.

Bishop and Larkin (1994) reported that the support that secondary mathematics (and science) teachers in metropolitan Melbourne were likely to receive from ESL staff was one of support rather than collaboration. They concluded from their questionnaire study, that the support and the institutional constraints operating within the ESL field almost determined that the program would have "little or no effect on the way the subject teacher teaches the class or assesses student progress" (p. 103).

Those staff who had received some ESL inservicing were clearly in the minority, so that a majority of staff in the areas of mathematics and science education are not aware of the special issues and needs, nor the appropriate teaching strategies for NESB [Non-English Speaking Background] students. Where changes have been implemented, they tended to be at the level of individual teachers rather that at the whole faculty level. They concluded that more professional development was needed for subject teachers if the access and success of NESB in mathematics is to be improved.

Galligan (1993) reported on a preliminary study of the difficulties that some pre-tertiary students from non-English speaking overseas countries face in expressing their mathematical knowledge in English. The author argued that even though mathematical language reduced the context of the problem, hence making it easier to handle in a second language than other knowledge, some words and phrases may cause some difficulty. Slight changes in wording of mathematics may help avoid many difficulties students face.

Finally, Clarkson and Thomas (1993) reviewed research and policy documents from Australia and overseas on learning mathematics in multilingual contexts. The authors concluded that students' learning may be enhanced in multilingual programs. Even in cases that this is not practical, the use of the child's first language should be encouraged. However, the authors also called for emphasis on development of English proficiency and for increased emphasis on research on practice and theory of teaching students from non-English speaking backgrounds.

It is probably safe to conclude that this area of research has not attracted a significant amount of study even though there is an increasing awareness in Australia of the multicultural and multi-ethnic constituents of the Australian population.

Micro Context—Schools, Teachers, Classrooms

The micro context is more concerned with the smaller contexts such as schools, classrooms, and individual lessons. Such studies examine these smaller contexts as microcosms of the wider social and cultural milieux.

Undertaking an ethnographic study, Zevenbergen (1995) examined the practices in three schools for their contribution to the construction of social difference. This study was a critical analysis of the context of learning in and through which students became constructed as learners of mathematics. She found that there were substantially different practices at the three sites which could be seen to contribute to the subject positions open to the students. Considering the broader context of schools and learning, Zevenbergen (1994c, 1995) argued that schools conveyed an ethos which would be more empowering for some students than others. For the students at the independent, elite middle-class school, the ethos was projected through rituals, such as school assemblies, and mathematics lessons which were heavily imbued with discourses of competition, meritocracy, and middle-class cultural values. In contrast, the same rituals at a working-class, government school were imbued with discourses of collaboration, social values, and good citizenship (Zevenbergen, 1994c). Such practices can be seen to contribute to students' sense of self, mathematics, and world views which impact upon their later views of the world and success.

At the level of the classroom, Zevenbergen (1993a) argued that the practices of classroom interaction offered subject positions which were gendered, and less empowering for girls. The interactions in a Grade 1 classroom were deconstructed and it was shown that the subtleties in language were gendered. The teacher offered subject positions to students in which boys were perceived as "clever" while girls who undertook the same task and produced similar, if not the same, responses, were perceived as "trying hard." Further, Zevenbergen (1992) found that teachers often interacted with students in ways which reaffirmed their perceptions of individual students. She argued that the practices both at the classroom interaction level as well as at the school level contributed to the construction of mathematical identity. When students were perceived as being talented, the teacher interacted with them in ways that were likely to produce the expected outcome. When the school later "streamed" students based on subjective assessments, students would be reified according to perceptions, both the teacher's and their own. Such practices can be seen to contribute to the differences which were manifested in later years of schooling. Rather than the observed differences, it was argued, being biological or psychological they were socially constructed and often socially regulating.

The practices at school also regulated student progress. Deconstructing school organisational practices revealed that differences in resource allocation, timetabling, relationships with parents and with the past, and staffing arrangements all impacted on the mathematical practices of the school and classroom. These impacted upon the learning outcomes, particularly when the outcomes were considered against externally driven assessment practices. Zevenbergen (1995) found that the middle-class schools were more likely to align their mathematics curriculum and teaching/learning and assessment practices with those of the conservative assessment practices of Year 12. In contrast, the working-class school adopted a more constructivist approach to learning mathematics whereby students were actively involved in learning for understanding. To this end, this latter group of students would be less likely to adjust readily to the formal, independent, and solitary mathematics learning that they would experience in their secondary mathematics. Accordingly, these students would be likely to experience greater difficulty in the transition from primary to secondary school cultures, therefore further contributing to their disadvantage. Zevenbergen (1994a) argued that the practices of mathematics serve as a social filter which regulate the population in a way which reinforces the hegemony of the status quo.

Classroom Interaction and Teacher Characteristics

This section focusses on research on interactions in mathematics classrooms and how these might explain classroom performance. It includes studies of whole classrooms and of groups within classrooms, where interactions were observed directly, and studies of teacher-student interactions.

As part of their ongoing study of the social context of secondary classrooms, Atweh, Christensen, and Cooper (1992) developed an instrument with which to record classroom interaction, the Structured Classrooms Interaction Schedule (SCIS). The instrument was used to observe Year 9 classrooms in four private schools representing different socioeconomic class and gender: low socioeconomic girls, low socioeconomic boys, high socioeconomic girls, and high socioeconomic boys. Each class was using the same text to complete the same topic in algebra. All lessons on this topic were observed using the SCIS instrument, field notes were also taken of the classes, and the teachers, mathematics coordinators and principals of each school were interviewed as were a selection of students. The results of the study (Cooper, Atweh, Baturo, & Smith, 1993) described and explained differences in delivery of the mathematics curriculum that occurred due to gender and social class. The data gathered by SCIS, field notes and interviews was analysed in terms of the communication exhibited between teachers and students.

The authors detailed the differences between schools in terms of background of students, curriculum, tertiary aspirations of students, and mathematical ability of teachers and students, and then described the differences in how the four teachers interpreted the text or taught the topic to their students in terms of approach, emphasis, expectations, achievement, and communication. They argued that their data showed that: in the low socioeconomic girls school, mathematics learning was a meaningless ritual or charade with the work being covered in a perfunctory manner with little expectation of understanding; in the low socioeconomic boys school, mathematics learning was for a select few with correctness of answers being used to determine who would go on to the harder examples; in the high socioeconomic girls school, mathematics learning was a democratic activity where students were free to choose from amongst a variety of alternatives; and, in the high socioeconomic boys school, mathematics learning was a sign of their membership to the elite and something they would use in later life and for which they needed to be self-reliant. They argued that expectation of success and focus on personal development was gender based while dependence and independence of learning was social-class based. Overall, the teachers in the four classrooms presented different views of mathematics, based on gender and socioeconomic status, in their communications with their students.

Atweh and Cooper (1995) reported in more detail on the classrooms from the two girls schools in terms of the construction of gender and the effect of social class on girls and mathematics. They discussed the two classrooms in terms of the construction of mathematics and the construction of the learner. They argued that the low socioeconomic school girls had come to see mathematics as "bits and pieces," as something which is learnt by following worked examples and being corrected by the teacher, and that they had constructed learning as "alternative needs" to take account of the idea that there were more important concerns than studying mathematics; while the high socioeconomic school girls had constructed mathematics as a collection of strategies and a system of generalisations and justifications, and had constructed learning as "balancing needs," to include a great concern for understanding. Atweh and Cooper concluded that mathematics teaching is a gendered practice with social-class related differences. In summary, they argued that the low socioeconomic school girls' perceived need was for "useful" mathematics for "everyday life," while the high socioeconomic school girls' need for mathematics was for university entry.

Outlining a theoretical framework for understanding and promoting collaborative problem solving, dialogue and metacognitive activity, Goos, Galbraith, and Renshaw (1994) suggested that the classroom be considered as a "community of practice." They argued that to create such a community three interrelated contexts needed to be considered: teacher-student interactions (which they proposed as involving scaffolding); student-student interactions (which they proposed as involving collaboration); and individual reflection. They expanded and discussed each of these contexts and presented a model of a classroom community of mathematical practice which interrelated teacher-student scaffolding, student-student collaboration, and individual reflection with metacognition.

Student-Student Interactions

Undertaking a study of students talking about mathematics in a third year tertiary chaos theory course at LaTrobe University, Davis, Jones, and Pearn (1994) raised three issues. The first was that although students may be using a common language, they may have different schemas. They presented an example of two students discussing intervals and iterations and showed how they had different conceptions of iterations of intervals. The second issue they raised was that "teacher telling" can prevent learning. They presented a discussion between a teacher and a student that, although well intentioned, appeared not to help the student. The third issue they raised was that definitions are crucial. They presented a dialogue between a male and two females where the male had no definition of "sensitive dependence" and the females could not articulate their definitions in a meaningful way for the male. They concluded that the issue of verbal communication and shared meaning is paramount in students constructing their own meaning and requires attention for effective teaching. They raised the question, "To what extent are teachers' and students' communications lonely voices in the dark?"

Higgins (1994) used protocols from a group of four children playing a game to highlight key factors of effective small group instruction. Higgins observed a teacher introducing a game where children have to place cards appropriately to form relations between the objects on the cards, and she recorded the discussions of four children as they played the game. This observation was one in a large study which videotaped 113 half-hour mathematics lessons from eight Year 5 and 6 classrooms. She concluded that the game was effective as small group instruction and argued that this was because it contained the following key factors: (a) Discussion was generated, (b) children were encouraged to think about the mathematical idea embodied in the task, (c) all children were given opportunities to participate in multiple subtasks, (d) the children wanted to help each other, (e) and the children mutually established the nature of the groups' interactions.

Group Interactions

As Stacey (1992, p.261) stated, "cooperative group work is an increasingly popular strategy for teaching mathematics." This section considers studies that investigated how groups interact. Stacey studied video-taped protocols of seven single-sex groups of three students (from Years 7 to 9) as they attempted to solve three problems, in an attempt to understand why students in groups are not more successful in solving problems than students on their own. The three problems had the same structure and required the groups to find elements of sequences for which there is a constant difference between consecutive terms, but a different-sized starting number. The students were directed to find the 10th or 20th terms and then the 100th or 1000th term. Stacey had previously identified the common error for this type of problem, which she called the "whole-object" method, where students believe that the 20th term is the 5th term multiplied by 4.

Before, videotaping the seven groups of students, Stacey had tested eight classes of Year 9 students at two schools with a 45 minute written mathematical problem solving test. Four classes took the test individually and four in groups of three. She reported that the performance of the groups at both schools were slightly worse than the performance of the individuals and this was not due to the groups being slower through having to discuss the problems.

Stacey reported that the two Year 9 groups answered the problems correctly but the other groups made the "whole-object" error most of the time. She found from the protocols that the groups had plenty of ideas and both the correct and incorrect solution methods were raised in all groups. However, she found that the groups (other than Year 9) did not choose well the method to follow. She argued that this was because the incorrect method was simpler arithmetically than the correct method and easier to express, understand, and calculate. She found that the groups did little effective checking and tended to pass over correct ideas, that there was little time for students with doubts to formulate and articulate them, and that queries were dispelled by repeating the incorrect calculation. She argued that easy accessibility appeared to be the prime reason for selecting a method and that her findings supported warnings against too great an emphasis on group problem solving and had disturbing implications for the development of proof.

Gooding and Stacey (1993a) studied the characteristics of small group discussion in reducing misconceptions related to division. A composite class of 28 Year 5 and 6 children were placed into groups of four and video-taped as they completed division activities which focussed on the following four interrelated conceptions about division: (a) Division is not commutative, (b) division of a smaller by a larger number is possible, (c) dividing by a number between 0 and 1 gives a larger number, and (e) the two notations for division work in opposite directions. All members of the class were given a pre-test and a posttest in Week 1 and Week 3, respectively, before and after the group work.

Gooding and Stacey reported that five of the groups improved their division knowledge and two groups did not. They denoted the improved groups as "effective." They reported that members of the effective groups interacted more and helped each other more by responding and explaining to a greater extent. They argued that there was also a different pattern in the interaction between effective and ineffective groups. They reported that students in effective groups: talked with more mathematical content; explicitly discussed division ideas; worked more together; proposed ideas, gave explanations and refocussed discussion more; and responded to questions more. They drew the implication that discussion was effective in overcoming misconceptions, but that teachers need to ensure discussion engages with the task.

In another report of the same study, Stacey and Gooding (1993b) provided more detailed information on individual children who improved or failed to improve their division conceptions. They related the students' progress to characteristics of the students. As a result, they argued that learning in the groups was facilitated by the following factors: adequate background knowledge, appropriate explanations, immediate use of newly corrected ideas, and sufficient sharing of ideas in order to not be excluded nor ignored. They drew the following two implications for group work: (a) Students with a wide range of background knowledge should not be put in the same groups, and (b) whole class discussion is useful in supporting group activity.

Gooding (1994) reported on a similar study, using the same methodology and instruments as Gooding and Stacey of four single sex groups of four taken from a class of 26 Year 6 children. Gooding found that the groups improved their division knowledge in relation to the remainder of the class. She focussed in the paper on one group of boys which made the least progress and gave detailed protocols of what each of the four children did. She found that the problem did not appear to lie in the grouping; it was an acceptable group of children who displayed adequate knowledge and good will. She argued that the reason for this group's poor progress lay in the wide differences in the group members methods of participation and learning. Quicker and more confident boys did the work for others. A boy gave the appearance of being an active member of the group when he was unable to participate through lack of knowledge and confidence, and consequently did not learn. Two other boys did likewise but were picked up in a revision exercise. She concluded that teachers have to be aware of the actual group-interaction processes of the groups they form to ensure that they are not fooled by the appearance of activity.

Cooper and Smith (1993) compared the effectiveness of two approaches to group problem solving: (a) non-assigned group activity, involving teacher intervention only in posing problems and follow-up discussions; and (b) role-assigned group activity involving direct teacher intervention through training in social interaction roles as well as posing problems and follow-up discussions. They studied 36 children from two Year 5 classes in a predominantly low socioeconomic status government primary school. The children were divided into 12 groups of three and assigned at random to the two treatments. The groups were pre- and post-interviewed while solving a puzzle and an open problem (no set goal). The groups undertook five one-hour problem solving sessions in the two treatments. The role-assigned groups were assigned, in rotation, the roles of leader/decision-maker, recorder/reporter, and checker. The interviews were videotaped as were selected groups during the treatments. As well, the treatment classes were observed and field notes taken. Due to the transient nature of the school population only nine groups (4 from the non-directed and 5 from the role-directed) were intact for pre- and post-interviews.

Cooper and Smith found that the role-directed groups displayed improvement in the following processes: planning; cooperatively acting, monitoring, modifying and evaluating, and showing increased metacognitive abilities. They also found that the role-directed groups showed stronger task commitment and more sustained collaboration than the non-directed groups. In fact, after four sessions, the non-directed groups had broken down and could no longer cooperatively act on problems. At the same time, the role-directed groups were able to continue to develop sophisticated solutions, particularly to open problems. Cooper and Smith hypothesised that different social interaction roles effected different metacognitive processes and proposed an analysis of group roles in terms of the metacognitive characteristics they enhance.

Reporting on the same study from the perspective of the communication between children, Cooper, Atweh, Baturo, and Smith (1993) noted the changes to each group from pre- to post-interview. The role-assigned groups showed the greater improvement in (a) problem solving, higher-intellectual functioning, and metacognitive processes, and (b) cooperative-behaviour persistence, group cohesion, and independent behaviour. The non-assigned groups were exhibited behaviour that was dysfunctional, acting as individuals, not persisting, interfering, loafing, and fighting. Only one role-assigned group failed to improve because one member of the group was too dominant. The reasons for the greater improvement of role-assigned groups, according to the authors, were that the social-interaction roles improved children's oral communication—the checkers tended to ask questions, the recorders/reporters tended to paraphrase, while the decision-makers tended to direct others. They concluded that the data appeared to indicate that groups should have communication frameworks that enhance two-way student-student interrogation, questioning, and posing of alternatives to be successful.

Teacher Factors

Mousley (1993) studied teacher beliefs and intentions and classroom interactions for classes undertaking the same activity in 11 classrooms ranging from Grades 1 to 9. The teachers were all volunteers who claimed to practise largely "activity-based" or "discovery" teaching. The common task the teachers were given for the students was to make a regular shape which holds one cup of birdseed from a given piece of cardboard and then to make a similar shape which is twice as big. The teachers were interviewed before the lessons on their planning and intentions. The lessons were observed and the teachers and groups of students videotaped. A few weeks later the teachers were again interviewed with video snippets used to assist recall and discussion.

In this study, Mousley reported a variety of different interactions in the classrooms based on differences in teacher beliefs and intentions. Some teachers interpreted the task narrowly and imposed limitations on student activity. Mousley discussed the classroom interactions of two such teachers. The first interpreted the task as focussing on the formula $V=L \times B \times H$ and changed the directions to making rectangular prisms (boxes). The second shaped the lesson so that it could be used to "prove" a rule she wished to explain. She directed the students to make cubes of 5cmx5cmx5cm and 10cmx10cmx10cm and to use the birdseed to compare volumes; she then used the finding that the volume had increased eightfold as an introduction to an explanation of why this is so. Other teachers saw the task as an opportunity to see what their students could do. Mousley described the classroom interactions of a teacher who left everything open and unexplained. Mousley reported how this lesson left the students free to negotiate their own meanings and facilitated cooperative learning. However, she also reported some problems with the open approach: slow progress of some groups, distractions, lack of focus on task, lack of commonality of learning, and lack of shared meaning of ideas. She concluded that the framework set up by the teachers framed both the text (language, gestures, etc.) and context of the lesson and these in turn shaped individual and group meanings.

Student Perceptions

Frid (1993) also interviewed 17 students enrolled in introductory mathematics courses at three tertiary institutions in Western Canada on their perceptions and experiences of the communication of mathematics in classrooms. The interviews involved the students in written and oral responses to a number of relatively open-ended and focussed questions relating to perceptions of mathematics, ways of determining correctness, attitudes towards mathematics, and their impressions of the role of symbols, technical, and everyday language in their mathematics learning. She described students' impressions of mathematics and learning as different from other subject areas in that they see mathematics as not real and relevant, requiring mastery of the use of mathematics language and symbols, and not amenable to personal knowledge. She also described students' impressions of the nature and role of language use in mathematics in terms of a lack of sense of meaning for symbols and a problematic role for everyday language. She contended that language is an essential component of mathematical meaning which should be facilitated by relating students' informal language to the formal language of mathematics through "social sharing." Frid (1994) investigated the relationship between students' classroom experience and the manner in which they construct mathematical meaning in two Year 5 classes from two government schools in Perth. Six lessons from each class were videotaped and, for each lesson, two different students, selected to represent the variety of children in the classes, were interviewed using the videotaped lesson as a stimulation to recall. Frid reported that, for the two classes studied, the students saw "negotiation of meaning" through conjecturing, criticising, explaining, testing and refining as the responsibility of the teacher. She reported that the students saw their role in negotiation as minimal and did not regard themselves or their peers as sources of knowledge. She discussed how the students expected the teacher to describe and explain procedures clearly which left no need for negotiation. She argued that the students saw their role as one of "ratification." Frid concluded by proposing directions for research that might uncover what she saw as crucial to understanding lessons, the intent and beliefs of the teacher.

Frid (1995) interviewed 40 students, 19 teachers and 4 administrators in two high schools to find out: (a) students' and teachers' beliefs concerning what mathematics is; (b) teachers' and students' beliefs on the intentions of mathematics study and why mathematics is in school programs; and (c) teachers' perceptions of student thinking about mathematics and how it compares with what students actually think. She reported the findings under the headings: What is mathematics, why is mathematics studied, social status of mathematics, unity of mathematics, career expectations, and mathematical interest or disinterest. Her conclusions were that students "understand" mathematics in context (by this she meant that students define mathematics in a school and in a personal sense and they do not separate mathematics from their personal or social contexts), incorporate wider cultural and community perspectives than educators focus on, and understand mathematics in terms of their career aspirations.

McDonough and Pavlou (1994) studied children's perceptions of effective learning environments in mathematics in Australia and Canada. In this, they administered an instrument developed by McDonough (1992), the Pupil Perceptions of Effective Learning Environments in Mathematics (PPELEM), to a random sample of 221 girls and 206 boys selected at random from 26 multi-age and 25 conventional classes from nine suburban primary schools in Victoria. The instrument gathered information on children's preferences for location, instruction type, adult role, tools, adult presence, and task type with respect to mathematics teaching.

McDonough and Pavlou described a range of findings with respect to Australian children and to the factors identified by the instrument. In summary, they found that all children preferred the classroom as the location for learning (although lower grades had a lesser preference for home), the multi-age children preferred group learning (with and without an adult) while the conventional class children preferred individual and whole class instruction, half the children perceived learning as not including an adult, the multi-age children preferred text more than the conventional children, female children preferred materials more than male children, and Year 2 children had a greater preference for exploratory activities than other grades.

Conclusion

Over the past four years since the previous review of research, it is noteworthy that there have been noticeable trends in the research in this area. Insofar as research into or with Indigenous people of either Australia or New Zealand, there were very few studies. The number of research studies on Indigenous Australian issues has remained constant, both in terms of the amount of research and the number of researchers who work in the area. Perhaps one of the more significant events in the past fours years was the inclusion of a workshop at the 1995 MERGA conference by a group of Yolngu women (Indigenous Australians from Arnhemland in Northern

Territory) who shared their work in reconceptualising the mathematics curriculum to incorporate their ways of knowing the world. Unfortunately, the research in this area is minimal and is still considered marginal. One of the significant issues is ownership; until researchers can reconcile the concerns that many Indigenous people have in relation to their cultural heritage and artifacts, this area may remain marginal. This is an area of considerable ethical importance, and perhaps it is timely for mainstream researchers to foster environments in which Indigenous people are given further opportunity and support to facilitate a sharing of their work within mainstream forums.

Second, the past four years has seen an increase in the theoretical issues which are integral to the area of sociology of education as well as to the field of mathematics education itself. In particular, the number of papers which develop critiques of the field and discourses within the field have made a substantive challenge to dominant views. Again, these papers may be considered marginal, but their increasing number suggests that there are growing concerns about mathematics education which must be addressed. As is often the case, for worthwhile change to occur, people must work in the margins to challenge the dominant views and it is perhaps this contribution that such papers make to the field.

Third, perhaps one of the more significant areas of growth in the past four years has been in the area of ethnomathematics. As an area of research, it has grown both within the Australasian context as well as internationally. To this end, this area has been allocated its own chapter within this publication.

Fourth, there has been a decline in the number of quantitative studies. While many factors may contribute to this trend, it is important to note the demise of such studies. Qualitative studies contribute substantially to indepth analyses of particular situations, but large scale trends do need to be monitored carefully. This trend does need to be monitored carefully so that future qualitative studies are not founded on false assumptions or outdated statistics. The dialectic between qualitative and quantitative studies is extremely important to ensure a balanced and informed research program.

Fifth, there is an increase in the number of studies of classroom interactions. The studies in this area have shown the intricacies of classroom dialogue and its role in the construction of mathematical meaning. Furthermore, such studies have extended the concept of "mathematical meaning" to encompass the individual construction of mathematical knowledge as well as the social construction of students as mathematicians. This conceptualisation extends the individual meaning making to encompass the social and cultural contexts within which meaning is constructed.

Last, is our redefining of context within a sociological framework. This conceptualisation represents a radical change from mainstream mathematics education which is dominated by behavioural science ideologies. Within such a conceptualisation, we have been able to recognise, value, and legitimise the integral role of the context in the construction of meaning. This means that many studies which considered factors which could be seen as contextual remained within the dominant psychological discourses and, as such, did not recognise the sociological domain of teaching and learning.

The contribution which the studies outlined in the chapter have made to the field of mathematics education reinforce Walkerdine's (1988) argument that the context and the individual are mutually constitutive. The studies which constitute this chapter have shown that the context, both social and cultural, plays an integral role in the construction of learners of mathematics, so much so that it is not possible to consider cognition without considering the context within which this individual is located. As noted in the Review of Research for the previous four year period, the questions arising from social theory have posed substantive challenges to the dominance of literature based within the behavioural sciences. This recognition represents a significant area for future directions in mathematics education research.

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Ethnomathematics

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Introduction

A summary of the research to date in this field poses some problems, not the least of which relates to the nature of the topic. Bishop (1988), who has done extensive work in the field, has said "the term ethnomathematics is not well defined" (p. 180). As a consequence, research in the field may be reported elsewhere in this volume in one of the overlapping regions of social and cultural aspects of mathematics education.

Reports of research in ethnomathematics appear in the fields of constructivism, intuitive mathematical knowledge, the history and pedagogy of mathematics (Booker, 1995) and in other content areas. For example, the extensive work of Bell (1993) is in the field of the linguistic and cultural aspects of mathematical learning in Australasia and South East Asia. The work of Lean (1993) who documented over 2000 counting systems of Papua New Guinea and Oceania is essentially anthropological in nature. The work of Peard (1995) into the ethnomathematics of gambling within the Australian social context is in the content area of probability.

Barton (1993) discussed the context within which mathematics educators have begun to talk about the cultural relativity of mathematics. He reported that ethnomathematics leads teachers to question *what* mathematics they are teaching. He noted that classical theories do not offer any help in seeing how there might be culturally different mathematics. "The result is that writers on ethnomathematics have developed a variety of conceptions as they try to reconcile cultural mathematics with the ... rational conception of mathematics" (p. 2).

Structure of the Review

In view of the fact that there are several meanings associated with the term ethnomathematics, the review starts with some definitions of the term. The Australasian scene is then examined from three perspectives. First, research into the ethnomathematics of some cultural practices of indigenous peoples in non-industrial cultures is reported. This is followed by a report of research into the ethnomathematical practices of some identifiable subgroups within some industrialised societies. Finally, research concerned with the influence of culture on mathematics education is considered before generalisations are drawn.

Definitions of Ethnomathematics

D'Ambrosio (1985) defined the term *ethnomathematics* as "the mathematics which is practised by identifiable cultural groups, such as national tribal societies, labour groups, children of a certain age bracket, professional classes and so on" (p. 45). He argued that its identity "depends largely on focuses of interest, on motivation, and on certain codes and jargons which do not belong to the realm of academic mathematics" (p. 45). Nunes (1992) observed that D'Ambrosio's use of the term ethnomathematics, referred to forms of mathematics that vary as a consequence of their being embedded in cultural activities whose purposes are other than the doing of the mathematics. This working definition is in keeping with that employed by other researchers in the field.

Borba (1990) claimed that the "ethnomathematics developed by different groups are likely to be more efficient at solving problems related to their cultures than academic mathematics" (p. 40). After arguing that ethnomathematics and traditional education are really compatible, Borba went on to say "ethnomathematics is developed by the cultural groups' interest . . . which is natural because it was generated by the members of the cultural group in response to their own situation" (p. 41). More recently, Borba (1992) reported ethnomathematics as "a field of knowledge intrinsically linked to a cultural group and its interest being in this way tightly linked to its reality ... and being expressed by a language, usually different from the one used by mathematicians" (p. 134).

Zepp (1989), in noting that the term is difficult to define precisely, suggested that it is useful to consider what the term is not. He suggested that:

> It is not a collection of interesting folk games, measuring techniques, or counting systems used by various "primitive" cultures ... nor is it a doctrine which states that differing races have differing mathematical abilities. Rather, the term refers to ... the ideology that mathematics education should be relevant to the goals and aspirations of students, whatever they may be. (p. 211)

Peard (1995) reported that the term *ethnomathematics* could be applied to that particular informal mathematics that is implicit in the everyday activities and ways of thinking of reasonably well-defined cultural or subcultural groups in secondary mathematics classes. These include children who assist parents frequently in activities involving calculations, classification, and measurement in small businesses. He maintained that the typical secondary mathematics class represents the coming together of a whole range of different kinds of ethnomathematics: that of sport, of music, of small business operation, of track gambling activities, and so on.

The Australasian Scene

In the present review the term *ethnomathematics* will be taken to mean the inherent mathematical ideas that arise naturally out of cultural practices and norms that may be identified among the variety of subcultures within Australasia, and the review will report the research that has been done in this field. Much overseas research in the field has focussed on what students know before instruction as a result of interaction with their social background in an endeavour to determine what understanding mathematics means to students who are being taught new aspects of mathematics (e.g., Carraher, 1989; D'Ambrosio, 1992; & Nunes, 1992). In general recent research in Australasia in the field has tended to follow this direction.

Australasian society should be viewed as a complex mixture of various subcultures with vastly different social values and life styles. Thus, in reporting the results of research into any ethnomathematical concepts it should be borne in mind that the conclusions will, of necessity, be confined to the cultural group or subgroup identified and defined according to focusses of interest, motivation, codes and jargons, and so on.

Ethnomathematics in Non-industrial Cultures

Research by Australian mathematics educators into ethnomathematics of non-industrial cultures includes studies of navigation, geometric concepts of space and craft work design, games of chance, and counting systems.

Barton (1992) examined the history of navigation systems, including those of Polynesians, and the mathematics used in their systems. His explanation of its development highlighted the intuitive nature of the mathematical knowledge used by native navigators. His paper provides a philosophical basis for ethnomathematics by explaining how it developed in navigation, how this development gave it legitimacy, and how it is integrated within other mathematical cultures.

Harris (1992) researched the ethnomathematics of Aboriginal Australians, focussing on the contexts in which mathematics is taught to Aboriginal students, and examining their concepts of space to determine how these differ from those of non-Aboriginal students.

Stillman and Finau Palaki (1995) examined the mathematics embedded in the design of Tongan craft work as a starting point for classroom mathematics. They observed that these cultural activities clearly had a purpose other than the doing of the mathematics. In fact the design process was not recognised for its mathematical nature but viewed as "women's work" in Tongan society.

Stillman (1995) used ethnomathematics as the window through which to look at games of chance played by the Igbo of Nigeria. She illustrated how teachers and students in the upper secondary school drew on ethnographical studies to enhance their teaching and understanding of the mathematics implicit in various games.

Bishop (1995) has discussed and paid tribute to the work of Glendon Lean who spent from 1968 to 1989 working at the Papua New Guinea University of Technology. Over that period Lean interviewed students and informants from all over the country, and collected and documented over 2000 different counting systems of the region. Bishop (1995a), in a tribute to Lean, whom he described as "a highly valued colleague who is sadly no longer with us" (p. 2), described the work as "a monumental one which not only contributes to humankind's store of mathematical knowledge, but which also demonstrated what anthropology can offer ... mathematics education" (p. 2).

Lean (1993) produced a doctoral thesis which Bishop (1995a) described as probably the most comprehensive study of counting in traditional societies ever undertaken and the most extensive database of Papua New Guinea and Oceania counting systems ever compiled. In doing so he challenged some of the commonly held views of these systems. These included, for example, the view that counting was invented under special circumstances in one of the ancient centres of civilisation from which it was diffused all over the world, and the myth that some primitive cultures have number words for only one, two, and many. He reported that he was unable to find a single instance of a language which has only the numerals 1 and 2 and which terminates precise enumeration at 2. Bishop points out that Lean's work could give us cause to reflect on some issues in ethnomathematics:

> Ethnomathematics research needs to be carried out *in* the communities concerned and with due reference to the languages of those communities. Ethnomathematics is situated in the linguistic and social context of the community and can only be abstracted from those contexts after careful analysis.
>
> Ethnomathematical research benefits from a regional perspective, because of the language and social relationships involved. Abstraction can benefit by being related to generalised situations which regional data can offer. (p. 9)

Ethnomathematics in Industrial Culture

It has been noted that a common element of projects in ethnomathematics and self-generated mathematics is the legitimisation of the learners' experiences and intuitive mathematical knowledge, leading us to re-examine the question "What is mathematics?" (Barton, 1993, p. 2). Research by Australasian mathematics educators into ethnomathematics in industrial cultures has tended to focus on this legitimisation.

Grier (1995a) explored a range of out-of-school practices that addressed the question "What is mathematics?" The participants of her study were adult TAFE students who were attempting to upgrade their mathematics. Through clinical interviews with these students she concluded that "many out of school mathematical practices appear to be permeated with number sense and operationalised through active problem solving" (p. 299). Grier (1995b) also examined the mathematics of adult basic education students' experiential worlds and their in- and out-of-school mathematical practices, focussing on activities such as measurement.

Balatti (1995) examined the ethnomathematics of financial planning in an advanced capitalist society in an area that impacts on the life worlds of its members - savings and investment. The research focussed on the literature produced by superannuation, insurance, and trust industries for the purposes of client communication, and offered an interpretation of the ethnomathematics of the financial planning industry.

Peard (1994) identified a group of Year 11 students for whom an interest in track gambling was a strong social influence. This common interest included betting on horse racing, dog racing, and trotting events both on and off track. He claimed that track gambling is an identifiable cultural practice within the Australian social context, and reported that an interest in track gambling is widespread among certain sections of the student population. The study compared the ways in which probabilistic ideas were processed by individuals who were familiar with the out-of-school gambling contexts and compared them with the responses of students who were totally unfamiliar with such contexts. The study examined the reasoning principles underlying both written and oral mathematics of probability, the cultural conditions in which this mathematics was practised and, in particular, whether these daily life procedures are restricted to application within that situation or whether they transfer to traditional classroom situations.

One of the major findings of his study was that there were noticeable differences between the two groups in their understandings of the concept of mathematical expectation and its relationship with the concept of mathematical fairness. He concluded that the gamblers constructed knowledge in the manner described by the constructivists, and went on to conjecture that it was familiarity with track betting, which involves gamblers in the computation of the returns from placing bets at various odds, which leads to the development of intuitive understandings of the concepts of mathematical expectation and expected return.

He claimed that the results have particular implication for curriculum development. The concept of mathematical expectation features prominently both in the new Queensland Senior Mathematics Syllabus and in the National Statement (Australian Education Council, 1991). In the past, the inclusion of these concepts has been confined to the more academic courses in senior secondary mathematics. The study showed that these relatively sophisticated mathematical concepts can be understood, at least in part, by a good proportion of lower achieving students and the findings provided strong support for the inclusion of these concepts in the mathematical education of all students, regardless of social background or prior school achievement in mathematics.

Interestingly, Peard (1994) reported that despite the fact that much research into gender differences in mathematical achievement has shown that males perform noticeably better than females on higher level formal probability tasks, there were no strong overall performance differences according to gender in his study. The equal involvement of female students in what is often viewed as a "male oriented" activity (track gambling) was noted.

The Influence of Culture on Mathematics Education

Bishop (1988) has noted that there has been a growing acceptance of mathematics as a social construct. He is adamant and unequivocal that mathematics is not a value free phenomenon and has argued persuasively that mathematics is a pan-human phenomenon. He has said that as "each cultural group generates its own language, religious beliefs, etc., so it seems that each cultural group is capable of generating its own mathematics" (Bishop, 1988, p.180).

Bishop (1994) has continued to report his research in cultural conflicts in mathematics education. In noting that this acceptance of a cultural basis for mathematics generates conflict, he posed the question "Is there one mathematics appearing in different manifestations and symbolisations, or are there different mathematics being practised which have certain similarities?" (p. 15).

Barton (1993) also has expressed the need to provide a basis from which it is possible to argue for different simultaneous views of mathematics. He claimed that classical philosophies of mathematics do not do this and that it is necessary to shift the focus of philosophical attention from the objects and logic of mathematics to its processes. This involves the identification of cultural aspects and the ethnomathematical components of the processes being considered.

The influence of language on the learning of mathematics within different cultures has been researched by Australian educators. Bell (1994) reported the results of a crosscultural study of mathematical thinking in which he examined some of the interactions in the learning of mathematics which flow from differences in several Asian cultures and their languages. Later, Bell (1995) highlighted the differential results in mathematical attainments between Australian and Korean Year 6 students and reported results in differential conceptual structures for some geometry and fraction terms.

Begg (1995a) compared the mathematics curricula of five industrial countries of similar population to New Zealand. He considered the effect of differing language and cultural backgrounds had on the development of the curriculum in each country. He concluded that "all the curricula were written with ideas of constructivism but this was not made explicit because of the desire by the writers to respect teacher choice and the concern by bureaucrats to avoid conflict" (p. 93). Although culture was often mentioned in making connections, he found that in all of the societies there was no evidence of contributions to the curriculum from people other than mathematicians, statisticians, or teachers and there were no situations in which the students' backgrounds were considered.

Begg (1995b) also focussed on collaborative research between countries of different economic status and the issues involved. He reported that when countries have different status such as is the case in many aid projects, the issues that need to be considered include the effects of language and culture on such issues as curriculum content, assessment, and teaching approaches.

Conclusion

Research in ethnomathematics in Australasia is a relatively new activity. However, there is a growing body of knowledge relating to the social and cultural aspects of mathematics education, the mathematics which is practised by identifiable cultural groups, the mathematics that people bring to the learning environment with them, self-generated mathematics, and intuitive mathematics developed outside school. The problem is how best to identify and take advantage of the intuitive, unconscious understandings brought into the learning environment by students from backgrounds that include a variety of ethnomathematical skills and concepts.

Mathematics education research is nothing more than an academic exercise unless mathematics educators take deliberate steps to make the results of such research known to those having teaching and curriculum development responsibilities for school mathematics in Australia. The challenge to mathematics educators presented by the research in ethnomathematics is to make these findings known to curriculum developers and teachers so that ultimately more students will be able to link their personal worlds with the school mathematics they are expected to learn.

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Gender and Mathematics

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Introduction

For over twenty years, educators in many parts of the world, including Australasia, have been concerned about gender inequities in mathematics education. This concern has led to extensive research, seeking to document any gender differences in mathematics learning, find explanations, and develop strategies for change. Australasian researchers have been involved in this research from the earliest days. Much of their work has been documented in previous reviews published by Mathematics Research Group of Australiasia (MERGA) (Leder, 1984; Barnes, 1988; Leder & Forgasz, 1992).

During the last four years, world-wide interest in the issue of gender and mathematics education has been high. the International Commission on Mathematical Instruction sponsored a conference on the topic, held in Sweden in 1993, several international journals have produced special issues devoted to research in this area, and a number of books of collected papers have been published. All of these contained contributions from Australasian researchers.

The recent interest in gender issues has stemmed not only from a desire among researchers to reflect on and consolidate earlier work and determine priorities for future research, but more importantly, from alternative conceptions of equity, changing methods of inquiry and emerging paradigms for research, many of them influenced by feminist theories.

In Australasia the number of students undertaking post-compulsory schooling and tertiary studies has increased as a result of demographic changes and education policies. Participation of girls in mathematics has also increased but is still less than that of boys. In some states girls now achieve better than the boys overall in final school examinations. This has led to considerable, often uninformed, media interest in issues of gender differences in education, and demands for special programs for boys. A quotation from an article in a national newspaper gives an indication of current thinking on gender issues among sections of the media and the general public:

> Male brains are generally stronger in spatial reasoning and have a higher aptitude for mathematical, scientific and logical reasoning. Female brains are verbally oriented . . . Do the boys, as a target group, need extra resources to get back on top? (Woodley, 1994)

This chapter begins with reviews of research and theoretical perspectives, continues with studies of participation and achievement and analyses of factors which may contribute to gender differences, and concludes with a discussion of the impact of government policies.

Reviews of Research

Leder (1992b) gave a comprehensive review of work on gender and mathematics around the world, beginning with a historical overview of the education of girls. She went on to discuss the evidence for gender differences in performance and participation, and to analyse possible explanations under the main headings Environmental Variables and Learner-related Variables.

Other recent reviews of research in this area include Forgasz (1994), which focussed on the Australian educational system and social context, and Tyrrell, Brown, Ellis, Fox, Kinley and Reilly (1994), an introduction to gender issues in mathematics for New Zealand teachers. Purser, Depree and Wily (1994) published a comprehensive bibliography of research on gender and mathematics in New Zealand. Some reviews placed gender and mathematics in a broader context. Boyd (1993) included mathematics in a review of research on gender differences in gifted children. Zevenbergen (1994a), analysed dominant discourses in mathematics education, discussing the construction of inequality along lines of gender, class and race. Yates (1993) gave an overview of research on gender-based inequality in education, paying particular attention to mathematics and science.

Theoretical Perspectives

Approaches to Research on Gender

A number of researchers have proposed theoretical frameworks for research on gender and mathematics. Johnston and Dunne (1996) suggested Habermas' framework of three knowledge-constitutive interests as a useful analytical tool. They identified strands of research located within each of Habermas' interests:

1. Research which seeks to document differences or looks for biological explanations falls within Habermas' *technical* interest. This is described as a need for control and prediction of the natural environment, linked to positivist philosophy.

2. Research which seeks social explanations for gender differences falls within the *practical* interest, in which knowledge is seen as socially constructed and verified by consensus. Such research includes studies of students' perceptions and attitudes, the effect of parent and teacher attitudes and practices and the influence of sex-stereotyping.

The social and the biological are conflated in the production of the "essential" In both of the strands above, characteristics seen as "feminine" (collaborative, dependent, compliant) are set in opposition to those seen as "masculine" (competitive, independent, aggressive) but the masculine is seen as the norm and valued.

3. *Emancipatory* research assumes a critical position, transforming gender research from descriptions based on "common sense" categories to considering the construction of these categories, asking how practices are implicated in the production and of gender difference, and raising questions about the interaction between gender and power in our society.

Other researchers have proposed broadly similar frameworks. Atweh and Cooper (1995) classified approaches according to the role played by the *gender* variable. Gender seen as a fixed independent variable leads to questions about the effect of gender on achievement, participation, attitudes and learning styles. Gender seen as an intervening variable leads to questions about how gender influences teacher-student interaction, the way mathematics is presented to students, or how the contexts of learning interact with gender to influence achievement and participation. Finally, gender when it is seen as socially constructed becomes a dependent variable. Mathematical knowledge and ability become problematic. Questions for research include: How is the learner positioned with respect to mathematics- active or passive, interested or uninterested, capable or weak? What do students perceive as the nature and value of mathematics, and how it is learned?

Mayo (1994) suggested that research on social disadvantage in mathematics education falls into four families of questions. It may investigate faults in the psychological make-up of students; look for problems in the social surroundings of the child; ask whether the syllabus is slanted towards the interests and experiences of privileged groups; or whether mathematics is used as a tool to limit access to power. The first appears to fall into the technical interest described by Johnston and Dunne, the second and third into the practical interest, and the fourth into the emancipatory interest. Mayo highlighted the fourth of these, suggesting that *blame the victim* arguments are too prevalent and that the concept of *ability* is problematic. Zevenbergen (1994b) saw problems with a strictly psychological approach which focussed on the individual and ignored the wider social and cultural context by which the individual is constructed. Carroll (1995) believes gender differences are inherent in the discourses of mathematics Critical of research based on global assumptions about females, she calls for a broader perspective, rethinking taken-for-granted categories.

Theorising Gender Reform

Willis (1996) presented a framework for thinking about the relationship of social justice to the curriculum, which provides a useful strategy to address gender issues. There are four perspectives:

Remedial: The curriculum - content, pedagogy and assessment- is a given. The problem of disadvantage lies with girls who, because of their gender, are less able to benefit from the curriculum. The solution is special assistance to girls to remedy deficits in knowledge and skills, change attitudes, and increase motivation.

Non-discriminatory: The content of the curriculum is a given. The problem lies with teaching and assessment practices related more to the interests and experiences of males than females. The solution is to change these to include the interests and experiences of girls; to provide a supportive learning environment and more valid assessment opportunities.

Inclusive: The curriculum is a selection from many possible curricula. The problem lies with content and sequence which reflect the values of the dominant male culture and match the typical developmental sequences of (middle-class white) boys. The solution is to rethink the 'typical child' for whom the curriculum is developed and to provide all children with curricula which value their own experiences, interests, and needs.

Socially critical: The curriculum is actively implicated in producing and reproducing gender inequality. The problem is with the way the mathematics learner is constructed through the curriculum, and the way mathematics is used to produce and support privilege. The solution is to challenge the hegemony of mathematics. Girls need to understand how they are positioned by mathematics, and how to use mathematics in the interests of social justice.

Willis emphasised that this is an oversimplification, but is still a useful framework for thinking about and critiquing strategies for addressing gender justice. It helps people to analyse their own practice, and can help to bring about changes in perspective. While the views of the curriculum and the sources of disadvantage are non-overlapping, strategies for dealing with them may be consistent with more than one perspective.

Each of these categories inspires particular types of research, from which are derived suggested solutions. Links can be seen between Willis' perspectives and the categories described by Johnston and Dunne (1996): the remedial perspective is related to the technical interest, the non-discriminatory and inclusive perspectives to the practical interest, and the socially critical perspective to the emancipatory interest.

Participation of Women and Girls in Mathematics

Secondary School Level

Participation rates of girls and boys in mathematics courses are monitored regularly by both systems and individuals, and provide a gauge of the effectiveness of intervention initiatives. Two groups (Ainley, Robinson, Harvey-Beavis, Elsworth, & Fleming, 1994; Malone, De Laeter, & Dekkers, 1994), studying enrolment trends across the country, reported an increase in the number of students staying on at school, the increase greater for females than for males. But Malone et al. noted a corresponding decrease in the proportion of the cohort choosing advanced mathematics, so that the overall pool of students studying it has remained fairly stable.

The gender effect is twofold. Firstly, boys study more mathematics than girls, and secondly, girls choosing mathematics are over-represented in lower level courses not recognised for university entrance. In general, the percentage of the cohort choosing the highest level mathematics has declined, but boys outnumber girls in these courses by almost two to one.

In Victoria, one of the aims of a new course introduced in 1991 was to address equity issues and reduce the filter effect of mathematics. Subsequent changes to that course have focused attention on gender differences in enrolment patterns and achievement levels (Vale, 1992; Cox, 1993, 1994, 1995a, 1995b; Rowley, Leder, & Brew, 1994). These changes did not increase the amount of mathematics taken by girls (Vale, 1992; Cox, 1995b).

Course structure and rules about subject choice affect enrolments. New junior secondary courses in Western Australia were introduced in 1988 with equity as an explicit goal (Johnston, Rennie, & Offer, 1993). Initially students could choose not only which mathematics units to study, but also how many. The effect was that girls chose less mathematics than boys, and there was a worrying decrease in the overall amount of mathematics studied. It was then recommended that all students study a minimum of 160 hours of mathematics per year. Although the average amount of mathematics chosen is still less than 160 hours, there is now no gender differential in any of the courses (Johnston, Rennie, & Offer, 1993). However, this gender balance has not carried over to Year 12, where the ratio of females to males was about 1:2 in both 1990 and 1993.

University Level

Although the percentage of women in tertiary courses has been increasing, their participation in mathematics is not growing as rapidly as in other areas (Gaffney, 1995). Around Australia by 1993 women made up about one third of the initial enrolments in mathematics majors (Cobbin, 1995). The proportion varied considerably from institution to institution, influenced by factors such as prerequisites and ethnic background. In New Zealand in 1992, female enrolments decreased between the first and third stages of an applied mathematics course, from 44% to 28% of students, and where there was a choice between a terminal and a continuing course, proportionately more women enrolled in the terminal course (Reilly, 1993).

While the number of women completing higher degrees in mathematics has been increasing steadily but slowly, these numbers are not reflected in academic appointments at lecturer level and above. Although more women now gain doctorates in mathematics, tertiary mathematics departments remain very male-dominated. In New Zealand in 1990 nearly 20% of doctoral graduates were women (and the numbers are increasing) but, although there are many well-qualified women, less than 10% of academic staff in mathematics were female (Thornley, 1995). In Australia there is a gender imbalance in academic staff appointments across all faculties. Less than 20% of senior lecturers and above are female (Cobbin, 1995).

Some Explanations

The important question is why girls opt out of mathematics when it is no longer compulsory. Higher-achieving students were more likely to choose higher enrolment in mathematics (Ainley et al., 1994) but fewer females than males with demonstrated high capacity for mathematics chose it (Sheret & Ainley, 1992). Perceptions of talent appear to affect choices more than actual talent and these perceptions are strongly influenced by gender (Watt & Bornholt, 1994).

To maintain perspective, it is important to note that factors other than gender, (language background, parental education, and socioeconomic status) are more highly correlated with enrolment. For example, the gender balance in subject choice for Aboriginal Australians is the same as that of white Australians, although overall enrolments are notably less (Ainley et al., 1994).

Relevance to future work discriminated most strongly between males and females as a reason for choice of courses (Ainley et al., 1994). Tertiary enrolment patterns show that females generally choose careers which require less mathematics. In seeking explanations for gender imbalance in subject choice, Johnston (1995) focussed on girls, rather than making comparisons with boys. Girls who chose mathematics because of career aspirations generally had well-developed, usually high, aspirations. Another reason given for subject choice was to keep career options open. A television advertising campaign to encourage girls into mathematics had an impact, but although girls chose more mathematics, they did not in fact choose the mathematics needed to keep career options open. Few of the girls thought mathematics was useful and practical.

Willis (1995b) pointed out that analyses by mathematics educators of the choices of high-achieving girls tend to be rather egocentric. The focus is restricted to whether or not they choose mathematics, "overlooking the real possibility that girls may more often have a wider range of choices and may have inspected the subject and found it wanting" (Willis, 1995b, p.269). The remedy may lie in improving the product rather than convincing the customer of its desirability.

Gender Differences in Mathematics Performance

Studies of performance in mathematics have been carried out at many levels from early childhood to university, but caution is needed in interpreting the results. The group studied may not be representative of the school population, or students' test performances may not be fair measures of the full range of their mathematical capabilities. This issue is taken up in more detail in the section on assessment later in this chapter. Overall, as we shall see, the pattern of gender differences in performance at primary, secondary, and tertiary levels conforms with that reported elsewhere (see Leder, 1992b). Similarities in the performance of females and males are much greater than the differences. Where differences are found, males tend to outperform females, but not consistently. Differences favouring males tend to be greater in more highly selected groups, but this is explained in part by the fact that males choose to study more mathematics than females do.

Early Childhood and Primary Level

There are few overall gender differences in mathematics performance below secondary school level. Differences have been found, however, in certain groups of students, or in specific skills.

Young-Loveridge (1992b) used clinical interviews to assess the number concepts of five-year-olds entering school, with follow-up assessments annually for four years. She found no overall gender differences, but significant differences in the rate of progress of children initially in the lower third of the distribution. By age nine the boys were achieving substantially better than the girls, and the poor performance of the girls was extremely stable over time. Young-Loveridge (1992b) saw these girls as at-risk within the school system unless steps were taken to help them. Boys benefited more than girls from intervention procedures devised to improve the number skills of five-year-olds (Young-Loveridge, 1993).

It has been suggested (see Leder, 1992b) that gender differences in mathematics may be related to differences in spatial skills, resulting from differential play activities. Peters (1992) investigated the spatial abilities and play activities of six four-year-old children. Those who scored better on the spatial tasks spent more time in play judged to be high in fostering spatial skills. Peters concluded that young girls possess a wide range of spatial skills. They play with equipment likely to develop these skills, even though such equipment is traditionally regarded as belonging to the male domain. She suggested that positive action is needed from pre-school teachers to ensure that all children benefit from the full range of opportunities offered.

Secondary Level

Some recent studies of gender differences in performance at secondary school level have focussed on specialised topics. Reported gender differences in the solution of algebraic word problems led Low and Over (1993) to carry out experiments to investigate how students process the information given in such problems. Boys outperformed girls in deciding whether a problem contained insufficient, sufficient, or irrelevant information, and in solving problems containing irrelevant information. Boys seem more likely to adopt a *top-down* strategy, beginning by identifying the problem structure, whereas girls may use a *bottom-up* strategy, trying first to relate all the given information.

Students' communication skills in mathematics and their use of mathematical vocabulary were the focus of another study which revealed gender differences (Malone & Miller, 1993). Either females performed better in explaining the meanings of mathematical terms using words, symbols, or diagrams, or there were no gender differences. The authors suggested that this result was fairly predictable, in view of research showing females' superior performance on tests of verbal ability.

Surveying recent changes in mathematics education in New Zealand, Clark (1992) reported a replication of the Year 8 section of the 1981 Second International Mathematics Study. Compared with the 1981 results, there was a slight improvement in scores, greater for girls than boys. There were no significant gender differences overall, or on any of the subtests. Clark concluded that, in the intervening years, intervention programs and a greater awareness of the gender issue had been effective in improving New Zealand girls' performance in mathematics.

Performance on high-stakes end-of-school examinations has attracted the attention of a number of researchers because of their influence on access to higher education and decisions about further study. In New Zealand, Blithe, Forbes, Clark, Robinson, and Whitwell (1994), Morton, Reilly, Robinson, and Forbes (1993) and Reilly, Morton, and Lee (1995) analysed participation and performance in two final-year mathematics examinations. In both, gender differences favouring males disappeared when other relevant variables, such as type of school and amount of mathematics studied, were taken into account (Morton et al., 1993). Gender differences in performance appear to be a consequence of more girls studying only one mathematics subject. A question-by-question analysis of one paper (Reilly et al., 1995) showed that males performed better on questions requiring more preliminary analysis, while females were more successful on those relying on acquired knowledge. The analysis was limited by the need to work with existing examination questions rather than purpose-designed items. Similar results held for the more highly selective scholarship examinations (Blithe et al., 1994). Gender differences in performance in university mathematics courses were few. Those observed favoured females, but the group of females is more highly selected as a result of lower participation at school.

Competitions

Mathematics competitions are another source of data about the performance of males and females in mathematics, but caution is needed in interpreting the data. Not all students choose to take part, so the sample is not random and probably not representative of the whole school population. Factors influencing the decision to take part could be related to gender. We also need to ask whether any gender differences resulted from the style of questions or bias in problem context.

Studies of the performance of competitors in the Australian Primary Schools Mathematics Competition (Leeson, 1995b), the Australian Mathematics Competition (AMC) (Atkins, Taylor, Leder & Pollard, 1994), and a problem solving competition for 11 and 12-year-old students in New Zealand (Curran, 1993) all found that boys performed better than girls of the same age. However, girls' performance relative to boys improved steadily over a period of ten years (Atkins et al., 1994). Girls who were unusually young for their grade nearly always outperformed boys in the same grade who were unusually old, illustrating once again the greater magnitude of within-gender differences compared with between-gender differences (Leder & Taylor, 1995a).

Leeson (1995b) identified topics in which boys performed markedly better (including rates, percentages, and estimation) and others on which girls performed better (number patterns, reflection of a shape in a mirror line, and identifying geometrical shapes within a larger figure). Leeson (1995a) sought to explain the gender differences in estimation of measurements, but found no significant gender differences in the strategies students used.

Bishop and Clements (1994) adopted a different approach to understanding gender differences in performance. They asked mathematics education researchers and experienced practising teachers to predict which questions in the Hunter Region Primary Mathematics Competition would produce significant gender differences, and in which direction. Neither group performed better on this task than could be achieved by random guesswork. The authors conclude that, "despite at least a decade of great interest in gender difference in performance and participation in school mathematics, mathematics education researchers and practising teachers have not made much headway in being able to predict whether given pencil-and-paper, multiple-choice items will yield gender-related performance differences" (Bishop & Clements, 1994, p.111). A further study (Clements & Bishop, 1995) showed that "experts" with extensive knowledge of the research literature, or long experience in constructing competition questions, could predict results with greater success. Given the continued popularity of mathematics competitions, in which boys gain most of the prizes, and the widespread use of multiple-choice tests in mathematics generally, the authors suggest that we need a better understanding of why the gender differences occur so that we may take steps to achieve change without compromising the quality or breadth of the curriculum.

Assessment Issues

The question of fairness arises in relation to any attempts to measure student achievement, including competitions. We have to ask whether any observed gender differences in performance reflect real differences in the mathematical understanding and skills of males and females, or whether they are artefacts of the method of measurement and the types of questions used.

The AMC has a multiple-choice format, to enable processing of the extremely large number of entries. Leder and Taylor (1995b) used data from this competition to investigate the role played by guessing in multiple-choice tests which have a penalty for an incorrect answer. Sensible guessing can increase scores in tests of this type. They found that most questions were read by more females than males, but answered by more males than females, and concluded that males were more confident than females about answering questions. Attempts to derive a measure of risk-taking behaviour ran into problems resulting from conflicting ways of defining risk-taking.

Concerns about gender-fair assessment motivated the Equity in Mathematics Education Group in New Zealand (Blithe, Clark & Forbes, 1993; Clark, 1992; Clark, 1993; Clark, Forbes & Blithe, 1994; Blithe & Clark, 1995; Blithe, Clark, & Forbes, 1995; Forbes, 1995a). Reviewing research from New Zealand and other countries, they note that females may be disadvantaged relative to males by multiple-choice tests and by restricted-time tests. Higher status is often accorded to topics such as applications of calculus, on which males are more successful, than to others such as statistics, on which females tend to do better. After a careful analysis of data from New Zealand secondary school and university examinations, the group concluded that females are disadvantaged by traditional timed examinations, and perform better if assessed internally by assignments or projects (Forbes, 1995a). In examinations with a choice of questions, males and females tend to make different choices, although all tend to answer questions in the order presented and to score better on questions answered early in the examination (Blithe et al., 1993).

Problem context may also be a source of female disadvantage. Girls are found to perform better on problems set in contexts which are familiar or of interest to them, for example people-oriented questions; and avoid, or take longer to complete, items set in less familiar "male" contexts (Clark, 1992). Males appear to be less affected by the context of a question. These observations, together with documented gender differences in a number of mathematics topics, suggest that much more attention needs to be paid to the order and choice of questions in traditional examinations. Blithe et al. (1993) expressed the view that "traditional [assessment] procedures are gender and culturally specific and in general reflect the world of the mathematics examiners (in New Zealand predominantly middle class pakeha men)" (p. 29) and concluded with a plea for a broader range of assessment procedures, to avoid disadvantaging any group. As internal assessment procedures test different skills from examinations, they believe that the use of examination results to moderate internal assessments cannot be justified. Clark et al. (1994) pointed out that "A reduction in gender differences in mathematics for one group in the population (Pakeha) does not necessarily lead to a reduction in another group (Maori)" (p. 177) and recommended further investigation of the needs of subgroups of girls, by both ethnic group and socioeconomic status.

A system-wide change in assessment practice took place in Victoria in 1991, with the introduction of the Victorian Certificate of Education (VCE) for students in Years 11 and 12. The VCE mathematics assessment incorporates a broader range of types of assessment than have traditionally been used in mathematics, including an Investigative Project and a Challenging Problem (Victorian Curriculum and Assessment Board, 1990). An important goal of the course designers was to encourage more equal participation in senior secondary mathematics by females and males (Vale, 1993). The project and the problem each require a written report of work carried out over an extended period of time, which is internally marked by the school and then subjected to a verification procedure.

Analysis of data for the first two years of the VCE showed that, in general, girls performed better on the alternative forms of assessment which required perseverance, ability to do independent work, and good research, communication, and problem solving skills. Males tended to do better on more traditional tests (Cox, 1993, 1995a; Leder, 1994; Vale, 1992, 1993). Leder (1994a) is careful to point out the substantial overlap in the scores, showing that within-group differences are considerably greater than between-group differences. Another feature of the VCE is specified work requirements which must be satisfactorily completed before a certificate is awarded. Cox (1993) observed that significantly more males than females failed to complete the work requirements. Studies of participation linked to achievement illustrate the importance of the amount of mathematics studied in influencing gender differences (Vale, 1993).

In 1993, the VCE Mathematics assessment was changed, reducing the workload for students. One of the assessment tasks on which girls were likely to outperform boys was eliminated. Cox (1994) noted "a general shift in favour of males" (p. 26) but pointed out that the differences were small and not of great educational significance.

Further changes in 1994 essentially retained the assessment style. The pattern of achievement changed, with males outscoring females only on one test in one of the three courses (Cox, 1995b). Overall, females did better in two of the three courses, with no significant difference in the third. More males than females failed to complete courses satisfactorily. It is important to note, however, that participation of females in VCE mathematics is less than that of males, especially in the most demanding course, where males outnumber females two to one. Cox pointed out that, while the VCE assessment does appear to have some advantages for girls, their apparent higher achievement may be inflated because the group of girls is more highly selected. Together with the New Zealand studies reported above, the VCE studies mount a strong case for the proposition that introducing a broader range of assessment practices would be a move towards gender equity.

Bannister (1993) reviewed gendered patterns of performance on tests and examinations across disciplines, and identified mode of assessment, subject choice, and girls' lack of confidence as explanatory factors. She noted girls' lower attainment on multiple-choice tests and tests requiring mathematical skills, and better performance on long-answer and essay questions. Rejecting psychoanalytic explanations such as differences in cognitive style, Bannister argued that one effect of gender difference research is to make the differences between males and females seem an essential part of their nature. She suggested that more attention should be given to studying how social relations and teaching and learning practices lead to the production of gendered attainment.

Zevenbergen (1995) argued that assessment is a political practice through which the marginalisation of many students is achieved and legitimated. It acts as a social filter:

> the distribution of students who do well at mathematics is skewed towards the dominant social groups so that students who are white middle-class males are more likely to succeed in mathematics than students who are not in this group. (Zevenbergen, 1995, p. 832)

Practices and discourses in mathematics education are instrumental in the construction of gender differences, and exposure to such practices is less empowering for females than for their male peers. Zevenbergen suggested that we need to rethink assessment practices, not only from a technical perspective, but at the level of ideology. This, she argued, can be done through "a critical examination of the beliefs and ideologies that structure our work and through reflecting and implementing changes based on those outcomes" (p. 836). More inclusive assessment must incorporate the social context of the student, taking into account differences in language, power relationships, cultural practices, and beliefs about school work.

The Effect of Cultural Background

Zevenbergen's observations above are particularly relevant for students belonging to racial or cultural groups which differ significantly from the mainstream Anglo-Celtic middle-class, and emphasise that it is important not to regard girls as a homogeneous group. Where there are differences in cultural background, social status, and in some cases language, differing patterns of participation and achievement may be expected. A number of studies have looked at the impact of cultural influences on gender differences in mathematics.

Kaeley (1995) reviewed studies of participation and performance in developed and developing countries, noting that poverty in developing countries may interact with cultural norms according lower status to women, so that families which can afford to send only one child to school invariably send a boy. Gender differences in performance in mathematics are more variable. Kaeley gave examples of developing countries, and ethnic minorities within developed countries, where women have more power and authority than men. They include West Indian communities and matrilineal communities in Papua New Guinea and some Pacific island nations. In these societies, either females outperform males in mathematics or there are no gender differences.

Higher achievement by girls than boys at primary school in the West Indies was the focus of a study of fifth grade Jamaican children (Mitchelmore & Clarke, 1993) which sought to investigate relationships between school achievement, gender, and health and nutritional factors. Variables showing significant gender differences in favour of girls included long-term nutritional status, blood lead levels, and ownership of exercise books. These were also the strongest predictors of achievement, and seemed to be caused by whatever influences cause gender differences. The authors speculated on reasons for these differences, suggesting that they may be related to gender differences in behaviour, leading to different energy requirements and personal hygiene.

Suthankar (1995) identified social and cultural issues in Papua New Guinea that disadvantage girls in mathematics education. Substantial gender differences in participation in education generally are attributed to cultural attitudes which see women as inferior. Expectations of docility inhibit girls' success in mathematics. Attempts, such as competitions, to encourage girls' participation did not take into account deeply instilled cultural values and failed completely.

Young-Loveridge (1993) found that Maori girls aged five to seven years were substantially ahead of Maori boys of the same ages at every assessment stage in her study. But Forbes (1995b) found that, among 13 year-old students, Maori girls performed less well than Maori boys and were more ambivalent than other students about the usefulness of mathematics. These differences increased throughout secondary schooling and were not explained solely by low socioeconomic status. Forbes concluded that "strategies to increase the participation and achievement of girls in mathematics in New Zealand are having a positive impact on girls of European descent, but are failing to meet the needs of Maori girls" (p. 115). She thinks Maori involvement and ownership of the intervention programs will be needed before any impact on the achievement and involvement of Maori girls can be expected. The apparent contradiction between the results of Forbes and Young-Loveridge may be a result of life experiences of Maori girls between the ages of seven and thirteen, or could be due to different types of test, methods of test administration, or sampling methods.

Psychological Studies of the Learner

One approach to explaining gender differences in participation and achievement in mathematics involves psychological studies of learner-related variables. These include investigations of attitudes towards and beliefs about mathematics, perceptions of achievement, and attributions for success and failure in mathematics.

Attitudes and Perceptions of Ability

Relich and his colleagues (Relich & Way 1992; Relich, Way, & Martin, 1994) have developed an instrument to measure the attitudes to mathematics of pre-service student teachers. Two main scales measure general attitude to teaching mathematics, and mathematics self-concept. Using this questionnaire with a large sample of student teachers, Way and Relich (1993) found that males displayed more positive attitudes and registered higher self-concept than females. While attitudes improved with years of study, self-concept appeared to be a stable construct, not easily changed.

Self-concept of mathematical ability has been identified as a major factor influencing participation in mathematics (Leder, 1992b). It is therefore useful to explore how and when perceptions of ability are formed. Boys in primary school (Forgasz, 1992a) and high school (Bornholt & Cooney, 1993) tend to over-estimate their levels of achievement and have higher expectations than girls. A further study (Bornholt, Goodnow, & Cooney, 1994) identified two sources of these gender differences: the cultural stereotype that males are more naturally talented in mathematics and girls in English, and a tendency among males to overestimate their performance on any specific task in comparison with females. In English, these two tend to cancel one another out, whereas in mathematics, they combine to produce a higher perception of general performance on the part of males. Expectations of achievement differ according to whether they refer to a specific task or events in general, with the latter more likely to be affected by cultural stereotypes. These results were partially supported by Fullarton (1993) who found that, although secondary school students performed at the same level on pencil-and-paper mathematics word problems, the males were more confident that their answers were correct.

Young-Loveridge (1992a, 1992b) interviewed nine-year-old New Zealand children, asking them to tell her about their feelings about mathematics, how good they thought they were at it, whether they thought it was important, and if so, why. Boys were more confident of their ability in mathematics than girls. Among the low-achieving group, the boys showed unwarranted optimism, many believing they were good at mathematics, while the girls showed unwarranted pessimism, expressing extremely negative and hopeless feelings about mathematics. All children thought mathematics was important, but boys and girls gave different reasons. Boys mentioned a much wider range of occupations in which mathematics would be used, while girls more often mentioned domestic activities, like shopping and child rearing.

In Leder's study (1992a), more boys than girls in Years 7 to 12 claimed that they would be willing to volunteer an answer to a question to which they did not know the answer. Also boys more than girls indicated that they preferred working on difficult problems. This suggests that boys are more inclined to take risks in mathematics. On the other hand, girls seem to be more prepared to take risks by guessing on multiple-choice questions, a result which appears to conflict with results reported earlier. This could be a consequence of teachers instructing girls in how to take multiple-choice tests, or the fact that this was a self-report measure rather than an observation of actual behaviour.

Attributions for Success and Failure

Previous research (reviewed by Forgasz, 1994) has found that both males and females believe effort is important for success. But when gender differences in attributions are found, males are more likely than females to attribute success to internal, stable causes such as ability, and to attribute failure to unstable causes such as (lack of) effort or luck. This is a functional pattern, tending to encourage confidence and persistence. Females are more likely to reverse this pattern, attributing success to effort or luck and failure to lack of ability. This can lead to feelings of helplessness and lack of perseverance in the face of difficulty.

Young-Loveridge (1992b), Glassey (1993), and Southwell and Khamis (1994) all investigated attributions, obtaining results which substantially confirmed the pattern described above. However, Young-Loveridge found that, among nine-year-olds, low-achieving boys exhibited the less functional pattern normally typical of girls. At this age, only the high-achieving boys showed the more functional pattern usually typical of males. The personality of the teacher emerged as important in the Southwell and Khamis study. Girls, more than boys, believed that good results depend on whether or not the teacher either likes them. When they get a wrong answer, females are more likely than males to start all over again, rather than use the work already done.

Learning Preferences

Owens, Nolan, and McKinnon (1992) found a generally consistent pattern across Australia, New Zealand, the USA and England of girls preferring cooperative learning and boys preferring competitive and individualised learning. A similar study of teachers' preferences (Owens, 1993) found that mathematics teachers were strongly inclined to the use of competition. Taken together, these results suggest a mismatch between girls' preferred ways of learning and their teachers' beliefs about effective teaching. It seems likely that this results in many girls finding a competitive culture in the mathematics classroom which is not to their taste.

Malone and Cavanagh (1995) investigated the contrasting cognitive preferences of girls and boys. Academically capable girls who chose alternative mathematics courses rather than the highest level courses had a preference for being feeling, caring, people-oriented, punctual, orderly, planned, and decisive, and they had a negative preference for being innovative, theory-and-solution-oriented, logical, flexible, spontaneous, and easy-going. This suggests that, to increase the number of girls in the highest level courses, the classroom climate and mathematical experiences need to be seen as more people-oriented and more orderly. This, however, is opposite to the boys' preferences for an easy-going, flexible, and spontaneous climate.

Autonomous Learning Behaviour

Forgasz (1992b, 1995c) investigated links between psychological variables and classroom climate and processes. The Autonomous Learning Behaviour (ALB) model, proposed by Fennema and Peterson (1985), postulates that ALBs (choosing to do high-level tasks, working independently on such tasks, and persisting in them) mediate the relationship between internal beliefs about mathematics, the influence of the teacher, and outcomes such as participation and achievement in mathematics. Forgasz showed that mathematics classrooms where the teacher is takes a personal interest in individuals, where investigative skills are emphasised, and where students believe that they are active participants, seem to foster more ALB-related beliefs in the class as a whole and in individual students. Gender differences appeared when the data were analysed separately for class groups of male and female students. For females, the only classroom variable linked to high scores on some of the ALB-related variables was the personal interest of the teacher. Forgasz speculated that the differences may be due to males having more interaction with the teacher than females, as has been observed elsewhere (see, e.g., Leder, 1990). Forgasz (1995c) concluded that

> perceptions of particular classroom environment factors are linked positively to individual students' ALB-related beliefs. These classroom variables are more consistent with contemporary notions of effective teaching and learning of mathematics and appear to have similar impact on individual males and females. ... Particular classroom learning environments ... may be more conducive in promoting the ALB-related beliefs of the cohort of males in the class than those of the females. (p. 236)

Learning Environments and Classroom Discourse

As Forgasz and Leder (1994) suggest, differences occur both in learner-related variables such as cognitive and affective factors, discussed above, and in environmental factors including aspects of both society and school. Within the classroom, the learner's interactions with mathematics, the teacher, and fellow students are critical to our understanding of the complexities of gender equity.

Interactions Within Small Groups

Much of the research on gender has taken place in secondary classrooms. More attention needs to be given to early childhood, when many aspects of gender are constructed. Higgins (1995) investigated gender dynamics in early primary school, seeking to identify processes leading to the development of gendered achievement by observing small groups of children engaged in independent mathematics activities. She noted ways in which classroom power structures shape the learning environment to the detriment of girls. Boys *own* certain equipment and the girls give in to their demands. Girls' access to power is indirect, through playing the role of teacher. Competitive games benefit boys, who are motivated by competition. Girls are more concerned with keeping to the rules and supporting all group members, and abandon a game if one member becomes upset. Higgins suggests that, to benefit girls, more cooperative activities are needed, and teachers need to restructure the learning situation to support the needs of girls as well as boys.

Yelland (1993, 1994a, 1994b, 1995) observed pairs of Grade 2 children working on structured LOGO problem tasks in single-sex and mixed pairs. She observed the pairs working, recorded their discussions, and noted their approaches to problems. Girl-girl pairs spent more time planning moves and acted less independently of one another than the other pairs, who took turns to make moves, ignoring suggestions or comments from the partner. The girls tended to take longer and use more moves, but they were more likely to improve their strategies throughout the series of lessons. The nature of the task and the way in which it is set emerged as important. Girl-girl pairs tended to be anxious not to exceed the perceived guidelines of the task, and so tended to be more cautious, using lower numbers and avoiding risk-taking. This did not necessarily detract from their learning (in fact it may have enhanced it) but rather influenced the *success* outcomes of time and number of moves, making girl-girl pairs seem less successful. Differences in the criteria used to measure success may not be related to the actual amount of learning that has taken place. In one case study (Yelland 1994a) the girl-girl pair became more efficient during the series of tasks, while the other pairs did not alter their strategies. Gender differences in behaviour, observed with older groups, have clearly begun by Grade 2. The girls demonstrated rule compliance and avoidance of risk taking, and also co-operated more.

Forgasz and Leder (1995a), observing mixed groups of two boys and two girls at Year 7 level, found that group activities did not necessarily enhance the girls' learning. Group members tended to adopt gender stereotyped roles, the girls being more concerned with writing and presentation and the boys with experimenting and doing the mathematics. The form of the assessment (a common mark given to the whole group) rewarded one boy's lack of effort. Forgasz (1995a, 1995b, 1995e) also noted the importance of group composition for the effective functioning of the group. A student can easily be marginalised in a group where other friendships are strong, and one student in a group can be very disruptive if that student is a leader, and dislikes the task. The girls' rule-following behaviour was noticeable here too. The girls worked on the task even though they did not like it; the boys' non-acceptance of the task led to disruption and lack of involvement. An unfortunate consequence of this is that it may cause teachers to choose tasks which interest boys, to reduce disruption, even though girls do not like them.

The nature of the task affects students' responses. In a pilot study with one class of girls Forgasz and Romeril (1993) looked at girls' preferences when given a choice of tasks. They found differences between choices in class context and in assessment context. Enjoyment and interest motivated many girls in their choice of problems but few were motivated by challenge. There was no clear link between the time taken on the task and the quality of the solution, but students whom the teacher ranked as outstanding tended to take more time.

The Construction of Gender in the Classroom

Within the classroom teachers also react differently to boys and girls. Forgasz (1995b) noted one classroom in which boys often indulged in horseplay without major penalty but a girl was sent from the room for lesser behaviour. Zevenbergen (1993a), observing Grade 1 classrooms, noted similar teacher responses: boys were allowed to interject while girls had to obey the "hands up" rule. It is clear that actions in the classroom construct or support gender-differentiated roles creating a "gendered habitus" (Zevenbergen, 1993a). In one set of classroom observations the interactions conveyed the message that boys' contributions were more valued: the teacher's praise tended to attribute boys' success to cleverness and girls' success to effort. In contrast, Leder's (1993) study with older children found no overall difference in the nature of teachers' attributional statements, although there were clear gender differences in the students' attributions for success and failure. The construction of gender clearly takes place over an extended period of time and involves many different inputs from the child's social environment. The construction of gendered life occurs within the mathematics classroom and many practices have disempowering consequences for a significant number of girls. Zevenbergen (1993b) gave a clear example of gendered positioning in a Grade 5 class, illustrating the wide extent to which this occurs. Through this gendered habitus students learn to take on dispositions which will be influential in their success in schooling and their later positions within the social structure. Zevenbergen argued that, for females, a gendered habitus is paramount to constructing their marginality in mathematics. The Social Context Project (see Zevenbergen, Atweh, Kanes, & Cooper, this volume) also showed gender as a constructed variable.

Coeducational and Single-sex Provision

Single-sex schooling has been proposed as one way of meeting girls' educational needs, since girls in a single-sex classroom have the teacher's undivided attention, and the problem of students adopting gender roles in group activities is avoided. Some researchers have compared single-sex and coeducational schools with this in mind.

Smith (1994) studied two neighbouring schools, originally a boys' school and a girls' school, which, over a period of five years, went through a transition to coeducation. Students' self-concept seemed to improve after the change to coeducation, apart from an initial lowering during the year of transition. Boys had higher mathematics self-concept than girls. Student achievement remained about the same before and after the change. Teachers felt that students preferred coeducation, although there were greater classroom discipline problems for girls in this setting. Teachers believed, erroneously, that girls achieved better in single-sex classrooms. Another study, (Forgasz, 1995d) compared the attitudes of Year 7 students in single-sex and coeducational schools, finding no significant differences in girls' attitudes and attributions between the two types of school. Byrne (1993) also found little evidence to support the claimed benefits of single-sex schooling for girls, and argued that institutions need to "deal with the real problems head on, and not by yet again asking girls to take evasive action, leaving boys to remain unsocialised with impunity" (p. 199).

Gill (1992) noted that the greatest move was not towards single-sex schools, but single-sex classes for specialist subjects such as mathematics within coeducational schools, and she discussed some of the problems and disadvantages of this. Her own research indicated that girls in single-sex mathematics classes were more likely to rate their achievement realistically, while in coeducational classes girls underrated themselves and boys overrated themselves.

Parker and Rennie (1995) monitored a large project in Western Australia which used single-sex classes at some levels for science and mathematics. They looked at student attitudes, teaching and learning strategies, assessment, classroom interactions, teachers perceptions and beliefs, and general issues such as community responses and organisational difficulties. The single-sex boys' classes were an early source of concern. In the absence of girls, the boys' poor communication skills and discipline problems were more noticeable, and many teachers were unsure how to cope with these classes. The attitudes of both girls and boys in single-sex classes were more positive than those of their counterparts in coeducational classes, but when invited to comment, twice as many girls as boys spoke positively about the single-sex classes. Girls in single-sex classes reported the highest levels of interaction with the teacher. Although achievement was not investigated as part of the project, reports from teachers and some students indicated that the girls' improved attitudes to science and mathematics were reflected in improved achievement. Similarly, attention to communication skills in single-sex boys' classes helped to improve their achievement.

Broadly similar results were obtained by Leder and Forgasz (1994) who investigated the effect of placing Year 10 students in a coeducational school in Melbourne into single-sex classes for mathematics. These included a more positive response to the program by females, and problems arising from disruptive behaviour by boys. A year after the program there was a stronger recognition by both boys and girls that both sexes had benefited. Overall the program did little to reduce the gender gap. Some beliefs became more positive, but gender differences in performance and estimations of achievement either persisted or became more marked over time. Females' attributions did change, however; they attributed success less to ease of task and environmental factors, and attributed failure more to environmental factors. One positive benefit for girls was an increase in the projected enrolment of girls in the highest level course for Year 12 to a level well above the state average (Forgasz & Leder, 1995b). There was evidence that no group was disadvantaged as a result of the move to single-sex classes; if anything, the males benefited more than the females.

The Impact of Technology

The introduction of new technology brings a significant change to the classroom environment, with implications for gender equity. Reviewing research from many parts of the world, Sutton (1992) noted fairly consistent reports of girls having less access to computers than boys, although individual schools varied greatly. Boys are often regarded as being more interested in computers and more competent than girls. There is thus a risk that computer use will serve to exaggerate existing inequities, making it imperative to evaluate the introduction of computer technology into mathematics courses for gender differences.

Boers and Jones (1992) found that female students were less favourably inclined than males towards the introduction of graphics calculators into a first year tertiary calculus course. The females were more anxious about mathematics, and had more difficulty learning to use the calculators. When they had done so, however, anxious students became less afraid of mathematics and more willing to explore mathematical problems. The more negative attitudes of females were not detrimental to their performance. Indeed, for students majoring in mathematics or computer science, the introduction of a graphics calculator was associated with "a dramatic improvement in performance relative to males" (p. 7). A possible explanation is that nearly all students had their own calculator, so females were not competing with males for access. Females tend to have more reflective learning styles which the calculator could support, resulting in more reflection on mathematical ideas and improved understanding.

A follow-up study (Boers & Jones, 1993) sought to explain the superior performance of females in a course using graphics calculators and found that it was due to females' greater success on purely algebraic questions. Females showed a slight preference for algebraic solution strategies, and males for graphical strategies. Males were more likely to be successful in using a strategy involving the integration of algebraic and graphical information. They concluded that, as a result of the introduction of the calculator, males performed less well on a test which had as its major focus the assessment of traditional manipulative skills.

Barnes (1995) reported an evaluation of a calculus course involving extensive use of computer graphing. Both girls and boys found the computer activities enjoyable, motivating and effective in helping them to understand new mathematical ideas. Girls tended to cooperate better than boys, including work on computer-based activities. Groups of girls took more responsibility for their own learning, organising their computer work, and ensuring that everyone understood.

The Impact of the Wider Society

Mathematics in the Workplace and Home

Children derive many of their ideas about the importance of mathematics from observation of the world around them, noticing who uses mathematics and for what purpose, who is expected to be good at mathematics, and so on. Willis (1995b) suggested that

> perhaps one key to girls' lower participation in mathematics may lie in the evidence that both children and adults who think mathematics is equally important for girls and boys often still regard it as more important for adult men than adult women. (p. 268)

A study of the use of mathematics in everyday life and in employment (Knight, Arnold, Carter, Kelly, & Thornley, 1992) revealed that these perceptions of who uses mathematics are fairly accurate, that the use of mathematics in the wider world is, indeed, deeply gender-inflected. Knight et al. found clear gender differences in the everyday activities in which men and women in New Zealand use mathematics, and in the amount of mathematics used, reflecting traditional gender roles in society. A workplace survey revealed that men carry out a whole range of mathematical tasks to a much greater extent than women, again, reflecting the gender-segregated nature of employment.

Knight and Thornley (1995) took this a step further, asking whether men and women undertook the same everyday mathematical tasks in similar ways. In the contexts of supermarket shopping and gardening, they found that where a task could be carried out in several ways, women were more likely to take into account contextual constraints or to use informal methods, and men were more likely to use the formal methods of school mathematics. It is hardly surprising if children, observing this, conclude that school mathematics is more important for men than for women.

Studies of the use of mathematics in the workplace or home need to take into account the fact that people vary in their beliefs about what constitutes mathematics. In a study of changing parental attitudes to mathematics, Horne (1993) found that mothers had lower personal images in relation to mathematics than fathers. This included aspects of ease or comfort with mathematics, persistence in problem solving, and confidence. Among those interviewed, there was a strong perception that men used mathematics in the workplace and women did not. Women who did actually use a lot of mathematics and were quite good at it perceived their skills as common sense rather than mathematical. If tasks were practical, most women did not initially see them as mathematics.

Messages Conveyed by Textbooks

Textbooks influence the construction of mathematics and gender by the image they present of how and by whom mathematics is used. Past studies found extreme gender bias. Publishers have been made aware of this and textbooks have changed. Many teachers now believe that bias has been eliminated (Willis, 1995b). Two studies inspected recent textbooks to see if this is so (Clarkson 1993; Willis, 1995b). Both found improvements on the past, but had some reservations. Clarkson observed that only about a third of all text segments he looked at contained any references to people, with slightly more in primary school texts than secondary.

Willis (1995b) noted the use of non-sexist language, but found that portrayals of women and men were stereotypical distortions. Presenters and explainers were twice as likely to be male characters. Men were more likely to be given names and to be described in terms of their jobs, women more likely to be described in terms of family relationships. The textbooks reflect the gender divisions in society, demonstrating how people are differently positioned with respect to mathematics. Willis claims that "school mathematics can be understood as constructing the learner as a gendered subject, where gender is a classification that relates to the domestic division of labour" (Willis, 1995b, p.272). Given the gendered nature of society, it may be impossible to remove all bias from textbooks. Willis (1995a) suggested using the text as an object of study, asking students to use their data analysis skills to investigate how the text portrays men and women.

Messages Conveyed by the Media

Investigations of the portrayal of women in the media can help to illuminate the construction of gender in our society, and girls' attitudes towards participation and achievement in mathematics and mathematics-related occupations. "Because of the status of mathematics and the areas to which it leads, the popular construction of gender as reflected in the print media is incongruous with girls' successful participation in mathematics" (Leder, 1994, p. 103).

Leder (1992, 1995a) and Lynch and Leder (1994) have collected and analysed newspaper articles dealing with gender issues or women's lifestyles and achievements. Such articles appeared regularly, and tended to confirm traditional stereotypes by dwelling on difficulties experienced by women, or stressing family/career conflicts (Leder, 1995). About half presented negative images of women, or stressed the personal cost of success for women. Lynch and Leder (1994) suggested ways in which newspaper articles can be used to develop critical awareness, especially by girls, of the social construction of their world. This kind of activity could be even more powerful if the articles chosen for analysis included mathematical statements, graphs or data, and students were able to use their mathematical skills to critique statements about women and their role in society.

Women in the History of Mathematics

It is frequently suggested that teachers can help to change the image of mathematics as a male domain through lessons about women in the history of mathematics, or by having students research the subject for themselves. Unfortunately, the information in some popular accounts of women mathematicians is misleading and inaccurate (Deakin, 1992a, 1992b). To help redress this situation, Deakin gave accounts of the lives of seven women mathematicians who lived prior to 1900, and pointed out reliable sources of further information about them. Apart from having to overcome obstacles and prejudice, assisted by wealth and position, they had little in common except a passion for mathematics. He pointed out that the achievements of these women vary greatly, and deplored the uncritical adulation with which many writers treat them. "If the only model for girls were Kovalevskaya, she might well be perceived as setting an impossibly high standard. ... We need a diversity of role models, and the seven women go some way to providing it if we recognise their very disparate talents and interests" (Deakin, 1992a, p. 113).

Adult Women and Mathematics

A number of mathematics programs are specially targeted at adult women. These may be courses in adult basic education, re-entry programs in the TAFE sector, or mathematics courses for primary teacher trainees. Teachers in these programs are able to pay attention to the special needs of women returning to study. Several researchers have carried out studies with the aim of better understanding these needs (Beesey, 1995; Carroll, 1994a, 1994b; Feil, 1995; FitzSimons, 1993; Helme, 1995; B. Johnston, 1995). These are discussed in more detail in FitzSimons, this volume.

Policy Statements and Policy Studies

Gender Reform Initiatives, Evaluation and Critique

In Australia, the National Policy for the Education of Girls in Australian Schools (Commonwealth Schools Commission, 1987) forms the basis of policies to support the education of girls. A review of this policy after five years resulted in the National Action Plan for the Education of Girls 1993-97 (Australian Education Council, 1993), establishing priorities for reform. Although mathematics is not mentioned specifically, priorities for reforming the curriculum and improving teaching practice potentially impact on girls' mathematics learning. To monitor the implementation of the plan, regular reports are produced (e.g., Ministerial Council on Education, Training and Youth Affairs, 1994) in which both state and non-government systems report progress.

To examine the actual outcomes for girls, however, we need to look at evaluation projects. Some state education departments have studied the impact of gender policies. For example, in New South Wales, the Quality Assurance Directorate

(1994) reported little change in the proportion of girls participating in mathematics. Teachers felt that the gender equity strategy had not sufficiently impacted on teaching and learning practices. Schools had not embraced the need for the whole-scale and fundamental change in philosophy and practice required to achieve gender equity.

Milligan, Thomson, and Ashenden and Associates (1992) interviewed girls about their school experience. Among many findings, it was clear that girls leave school with very different patterns of knowledge, understanding and skills from those of boys, and receive conflicting messages about their post-school options. Johnston and Spooner (1993) focussed on subject choice in their interviews with students and other school personnel. They expressed concern that students choose mathematics because of its use as a filter, rather than because of its intrinsic value or interest. Some girls were pressured to choose mathematics despite feeling inadequate in the subject and being unable to see any relevance in what they were learning. Also of concern were the hierarchy of levels of mathematics, the stigma of being in the lowest level class, and attitudes of mathematics teachers towards low achievers which reinforce their sense of inadequacy.

In discussing a series of case studies of gender equity initiatives in secondary schools across Australia (Kenway & Willis, 1993; Willis, Kenway, Rennie, & Blackmore, 1992), Kenway and Willis pointed out that "enhancing girls' post-school options generally focusses on encouraging girls into subject areas which are seen to be linked to the 'non-traditional' jobs, that is, mathematics, science, technology" (p. 6). An implicit message is that young men make well-informed choices, young women do not, and that the problem is essentially one of changing the choices of individual girls. They identify four approaches to equity: changing choices, changing girls (for example by raising their self-esteem), changing curriculum, and changing the learning environment. The following are among the conclusions reached:

1. Enhancing girls' post-school options is nearly always defined in terms of paid work. Only rarely are issues of unpaid work, participation in public affairs, family responsibilities and leisure considered in any way.

2. Emphasis on non-traditional subjects and careers under-values those subjects and careers which are traditional for women.

3. Approaches to improving curricula are often superficial, changing the packaging rather than improving the product. Teachers did not regard fundamental curriculum reform as necessary, desirable or even possible, and only occasionally considered that their subject might be deeply gender inflected.

4. Some approaches which did seek to make the curriculum more inclusive of girls failed to take account of the differences between girls.

Bornholt (1993) queried whether the curriculum is intended to prepare students for the needs of daily life, or whether the real purpose is to provide for the needs of industry. The emphasis on increasing girls' participation in higher level mathematics and science applies to girls preparing for the professions, not to girls generally. The issue is essentially about status and power. Although policy documents point out the complex and multifaceted nature of the issue, school, regional, and system action plans tend to focus on a single cause. As an example, she cited the emphasis on increasing self-esteem. Her research found that girls were more realistic in their perceptions of performance on a recently-completed test, while boys over-estimated. She concluded that girls' low self-esteem was not the problem.

A study of women in science and technology in higher education in Australia (Byrne, 1993) identified a number of myths which influence policy making, and have been repeated so often that people have come to believe them, despite lack of evidence. Byrne found no support for the widely accepted idea that female role models are effective in encouraging young women to enter male-dominated areas of work or study such as mathematics. Mentoring, however, can be effective in recruiting and retaining women in these areas, and can be undertaken by men as well as women.

Mathematics Curriculum and Gender Equity

The document Mathematics in the New Zealand Curriculum (Ministry of Education, 1992) has a section on Catering for Individual Needs which makes specific reference to girls, pointing out the lower participation of females in mathematics at senior school level and beyond, suggesting strategies for involving them actively in the subject, and emphasising the need to encourage them to develop greater confidence in their mathematical ability.

The corresponding Australian document *Mathematics—A Curriculum Profile for Australian Schools* (Australian Education Council, 1994), unlike the New Zealand document, contains no explicit reference to gender, but Willis (1995c) has explained that the writers were strongly influenced by social justice considerations. There was a commitment to altering the conditions of schooling to ensure that all Australian children, whatever their gender, ethnicity, or social class, encounter the conditions necessary for success in mathematics.

Willis (1995b) described classroom practices involved in producing social inequality, suggesting that many students' experience of mathematics is gendered, and proposing ways in which our understanding of gender issues can be used to enhance the mathematics curriculum to make it more inclusive, without sacrificing standards. Inclusive pedagogy, Willis suggested, should encourage perseverance, should structure activities to provide increasing levels of challenge, and should avoid being too supportive. It should bear in mind the importance of context, but note that contexts which are familiar and meaningful to one group of girls may not be so to those of different backgrounds. Finally, a mathematics curriculum which is truly empowering for girls would address the ways in which mathematics is used to intimidate and mystify, providing students with the confidence and competence to use mathematics to analyse some of the myths about gender, and to question some of the realities of their own lives.

Curriculum development in Australia along the lines set out by Willis has been funded by the Department of Employment, Education and Training. Materials produced include an introductory algebra course for students in Years 7 to 10 (Lowe, Johnston, & Willis, 1993; Lowe, Kissane, & Willis, 1993; Lowe, Willis, Grace, &

Kissane, 1994; Lowe, Willis, Kissane, & Grace, 1994) and an introduction to calculus (Barnes, 1991, 1993). The latter is a series of units presenting calculus as a human endeavour, and emphasising its intrinsic value as well as its relevance to a range of social issues and situations familiar to students. Students are encouraged and guided to develop the mathematics for themselves through a sequence of small group investigations. The text is designed to be accessible so that students can read it for themselves and, by reflecting and writing about what they have done, take responsibility for their own learning (Barnes, 1994). An evaluation of a draft version of these materials (Barnes, 1995) found that they were generally successful in achieving the objectives. With only a few exceptions, students, both girls and boys, enjoyed working collaboratively, felt in control of their learning, experienced a sense of achievement and seemed to have increased in confidence.

Communicating the Implications of Research to Teachers

An important aspect of research in any area of mathematics education is communication of the results to school teachers and administrators. This is particularly the case in the area of gender and mathematics, where, as we have seen, there is a tendency to grasp for simple one-dimensional solutions to what is a complex, multi-dimensional problem. To be effective, this communication needs to draw out the practical implications of the research, and to be readily accessible to teachers. Some researchers communicate their results by writing articles for journals and newsletters of state and national mathematical associations, or by making presentations at conferences of teachers. For example, the Mathematical Association of Tasmania devoted a whole issue of its journal to gender issues (Langridge, 1992).

Ideally, trainee teachers should have adequate opportunity to consider issues of gender and mathematics during their training. Haynes (1995) described a course for primary teacher trainees at Auckland College of Education on Girls and Their Mathematics Education in which students made regular observations of children in a local school, interviewed children about their attitudes to mathematics, and prepared and taught a lesson designed to be enabling and motivating for girls. The combination of action research with theoretical study appears to be particularly effective.

Writing for both experienced teachers and trainee teachers at the secondary school level, Leder and Forgasz (1995) discussed implications for the classroom and strategies for action. Teachers are encouraged to undertake action research on such topics as their own interactions with their classes, the students' interests and aspirations, gender bias in teaching resources and achievement, and participation patterns in their own school.

Conclusion

The last four years have seen some shifts in the focus of research on gender and mathematics, which may result in part from the widespread public perception that much has been achieved. Compared with research reported in Leder and Forgasz (1992b), there has been more analysis of participation and performance data from, for example, the monitoring of government policies. There is also a greater tendency for researchers on a variety of topics to report data by gender, although much remains to be desired in this respect. In addition, there has been more research on the social context of mathematics learning, more focus on analyses and critiques of government policies, some attempts to develop new theoretical perspectives on gender equity, and less emphasis on psychological explanations of gender differences.

Cultural and socioeconomic background, perceptions of ability, career aspirations, perceived usefulness of mathematics to themselves, and cognitive preferences are all contributory factors in gender differentials in subject choice and may affect achievement. The tapestry is complex. In order to redress this problem action is needed on many different fronts: altering the environment of the mathematics classroom, changing the mathematics that is taught, providing career information in appropriate ways to encourage girls into a variety of careers, and trying to find ways of supporting girls whose language, ethnic, or social class backgrounds may combine with gender to affect their study of mathematics.

Returning to Willis' four perspectives on social justice in relation to education (Willis, 1996), we see the need for further research linked to the inclusive and socially critical perspectives. This is not to deny that there is still a place for remedial and non-discriminatory programs but rather to suggest that the focus should be directed to more holistic approaches. Leder (1995b) has pointed out the difficulty of trying "to capture and describe the complex classroom environment through any one classroom observation schedule" (p. 216) and has argued for more intensive and detailed studies which focus on small groups of students, student-student interactions, small group instruction, and using alternative methodologies. Methodologies with inclusive and socially critical perspectives are needed.

Programs to improve girls' learning of and participation in mathematics have benefited both girls and boys. Those which tackle curriculum, pedagogy, assessment and school organisation in a coordinated way provide the best opportunities to improve mathematics education for all. And, as Barnes (in press) points out, there is a need for more careful evaluation of these initiatives, looking at a much broader range of outcomes than participation rates or standard types of tests. They should include students' beliefs about the nature of mathematics and about themselves as learners of mathematics, the value they place on the mathematics they are learning, and the role they expect mathematics to play in both their professional and their personal lives.

The problem of gender inequity in mathematics education is clearly still unsolved. Research has been moving towards understanding the ways in which both gender and mathematics are constructed not only in the classroom but also in the wider community. Further work in this direction is needed if real change is to occur.

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Politics of Mathematics Education: Australasian Perspectives

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Introduction

Two difficult definitional questions confront anyone attempting to write a review of research on the politics of mathematics education. First, what actions and decisions are encompassed by the expression *the politics of mathematics education*? And second, what does the word *research* mean in the context of the politics of mathematics education?

Elaborating the Politics of Mathematics Education

Mathematics education takes place within a social and political milieu and *every* decision and action in the realm of mathematics education might be regarded as being *political*. In particular, decisions by education policy makers are intended to have an impact on mathematics education practices by influencing curriculum developers, textbook publishers, examiners, teachers, and students. It is often the case, too, that decisions made by others (teachers' unions, for example) are intended to influence education policy makers and politicians.

Barton (1995) has argued that "not only is mathematics education deeply political in ways which are specific to the subject, but also it [mathematics] is one of the most political of all subjects" (p. 165). The political importance of school mathematics might be attributed to a widely held view that "although the amount of recognisable school mathematics actually performed by people at home or in their work is very small, this subject is one of the greatest determinants of vocational or educational advancement" (p. 167).

In attempting to elaborate the domain corresponding to "the politics of mathematics education," Barton (1995) argued that regardless of the ways in which control over school mathematics is exercised, even if it is shared, there will always be an identifiable group which has some form of the control over the mathematics curriculum operating in a class, or a school, or a school system, or indeed across a nation. Control of the mathematics curriculum (taken broadly to include content, strategies for teaching and learning, and assessment and monitoring procedures) has been a key political issue throughout Australasia during the 1990s. (Australasian, in this chapter, refers to Australia and New Zealand.)

Barton's (1995) elaboration of the domain encompassed by the expression "the politics of mathematics education" provides justification for the following approach taken in this chapter: we shall concentrate on identifying and critiquing major forces within Australasia over the period 1992–1995 which *changed*, or at least had the *potential to change*, the content and nature of mathematics teaching and learning in this part of the world.

Research and the Politics of Mathematics Education

In regard to the second issue, concerning the meaning of research in the particular context of the politics of mathematics education, we recognise that there is a tension between (a) the need to report events and discourses which reveal the extent and nature of political undertones in the domain of mathematics education; and (b) the requirement that this review concentrates on actual research studies which have been carried out in Australasia during the period 1992-1995. We consider that our role is not only one of identifying and briefly discussing key issues, but also one of pointing to research which has been done with respect to those issues. Furthermore, we believe it is also our role to draw attention to important issues for which research needs to be carried out.

We believe an emerging issue is the impact of the Internet on research in mathematics education research, and of the way electronic mail in particular is being used extensively in political activity in the mathematical sciences. This new development raises interesting questions concerning the meaning of research in the context of the politics of mathematics education.

Legal, as opposed to moral and educational rights. Associated with the second question of how research is to be defined is a moral dilemma. Does a so-called research report represent a report on *genuine* research if it is carried out by carefully selected researchers who are given tight prescriptions in regard to what can be investigated and what cannot, how it is to be investigated, and what can be reported and what cannot? This difficult issue was raised directly by Ellerton (1995), in an editorial in the *Mathematics Education Research Journal.*

Mathematicians, mathematics educators, and mathematics teachers are not the only ones with expertise on curriculum matters, and are not the only ones who can be regarded as *stakeholders* in matters affecting mathematics teaching and learning. Students, parents, leaders of business and industry, for example, are also interested parties. This raises the question of who should be involved when important mathematics curriculum (or other mathematics education policy) decisions are to be made. When decisions are to made on a national curriculum framework, for example, is it legitimate, and is it wise, for a committee of education bureaucrats and policy specialists to decide on a policy, or structure, or philosophy (such as *outcome-based education [OBE])* *before* bringing in subject specialists and subject committees to provide the details which will give expression to the policy?

In commenting on such matters Ellerton and Clements (1994a) distinguished between legal as opposed to moral and educational rights. They argued that because

State and Territory governments have a *legal* right to define curriculum policy and to develop curriculum frameworks, it does not follow they have a *moral* or *educational* right to do this without involving other major stakeholders from the outset.

Education Policy Makers and Mathematics Education Policy

An Authority Model, With Superordinate and Subordinate Groups

Marsh's (1994a) historical account of the move during the period 1987-1993 towards a national curriculum in Australia provided much insight into the attitudes of education bureaucrats and politicians on the question of who should most influence mathematics curriculum and mathematics education policy matters.

Marsh, as Director of the Secondary Education Authority of Western Australia, had worked in Canberra during 1992 and 1993 on the Secretariat of the Curriculum Assessment Committee (CURASS), a body charged with the responsibility of coordinating the development of the national curriculum. He indicated that CURASS worked from a hierarchical "authority-innovation-decision-making" systems model which incorporated two groups of decision makers: there was the *superordinate group*, which made the major decisions, initiating, directing, and disseminating policies; and there was the *subordinate group*, which assisted the superordinate group in implementing policy (Marsh, 1994a). According to March those with most influence were State and Federal Ministers of Education, the Curriculum Corporation, AEC, CURASS, and the Directors of Curriculum of government school systems. Those with low influence included national and state professional associations, and university academics.

Crump (1995), a University of Sydney academic who reviewed Marsh's (1994a) book, described Marsh's and CURASS' acceptance of the authority-innovation-decision-making model as "alarming." Marsh's book reveals a disdain within the ranks of the decision makers for academics and national and state professional associations of teachers.

"Colonial Echo," or "Muddling Through"?

The move towards a national curriculum in Australia and New Zealand can be reasonably interpreted in terms of colonial echo. There can be no doubt that the national curricula developed in Australia and New Zealand (NZ) in the early 1990s were remarkably similar to the national curriculum which had been introduced in the United Kingdom (UK) in the second half of the 1980s (Ellerton & Clements, 1994b; Francis, 1993), and that standards-based approaches deriving largely from the United States of America (USA) are also increasingly influential in Australia (McDonnell, 1995; Pascoe, 1995).

Australian education policy makers have *consciously* applied overseas policy initiatives to Australian education systems. The quality movement in school and higher education in Australia, for example, has been imported largely from the UK and the USA. Since the mid-1980s, the Department of Employment, Education and Training in Canberra has been controlled by economists with little direct experience in education settings, but large knowledge of overseas trends in methods linking education and economics (Pusey, 1991).

Anyone who doubts that the USA and the UK continue to have a large influence on education policy makers in Australia should study the set of papers in the Proceedings of a conference on "Efficiency and Equity in Education Policy" held in Canberra in September 1995. There were seven plenary speakers at the conference, comprising two professors from UK, two professors from the USA, and three senior academics from Economics Departments in Australian universities.

In responding to the plenary paper on curriculum standards delivered by Professor McDonnell (Professor of Political Science and Education at the University of California), Scales (1995), the Chair of the Industry Commission in Australia, began by stating it was "striking how relevant Professor McDonnell's paper is to Australia." Scales then spoke of the parallels that could be drawn between Australian and USA' experiences with national curricula, and about the need for Australia to define carefully student learning outcomes and indicators in order that "an overall assessment of school system performance" could be made. He suggested statewide assessments of student learning were needed as such data would enable comparisons to be made between school systems.

Although there was not one mathematics educator present at this high-level conference, policies which were likely to influence the teaching and learning of mathematics in Australian schools were being formulated, mainly by economists, politicians, and visiting professors from the UK and the USA.

Truran (1994) argued against a colonial echo approach to writing the history of mathematics education in Australia and New Zealand, maintaining that colonial echo explanations are "too simplistic and too general" (p. 632). On the basis of his historical analysis of the introduction of probability into school mathematics programs in South Australia, Truran (1994) claimed that "muddling through" seemed to be a better explanation of curriculum development than either "colonial echo" or "mature development" (p. 638).

The evidence presented in relation to the Canberra conference suggests that although the process of muddling through in education policy occurs within Australia, those controlling the direction of the muddling still look to the UK, the USA and, increasingly, to the discipline of economics for inspiration.

Introducing National Mathematics Curricula

National Mathematics Curricula in Australia and New Zealand

There could be no more overtly political intervention in mathematics education than a government moving to impose on schools a national curriculum whose structure and content are opposed by many of the nation's leading mathematics and mathematics education researchers. This is precisely what happened in Australia and New Zealand over the period 1989–1993 (Barton, 1995; Collins, 1995; Ellerton & Clements, 1994b; Marsh, 1994a).

Collins (1995) maintained that from the late 1980s the Federal Labor Government in Canberra had "the whole education and training sector in its sights." According to Collins (1995), the Federal Government's attempt to control education has been "perhaps, the strongest, most determined attempt in the history of schooling in Australia to control and steer not only the whole school curriculum—both primary and secondary—but also all sectors of education" (p. 6). She went on to describe the structure which was in place for the purpose of changing the curriculum as "neo-corporatism," which she described as "a system of decision-making which bypasses parliamentary debate" (p. 8).

The Australian Education Council (AEC) set about developing national statements and profiles in eight key learning areas (KLAs), one of these being mathematics. However, on July 2, 1993, after a number of State and Territory Labor governments had lost office resulting in the balance of power on AEC being changed, the Council decided (by 5 votes to 4) not to endorse the national curriculum framework which had been developed over the period 1989–1993. Instead, the framework was sent back to the States and Territories for further consideration and development.

The AEC decisions of July 2, 1993, attracted considerable media coverage across Australia, creating an impression in some quarters that a major education opportunity had been lost purely on political, as opposed to educational, grounds. Regarding the July 2 decision, the then Prime Minister of Australia, Mr Keating was reported as saying that the move to abandon work on a national curriculum "was one of the most depressing outcomes ever of Commonwealth State meetings. ... If we can't even give our kids a national curriculum after five years of work on this by the Commonwealth and States ... then you do wonder if we can get to anywhere cooperatively" (*Australian*, July 6, 1993, p. 4).

Hannan, who had chaired the AEC's CURASS Committee in 1992, was another to lament the July 2 decision. In an article in the *Weekend Australian* of August 7-9, 1993, Hannan described the attempt to develop a national curriculum as "the most ambitious piece of cooperation among states to have been attempted in Australian education." He went on to say, that right up to the "final destructive meeting of Ministers early in July it had been running like a Rolls. It was on time and it was in very good nick." Hannan was clearly angered by the July 2 decision, and accused academics who had criticised the national statements and profiles—the main components of the nationally developed curriculum framework—as being "mostly mistaken or lying and only occasionally on the track."

Nationally-developed Mathematics Curriculum Documents

Despite the 1993 AEC decisions not to adopt the national curriculum framework and documents, early in 1994 the Curriculum Corporation published a set of nationally developed curriculum documents (national Statements and Profiles) for eight KLAs. Ellerton and Clements (1994b) claimed that these, and other associated curriculum materials which the Curriculum Corporation had published, effectively defined a *de facto* national curriculum. Most State and Territory Ministers of Education insisted that State syllabuses had, to a certain extent, to be in line with the nationally developed documents.

Although Ellerton and Clements (1994b) documented fully the processes which generated the national statements and profiles *for all eight KLAs*, most of their references are to documents related to the development of *A National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991) and to various Curriculum Corporation publications (1994a, 1994b; Olssen, Adams, Grace, & Anderson, 1994) concerned with the *Mathematics Profile*. In this chapter it would be inappropriate to attempt to do more than draw attention to the main arguments that Ellerton and Clements presented in their book. That will be done from time to time throughout the remainder of this chapter.

The national mathematics Statement and Profile in Australia incorporated an OBE approach to curriculum. It included not only statements of almost 200 *outcomes* for children in K–10 (the compulsory years of schooling), but also of over 1000 *indicators*. The outcomes were placed in eight levels, covered five major content strands, and were accompanied by an elaborate scheme for standardising the assessment and reporting of mathematics achievement across the nation (Willis & Stephens, 1991).

A similar move towards a national curriculum occurred in New Zealand, and the move was no less controversial there than in Australia. According to Barton (1995):

> Mathematics was the first subject to be developed . . . and . . . the politics of curriculum control were played out more strongly as different groups sought to gain their place in the total process of curriculum change. This was reflected in the convening of a meeting of many mathematics educators prior to letting the curriculum contract; the mass sacking of the first Advisory group appointed to oversee the writing of the curriculum document; submissions by the Education Forum (a subcommittee of the Business Round Table); threatened walk-outs of the group responsible for the NZQA mathematics Unit Standards; national meetings of Maori educators to determine the process by which a Maori curriculum might be established; significant Ministry of Maori Development funding; Kura Kaupapa Maori and Maori Language Commission interest in mathematics education; and the contemporaneous rise of mathematics educators in university mathematics departments. All this activity in the period 1990–1994 can hardly be coincidence, Mathematics was the board on which changes in curriculum development were played out. (pp. 170–171)

The fact that the moves towards national mathematics curricula in Australian and in New Zealand took place at the same time is hardly surprising given that ministers of education from New Zealand attended AEC meetings, and that leading education bureaucrats from all Australian States and Territories and from New Zealand met on a number of occasions for the specific purpose of developing for schools national outcome-based curricula in all KLAs (Ellerton & Clements, 1994b).

Although during the period 1993–1996 there have been many government-funded trials of the nationally-developed statements and profiles, it is difficult to interpret so-called findings of these trials because typically data were gathered from a small number of schools which *volunteered* to be part of the trialling process and which received additional support during the period in which they participated in the trials (Ellerton, 1995).

The processes by which the national mathematics curriculum in Australia was developed were criticised by the Australian Mathematical Sciences Council (AMSC), whose constituent members were the Australian Mathematics Society (AustMS), the Statistical Society of Australia (SSA), the Australian Association of Mathematics Teachers (AAMT), the Mathematics Education Research Group of Australasia (MERGA), and the Mathematics Education Lecturers Association (MELA). After discussing the roles of AMSC and each of its constituent members in mathematics education in Australia, Ellerton and Clements (1994b, 1994c) provided details of how AMSC and its constituent members were excluded from all policy decisions related to the introduction of the national mathematics curriculum.

The national associations in the AMSC objected to the national mathematics curriculum documents on both mathematical and mathematics education grounds. More fundamentally, they were adamant that mathematics curricula for Australia's schools were not *owned* by government bureaucrats, and that AMSC and its members should have been invited to be actively involved at all stages in the development of any national mathematics curriculum framework. The lengthy and at times acrimonious exchanges between the bureaucrats and AMSC and its member societies are fully documented by Ellerton and Clements (1994b).

Marsh (1994a) has described the actions of the mathematics and mathematics education communities as representing a kind of "paranoia." However, Crump (1995), in reviewing Marsh's (1994a) book, stated that he regarded the actions of the mathematicians and mathematics educators as "reasonable and fair given the rush with which it all occurred" (p. 320). According to Crump (1995), it was the lobbying of the mathematics community which, by inflaming the media, "struck a devastating blow to the national curriculum project" (p. 320).

In the USA, the development by the National Council of Teachers of Mathematics of the acclaimed and influential *Mathematics Standards* was developed with a cooperative approach which obviously valued the expertise of mathematics and mathematics education professionals. The subsequent concerns raised by some mathematicians about the Standards appear to have more to do with their not having taken the opportunity to be involved throughout the process, rather than with any attempt to exclude them. In the Australian case, however, national professional associations of mathematicians and mathematics educators were denied the opportunity to have a strong positive involvement (Pascoe, 1995).

A Difference Between the Australian and New Zealand Contexts

One major difference between national curriculum issues in Australia and in New Zealand is that the New Zealand Government, like the UK Government, has the constitutional authority to impose a national curriculum; the Australian Federal Government does not have that authority. After Federation in 1901 constitutional control of education in Australia was vested in the States and the Territories.

The development of a national curriculum framework in Australia, then, challenged the traditions by which the curricula of Australian schools had been controlled. This challenge had two main thrusts. First, the right of a State or Territory freely to define curricula for its own schools was called into question. And second, the role of national subject associations and university academics as vitally important stakeholders in school curricula was largely denied (Ellerton & Clements, 1994b; Gaudry & Thomas, 1993; Marsh, 1994a).

The UK National Mathematics Curriculum

Similarities Between the UK and Australasian National Curricula

Some of the parallels between national curriculum decisions in Australia and New Zealand and similar developments in the UK are striking (Ellerton & Clements, 1994b). As in the UK, the national curriculum movements in Australia and New Zealand were marked by government determination to decrease the role of Australasian academics in matters concerning school curricula, and to increase centralised curriculum decision-making powers of education bureaucrats (Marsh, 1994a). In fact, Boston, the NSW Director-General of Education, argued in 1993 that this was the way it should be because the State and Territory departments of education had the most experience in curriculum development (Ellerton & Clements, 1994b).

The same five major content Strands (Number, Measurement, Space [Geometry in NZ], Chance and Data [Statistics in NZ], and Algebra), as those incorporated in the UK national mathematics curriculum were key elements of the nationally developed mathematics curricula for both Australia and New Zealand. So too were the notions of *bands* and *levels*.

Has the UK National Curriculum Been Effective?

The obvious similarity in the structures of the Australian and New Zealand curricula with the UK national curriculum meant that there was some interest in the level of acceptance of the UK curricula by teachers and educators in the UK. Research, some of it carried out by Australian researchers (Groundwater-Smith, 1993; Truran, 1993), indicated that despite some initial optimism, teachers in the UK quickly found themselves straitjacketed by the national curriculum (Kelly, 1990; Moon, 1992; Truran, 1993).

According to Groundwater-Smith (1993), in 1990 only about one-quarter of the teachers surveyed in a large study conducted in the UK saw themselves facing more constraints and loss of autonomy. Just two years later, however, in 1992, almost 80% of teachers constructed such a scenario. In 1990, 55% of teachers felt that the national curriculum matched the needs of pupils *very well* or *well*, but by 1992, this percentage had dropped to 24%. And, although in 1990 only 17% of teachers felt that the national curriculum restricted their capacity to adapt their teaching to children's needs, by 1992, the corresponding figure was 51%.

Equity and the UK National Curriculum

Significantly, teachers in schools in low SES disadvantaged areas were the ones who had lost most faith in the claims made that a national curriculum would work towards achieving greater equity of educational outcomes. Furthermore, evidence collected by inspectors during more than 7000 lessons in the UK in 1991 and 1992 indicated that the introduction of the national curriculum in the UK had not resulted in improved classroom practice in mathematics. Truran (1993, p. 137) reported that:

- Less than five per cent of lessons were considered by inspectors to be very good, and a lot of work at Year 7 was pitched far too low.
- The use of assessment was not well-developed in mathematics. There was also little success in the cross-curricular use of mathematics, and not enough attention was paid to mental and calculator skills.
- The poor quality of senior staff responsible for implementing the mathematics curriculum was being blamed for unsatisfactory results.

Mathematicians' Reactions to the UK National Mathematics Curriculum

Neither the academic mathematicians nor mathematics educators in the UK were happy with the national mathematics curriculum. Clare (1995) reported that the London Mathematical Society and the Royal Statistical Society blamed the national curriculum for "a significant decline in the mathematical preparedness of undergraduates studying maths, science, and engineering at university." According to Clare (1995), the national mathematics curriculum failed to lay the necessary foundations and misrepresented the nature of the discipline.

A working party established by the London Mathematical Society, the Institute of Mathematics and its Applications, and the Royal Statistical Society strongly recommended in 1995 that a standing committee be established "to ensure that sound advice and adequate support are provided" (London Mathematical Society, 1995, p. 3) in mathematics at all levels. The School Curriculum Assessment Authority's response was that it was already "informed by the widest possible range of professional opinion" (Patel, 1995). When this arrogant response was read by mathematicians in Australia they immediately recognised similarities between the ways in which mathematicians in the UK and in Australia were being treated in regard to national curriculum issues.

Psychometric Considerations

Research has revealed that there are serious and unresolved issues concerning the reliability and validity of teachers' ratings of student levels of achievement (for a summary of this research, see, for example, Wiliam, 1993). Thus, it is difficult to share Masters' (1992) confidence that an adequate measurement system can be developed for the Australian profiles. That is still the case even if we accept Masters' (1994) argument that although "profile levels are fuzzy," this does not really matter because "holistic, on-balance judgments of students' broad levels of achievements are likely to be good enough for many everyday purposes" (p. 50).

As criticism of the UK's national curriculum mounted, supporters of the Australian Education Council's nationally developed curriculum documents (e.g., Francis, 1993) began to emphasise the differences rather than the similarities between the educational and political background contexts in the UK and Australia.

Accountability, Benchmarking, and Value-added Comparisons

National and Statewide Testing

The nationally developed curricula in Australia and New Zealand were not accompanied by the introduction of a national pencil-and-paper testing program. Indeed one of the architects of the profiles, the late Garth Boomer, stated that unless the national profiles were introduced, the State, Territory and Federal Governments would combine to introduce national testing (Ellerton & Clements, 1994b). Ellerton and Clements, on the other hand, predicted that the explicitly worded outcomes and pointers in the profiles documents would provide an obvious basis for the introduction of national testing.

It is significant, then, that during the period 1993-1996 there was a strong push within Australia and New Zealand for the introduction of national testing regimes based on the content and levels of the nationally-developed curriculum documents. Indeed, since 1993, all State and Territory government systems have continued to use, or have adopted, centralised testing programs, and clearly a *de facto* form of national testing has been implemented (Australian Education Union, 1995).

Teachers' unions, including the Australian Education Union, did not assume that the nationally developed profiles would provide a solid base for statewide, and eventually national, testing. Thus, in 1995, at a time when the unions were threatening to boycott statewide testing schemes, they were still supporting the introduction of outcomes-based education (OBE) programs based on nationally developed profiles (Australian Education Union, 1995).

Benchmarking, and Value-added Comparisons

The concept of *benchmarking* is becoming increasingly important in the context of national curriculum debates. Thus, for example, McGaw (1995), the Director of the Australian Council for Educational Research, after pointing out that the national curriculum statements and profiles and the State and Territory variants of them provided a new basis for monitoring educational performance, referred to the benefits, from an accountability perspective, of being able to make value-added comparisons based on full cohort testing at more then one year level. This approach has also been advocated by quality assurance personnel in the various State and Territory Ministries of Education (e.g., Cuttance, 1995).

National Curriculum and the Teachers' Unions

In 1993 teachers' unions and the Federal Labor Government agreed to build support for the national curriculum concept into enterprise agreements with unions. The Government also moved to provide funding to professional associations to support the national curriculum thrust. Some unions and professional associations accepted the Federal Labor offer of substantial additional funding and so committed their membership to cooperating with implementing the nationally-developed statements and profiles. (Bartlett, Knight, Lingard, & Porter, 1994; Crump, 1995; Maslen, 1993; Spark, 1995).

As the teachers and teachers' unions became more fully aware of what might be involved in the concept of profiling they began to feel threatened by the scope of the demands being placed on teachers. In the *Daily Telegraph Mirror* of June 1, 1994 the President of the NSW Teachers' Federation, Cross, was reported as saying that primary and secondary schools were in a "total state of confusion" over the introduction of new-style syllabuses and student profiles. Cross added that the vast majority of teachers did not fundamentally understand an outcomes syllabus (what children know) or a profile (a learning plan). Lanyon (1995), writing on behalf of a number of school principals, stated, bluntly, that the Federation "had got it wrong in agreeing to outcomes" (p. 19).

Practising teachers present at a January 1994 Summer School, funded by the Commonwealth Department of Employment, Education and Training (DEET), made it clear that they were concerned that the introduction of profiles would increase the workload of teachers. Cooper (1994), a teacher who attended the Summer School, stated that the description of profiles, by a speaker at the Summer School who had been involved in the development of the profiles, as a "hump of extra work" probably did not give a true image of just how much extra work would be required of teachers. "The visual image I perceive looks less the size of camel hump," he added, "and more the size of a barrow, one of the pre-historic grave-mounds" (p. 13). "I am not suggesting that profiles will drive teachers to an early grave, but they do represent a large increase in teacher workload."

Despite money having been contributed by the Federal Government to support a National Professional Development Program (National Education Forum, 1995), which assisted teachers in all States and Territories of Australia to adopt outcome-based approaches (or the State or Territory versions of these), many teachers have been overwhelmed by the expectations placed upon them (Eltis, 1995). Some have begun to say, openly, that their unions and professional associations had "got it wrong" (Lanyon, 1995).

Other teachers, especially some in trial schools, have been more favourably disposed to OBE approaches. However, even teachers in trial schools which received additional funding and resource support, were concerned about "the heavy demands upon time, energy and commitment," that had been placed on them. They claimed that if the approaches were to be maximally effective they would need more resources, especially in the form of books and time release (Bonser & Grundy, 1995a, 1995b). Likewise, supporters of the national curriculum (e.g., Willis & Kissane, 1995b), recognised that "outcome-based education makes considerable professional demands on teachers both individually and collectively" and is unlikely to occur on a wide scale without appropriate support and consistency in policies.

Many professional associations have never given explicit support to OBE approaches. For example, in August 1995, the Mathematical Association of New South Wales (1995) urged "all involved to consider these issues before thrusting teachers and students in NSW into a system which has been shown in England to be unwieldy, time consuming and not beneficial" (p. 4). Be that as it may, some writers have claimed (e.g., Willis, 1994) that OBE practices are likely to generate greater teacher professionalism, and that in particular the nationally-developed curriculum documents are likely to encourage teachers to adopt a wider range of assessment techniques and methods.

Research and the National Curriculum

Guttmann (1993), a mathematician at the University of Melbourne, pointed out that although it had often been suggested that "a national curriculum is, like motherhood, a good thing and that no rational person could object to the concept" (p. 16), it was worth examining the basic question of whether a national curriculum was desirable. Guttmann (1993) said that although he recognised that "it makes some good sense to identify common features in curricula across the States, and to seek to ensure that there are no major discrepancies" (p. 16), any national agreement on content should make sure that quality is not compromised, and that a degree of diversity is allowed.

Education Turbulence

A common argument used to support the introduction of a national curriculum was that, without one, children in mobile families are disadvantaged (see, e.g., Wilson, 1992). Although Guttmann recognised the salience of this argument, especially in an increasingly mobile population, he argued that it was possible that students who often changed school could, in fact, be *disadvantaged* by the existence of a national curriculum. Teachers following their local version of the national curriculum might be loath to vary their order and approach to meet the needs of newcomers, and hence educationally turbulent students (that is to say, students who had attended many schools) might be forced to repeat work, or they might never study particular topics at all.

There is, in fact, some research evidence in relation to the effects of educational turbulence on primary school children. In the mid-1970s, Mackay and Spicer investigated claims that the children of highly mobile defence personnel experienced educational disadvantages as a result of their mobility. After carrying out a large and carefully designed study they found that the academic performance of children in very mobile families was superior to that of children (also of defence personnel) of similar ability who were at the same grade levels but who had attended only one or two schools (Mackay & Spicer, 1975).

One can speculate on possible reasons for Mackay and Spicer's findings. For example, it is possible that the students from mobile families were faced with the challenge of linking what they had learned previously with what they were presently encountering, and the mental effort in building such links could have been beneficial. Whatever the reason, the conclusion to be drawn from the research findings is that one should not assume that the frequently heard mobility argument in favour of a national curriculum, is valid.

From that perspective, it is interesting that the *Schools and TAFE* policy of the recently elected Federal Coalition Government in Australia contained the following paragraph:

> While the Coalition does not support a Commonwealth-imposed national curriculum, a Coalition Government will support a level of consistency in basic skills taught across the nation. Parents have a right to expect that, in a nation of around 18 million people, their children will not be disadvantaged by moving interstate. (Hill, 1996a, p. 10)

Following this statement the Coalition indicated that it would "support the efforts of States and Territories to integrate common essential elements of the National Curriculum Framework into State and Territory syllabuses" (p. 10).

Linguistic and Cultural Influences

Another of the possibilities raised by Guttmann (1993) was that in a nation in which there were large language-background and cultural differences between children from a wide range of ethnic communities, curriculum diversity could be more desirable than curriculum uniformity. In fact, under such circumstances a common curriculum with common outcomes might be a recipe for inequality of educational opportunity. This point of view has been expressed by other critics of the national curriculum concept (see, for example, Ellerton, 1995; Lanyon, 1995; and the editorial in the *Sydney Morning Herald,* July 2, 1993).

Ellerton and Clements (1994b) stated that they had never found any advocate of a national curriculum who referred to data on the relatively "poor" performance of urban and rural Aboriginal students, summarised by Bourke and Parkin (1977). Those data are, almost certainly, best explained by cultural and linguistic factors. The pencil-and-paper questions from which the data originated were in English and related to concepts, skills, and events which were not culturally appropriate for many of the

Aboriginal students. It is at least arguable that the expectation of common outcomes would exacerbate rather than assist in solving the problem.

Thomas (1994) argued that Australian education systems need to develop mathematics education programs based on a clear view of the multicultural and multilingual nature of Australian society. She drew attention to a comment by Collins (1994):

> In particular, using the national curriculum for comparing and reporting of any kind ... is a dangerous exercise which will trap teachers inside a set of assumptions about the "normal" development of children's learning in classrooms. We can take a good bet on who will benefit from that, which classes of children will zoom along and which will straggle along behind in the "scientifically-devised" path of school learning. (pp. 16-17)

Willis (1995), on the other hand, argued that the *Mathematics Profile* would support principles of social justice in education.

Absence of a Research Base

In fact, supporters of the nationally developed curriculum documents have rarely attempted to provide an education research basis for the Profiles, but have tended to rely on the rhetoric of OBE enthusiasts such as Spady (1993). The Eltis Committee reported:

> A search of relevant ERIC literature from 1993 until the present reveals numerous entries (close to three thousand) under the umbrella term "outcomes-based education." A closer inspection of some three hundred of these reveals that most provide a description or definition of the approach alongside other restructuring reforms. Alternatively, they provide arguments for or against the approach. Few demonstrate substantive support for their use beyond general statements about improved test performance or better attitudes to learning by students, and about increased accountability ... It would seem that very few research investigations have studied the implementation and effects of using OBE models. (Eltis, 1995, pp. 15-16)

Three recent booklets published by the Curriculum Corporation, *CURASS Guidelines Papers* (Curriculum Corporation, 1994c), *Introducing Statements and Profiles* (Curriculum Corporation, 1994d), and *Using the Mathematics Profile* (Olssen, Adams, Grace, & Anderson, 1994), contain no reference to any research base which influenced those who, in 1989 and 1990, made decisions about the structure of the proposed profiles.

Ellerton and Clements (1994b) further argued that the OBE foundations of the *Mathematics Profile* were neo-behaviourist (see also, Marsh, 1994b). As such, it was out of line with contemporary mathematics education research and best practice. Neyland (1995a, 1995b), who was involved in the development of the New Zealand national mathematics curriculum, was another to make the same point.

More recently, the Eltis Committee (Eltis, 1995) and Willis and Kissane (1995a, 1995b) have reviewed some research into the effects of the adoption of OBE approaches for mathematics education. Strengths and weaknesses of the Standards movement in USA have been discussed (McDonnell, 1995; Pascoe, 1995). However, there are still no comprehensive research studies which have been published into the effects of OBE approaches on mathematics teaching and learning in Australia and New Zealand (Ellerton, 1995).

Recommendations of the Eltis Committee. One of the few arguably independent, and balanced, evaluations of the literature on outcome-based education was carried out by a NSW Government review committee headed by Professor Eltis (for comments on the review, see Owens, 1996). This committee recommended that the NSW "Board of Studies no longer be required to incorporate the national profiles directly into NSW syllabuses" (Eltis, 1995, p. I), and that with respect to both primary and secondary school mathematics, a limited number of outcomes, derived from current syllabuses, be developed and trialled, the focus being on knowledge, skills, and understandings.

Equity, Entitlements, and Outcomes

The rhetoric used in the UK in the late 1980s, that "all students were *entitled* to a broad, carefully planned curriculum," was adopted by Federal Education Ministers in Australia in the early 1990s (Marsh, 1994a). Those who were responsible for formulating the national curriculum statements and profiles in Australia and New Zealand tended to argue that although the school-based curriculum development regimes of the 1970s and 1980s had been needed to free-up cramped, selective curricula, the locally-developed curricula had gradually "become soft" (Boomer, 1992; Hannan, 1993).

Supporters of the national curriculum concept argued that the existence of the soft options tended to persuade teachers and school administrators that these would be "meaningful for kids from disadvantaged backgrounds." Hence, the argument went, two classes of students were created—those who opted to take, or were forced to take, the soft options, and those who took the more conservative combinations of subjects. Only the latter prepared students for possible entry to prestigious professional degree courses.

The OBE programs which were features of the national curricula in the UK and in various states in the USA provided tempting models for Australian bureaucrats and politicians to follow. However, the imposition of centrally prescribed curricula was not in line with the school-based curriculum development and teacher professional development programs of the 1970s and 1980s (Lanyon, 1995). Those developing the national curriculum, therefore, tended to use what Marsh (1994a) has euphemistically called an "authority model," in which the concepts of "entitlements" and "social justice" were frequently called upon.

Piper (1992) argued that the concept of an entitlement curriculum signalled a dramatic shift in the focus of the debate from issues of control and accountability to issues of warranty and consumer protection, from a focus on declining standards to a focus on guaranteed quality. This led to the concept of an entitlement curriculum: any society which requires students to attend school until a statutory leaving age should also offer them worthwhile things to do while they are there.

Thomas (1994), Collins (1994), and Lanyon (1995), on the other hand, have suggested that Australian education systems need to incorporate forms of education which take account of the multicultural and multilingual nature of Australian society.

Collins (1994) has argued that using the national curriculum for comparing and reporting of any kind "is a dangerous exercise which will trap teachers inside a set of assumptions about the 'normal' development of children's learning in classrooms." According to Collins (1994), under such a scheme some children would "zoom along," but some would "straggle along behind in the 'scientifically-devised' path of school learning" (pp. 16-17). This was because with such an approach teachers would be likely to feel pressured to try to teach prescribed concepts and skills to *all* children—even to those who were not ready to learn them, or who were not suited to them, culturally (Zevenbergen, 1994, 1995).

In a similar vein, Lanyon (1995) argued that student outcome statements tend to "deny the influence of any child's socioeconomic and cultural backgrounds and look simply at those who can and those who can't" (p. 16). Ellerton (1995) asked whether the national mathematics curriculum would be suitable for Australian and New Zealand learners from families outside what might be regarded as the Anglo-Saxon mainstream.

Jones, Kershaw, and Sparrow (1995) drew attention to differences between "Western" mathematics as represented in the curriculum of the Northern Territory Department of Education in Australia and traditional forms of mathematics in Aboriginal communities. They quoted extensively from Batchelor College documents in which it is made clear that language and culture are vitally important aspects of indigenous mathematics. (Batchelor College is a tertiary institution which specialises in the preparation of Aboriginal teacher education students for teaching in Aboriginal schools). According to Jones et al. (1995),

> fundamentally different world views such as those just described translate equally dissociate language structure and function and vice versa. ... English is not structured for the purpose, whereas relevant Aboriginal dialects are. In a very real sense culture is language. (p. 19)

Jones et al. (1995) referred to the work of Christie (1994) and Harris (1990) in elaborating the view that Western mathematical knowledge is essentially antithetical to indigenous forms of mathematics.

The crucial importance of cultural aspects of education was highlighted in unequivocal terms by Harris (1990); he argued that "the nature and degree of the difference between Aboriginal and European culture is so great that the only honest conclusion we can arrive at is that they are largely incompatible" (p. 9). According to Harris, the two cultures "are warring against each other at their foundations," and "it is harder to find what they have in common in cultural terms than it is to see the differences" (p. 9). Such a view cannot be easily reconciled with proposals to introduce a national mathematics curriculum into Aboriginal schools.

Jones et al. (1995) commented that the process whereby Aboriginal and other so-called non-western minority groups will have the right for their children to experience a curriculum which links with their personal worlds rather than being in opposition to it, is underway. Essentially, "cultural reaffirmation has translated into a decolonising process, particularly with respect to education" (p. 2). Western mathematics, together with its associated language and symbol forms, represents a highly specialised way of thinking, and to a certain extent that explains Bechervaise's (1992) description of mathematics as a foreign language.

Yet, the reality is that most Aboriginal children and, more generally, children from many other minority groups, have been required to attend school and to learn a western style of mathematics. Learning the language and symbol systems of western mathematics is an essential component of the enculturation (some may say indoctrination) process. Harris (1992) made the point well in talking about "school shapes." She stated that when she had been teaching Aboriginal children she had absolutely hated using attribute blocks:

> It seemed to me quite nonsensical to sit there asking Warlpiri children "Give me a square, show me a circle, can you find a triangle?" and so on. I just thought that if we had to classify, we should classify something that was meaningful to them. (p. 65)

According to Harris, an Iwaidja woman (on Croker Island off the coast of Arnhem Land) had described the certain plane shapes drawn as follows: a square was "flat, smooth"; a circle was "round"; an oblong was "long"; and a triangle "had sides."

One of the challenges facing mathematics education researchers is to explore further links between language, culture and mathematics education, and to develop political processes which ensure that their findings influence curriculum decisions.

Cultural Imperialism

Bishop (1993) argued that because there is such a strong link between culture and language, imposing mathematics curricula on minority groups is a form of cultural imperialism. Although it is certainly true that not all languages express Western mathematical concepts in an efficient way, it is equally true that many of the concepts in the indigenous languages and cultures would not be easily expressed in English or in other languages spoken by large numbers of people. As Bishop (1993) stated, "if countries and societies within countries are to engage in the process of cultural reconstruction then the language element in relation to informal, non-formal, and formal mathematics education is critical" (p. 20).

There is a danger that such is the power and prestige of western mathematics that well-meaning politicians, education bureaucrats, and education experts will foist on unsuspecting school children a type of mathematics learning and content which many are not ready, both from the cultural and cognitive points of view, to learn. The concept of an entitlements curriculum, and the idea arising from the effective-teaching literature that there is a best way to teach mathematics, will almost certainly continue to create so-called minority students who are regarded as disadvantaged (Zevenbergen, 1994, 1995).

Towards a New Millennium

Historical Perspectives: Mathematics for the Minority

Throughout the nineteenth century, and for much of the twentieth century, school and university administrators in Australia and New Zealand were under pressure to duplicate the mathematics curricula prescribed for corresponding education institutions in other advanced western nations. Clements (1992) argued that this pattern continued until well into the twentieth century, with mathematics curricula being developed for the minority, and examinations based on these curricula being used to establish *standards*. Contrary to the impression given in many reports there is evidence that standards have been well maintained over time (see Ellerton & Clements, 1991; McGaw, 1994).

Around 1960, in both Australia and New Zealand, only a small proportion of school students were in senior secondary classes, and a tradition had been established that only the most intellectually capable senior secondary students would take the advanced mathematics subjects. School mathematics had become a sieve, through which only the very capable could pass. Gradually, however, the proportion of students continuing through to senior secondary school increased, and this created altogether new challenges for those involved in formulating policy for mathematics education.

Mathematics for the Majority

By the mid-1990s retention rates in Australian and New Zealand secondary schools had increased to unprecedented levels. In Australia, 80% of students completed Year 12 (Gregory, 1995). The number of students in higher education in Australia had increased from 300 000 in 1980 to 600 000 in 1995 (Gregory, 1995). Although most of the senior secondary students chose, or were required, to study some form of mathematics, only a small proportion continued with higher studies in mathematics or the physical sciences at a university. And, unlike the situation in earlier times, those who did were often not among the most intellectually capable. In the mid-1990s, in spite of many more students completing Year 12, many universities have difficulty filling places in mathematics or the physical sciences.

A Serious Shortage of Mathematics Teachers

To make the present situation even more complicated, in recent years there have been relatively few mathematics graduates deciding to become secondary mathematics teachers. There is, in fact, in Australia and in New Zealand, a serious shortage of qualified mathematics teachers.

At a March 1996 meeting in Victoria of a group of mathematics educators known as Mathematics Educators Sharing (MESH), those present were addressed by Professor Northfield (from Monash University, Frankston campus), by Mr Jenz (a member of the Victorian Principals' Federation), and by Ms Wilson (on behalf of the Victorian Council of School Organisations). Following the meeting a statement, signed by Professor Leder (President, the Mathematics Education Research Group of Australasia, Inc.), Ms Mousley (President, Mathematics Education Lecturers Association), and Ms Thomas (President, Australian Mathematical Sciences Council), was released. Among the points made were that :

- an oversupply of teachers in recent years has masked a chronic shortage of trained mathematics teachers;
- the crisis is seeing schools reduce mathematics teaching time through strategies such as nine-day fortnights, and even four-day weeks in senior years;
- in many schools, teachers of well supplied curriculum areas are being put in front of mathematics classes, without the necessary training or qualifications. This happens most frequently in the critical lower-secondary years, and;
- new subjects and programs are eating into the time available for core English and Mathematics teaching.

Leder, Mousley, and Thomas (1996) added that since "mathematics is used in every field of small and large business and in day-to-day decision making and problem solving," it is imperative that urgent steps are taken to address the issues raised.

Quite clearly the serious shortage of qualified mathematics teachers in Victoria had the potential to become a serious embarrassment to the Victorian State government. Part of the problem, it seems, is that major issues associated with mathematics education at all levels are seen as being so strongly political that governments, facing huge blow-outs in their budgets, are doing their best to hide or mask embarrassing statistics.

Media Strategies

Education, especially mathematics education, has become such a politically sensitive area of concern that governments now go to some trouble to avoid potentially embarrassing situations. Those attending the March MESH meeting in were handed a copy of a leaked 7-page "Media Strategy" (Thomas, P., 1996) in which members of the Victorian Board of Studies were instructed how they were to handle public criticism of Board policies and practices relating to the State's controversial Learning Assessment Project (LAP), which includes whole cohort testing in mathematics.

The objectives of the Strategy were: (a) to ensure the benefits of LAP 96 were publicised throughout Victoria in a favourable manner; (b) to minimise criticism of LAP 96 in the media; (c) to ensure parents of Victorian primary school children received consistent information on the benefits of LAP 96 despite any efforts in the media by opponents of the project; and (d) to ensure that the Board of Studies retained control of communications in the public media, was not distracted by criticism, and was not drawn into prolonged debate in the media.

Most of the mathematics education problems faced by the Victorian government are to be found in all States and Territories in Australia, and in New Zealand. What is not clear is how the problems can be solved. Most mathematics educators, if not politicians and bureaucrats, agree, however, that attempting to hide the problems is not a sensible or morally defensible way to proceed.

Cooperation and Compromise

How are mathematics departments and mathematicians in Australian universities coping with the new state of affairs, whereby the backgrounds and calibre of students whom they receive from the schools are different from previous eras? The reaction of many has been to stand firm, asserting that it would be unwise to compromise standards. Sometimes, such a stand is accompanied by statements indicating that the mathematics programs in schools in the 1990s are less rigorous than was the case in the earlier periods.

A Compromise Solution for the Victorian Certificate of Education (VCE) Mathematics Concerns

In 1990 VCE senior secondary mathematics courses for Years 11 and 12 were introduced. For these, half of the assessment was based on project work and reports on extended problem-solving tasks (Stephens, 1991). The courses were strongly criticised by many university mathematics departments. Swedosh (1995), who worked in the University of Melbourne's Mathematics Department, conducted a study which showed that the new VCE mathematics subjects did not prepare students for the existing first-year Mathematics subjects at the University of Melbourne as had the Higher School Certificate Mathematics subjects (which the VCE subjects replaced). It could be argued, though, that the university courses needed to be brought into line with what the schools were doing, rather than vice-versa.

A review committee consisting of many of the major critics of the new VCE mathematics courses and of some of the main defenders of the new courses was established. The end result was a compromise, with VCE mathematics programs being accepted which are a combination of the old HSC courses (which tended to emphasise content and skills) and the new VCE emphasis on process and problem solving (Cox, 1994). From a "politics in mathematics education" perspective, a potentially serious and divisive problem was resolved by the interested parties being prepared to discuss the issues and having done that, being willing to compromise.

The Need to Cooperate

In the future mathematicians, mathematics educators, mathematics teachers, and education bureaucrats will often be called on to cooperate with respect to mathematics education curriculum and assessment issues. The expertise, integrity, and diligence of those who engage in debate over issues such as whether mathematical standards are falling in schools, or whether new courses are not preparing students adequately for higher mathematical studies, are beyond dispute.

Together, the interested parties need to work out solutions to problems associated with the "mathematics for the majority" situation which now prevails in the secondary schools. One of the main lessons of the national curriculum episode was that attempts by one or more of the groups to exclude other legitimate stakeholders will be counterproductive.

Policies, curricula, syllabuses, assessment practices and tertiary selection procedures are needed which give adequate recognition to the aspirations, needs, and thinking of the majority of children who enter primary and secondary schools while at the same time making sure that there is no dilution of standards.

The academics, for their part, will need to take seriously the view from many teachers in schools that abstract school mathematics curricula, which are only vaguely connected with the immediate needs of most learners, are no longer suitable for many secondary school students. Furthermore, the continued widespread use of pencil-and-paper tests based on these abstract curricula inevitably produce many mathematics education cripples, for whom the main lesson, learned from years of studying school mathematics, is that they cannot do it. A wider range of assessment procedures need to be used, at all levels (Clarke & Jasper, 1995; Morony & Olssen, 1994; Olssen, 1993).

There has been a strong belief among politicians and the business community that economic prosperity is intimately linked to having rigorous mathematics curricula. Another view, often propagated by professional mathematicians, was that standards of numeracy were falling, and that strong government was needed to reverse that state of affairs. Stakeholders need to discuss possibilities and cooperate in developing balanced solutions.

Business, Key Competencies, and Benchmarking

Alongside the national curriculum development, there was a second initiative known as the key competencies. This has been described by Collins (1995) as "a direct attempt to harness the post-compulsory school curriculum to ensure that all students improved those instrumental skills thought necessary if they were to become competent human resources for the Australian economy" (p. 10).

Like the statements and profiles, the key competencies failed to get endorsed at the July 1993 AEC meeting, but have since had considerable funding directed to assist their further development and acceptance. They appear to have had little effect on mathematics curricula in schools, but continue to receive considerable support from the National Industry Education Forum (e.g., Stanton, 1995), which includes among its membership the Business Council of Australia.

In 1995 the National Industry Education Forum, the Australian Council for Educational Research, and the Schools Council (which operated within DEET) jointly sponsored a visit by Professor Resnick to speak at a series of meetings on benchmarking in school education (Pope, 1995). The main thrust of these meetings was recorded in the June 1995 edition of *Unicorn*. What was clear was that the major players in this exercise were the same as those who had been the most ardent supporters of the profiles.

One of the authors (Thomas) attended the Melbourne meeting and it was clear that benchmarking against world's best practice, which may have some educational validity if approached in a sensible way, was not being seriously considered. What was being considered was benchmarking using the profiles.

In both Australia and New Zealand the situation had been reached where, irrespective of which political party held office, a tightly knit group of educational bureaucrats seemed to have convinced themselves that standards in schools could be significantly improved by the use of curricula and practices based on OBE principles. This group transcended party politics and its members tended to believe that desirable change was best achieved through the application of authority models, rather than through democratic, collaborative processes. What they said had considerable appeal to business and government.

It could be argued that, regardless of whether OBE practices in education are wise, the preference among some influential groups for trying to achieve change through authority models represents a threat to the present and future health of mathematics education and mathematics education research in Australia and New Zealand. A survey study by Swinson (1995) has shown that most Australian mathematics teachers only infrequently engage in professional development activities. The literature on changing schools indicates that it is only when teachers feel a degree of ownership of change initiatives that they are likely to change their practices in significant ways (Ferguson & Montgomery, 1994). Authority models are unlikely to generate the sense of ownership which is required.

An Example of Cooperation: The Strategic Review of Advanced Mathematical Sciences

The Australian Mathematical Sciences Council (AMSC)

Since the establishment, in 1989, of AMSC, there has been much more communication between Australian mathematicians, statisticians, mathematics educators and mathematics teachers (Ellerton & Clements, 1994b). The Council has met regularly and has enabled a collective voice to be put forward on many issues, including those associated with the development of the national mathematics curriculum. It would be wrong to suggest that this coming together of mathematicians, mathematics educators, and mathematics teachers has not generated tensions. Indeed, recently (January 1996) one of the foundation constituent members of AMSC, the Australian Association of Mathematics Teachers (AAMT), decided to withdraw from AMSC ranks. Two other societies however, concerned with Operations Research and

Bridging Mathematics, have become observers with the view to becoming full members.

One of the major benefits arising from the existence of AMSC has been that mathematicians, mathematics educators, and mathematics teachers have had a forum at which they were able to express their concerns, and to comment on how they believed these related to the concerns of other constituent members of AMSC. We have argued elsewhere (Thomas & Clements, 1994) that one result of the work of AMSC has been that mathematicians have gained a better appreciation of the work of mathematics educators and teachers, and vice-versa. In a brief case study we showed how the various interests represented in AMSC were able to combine to produce a much better result, for *all* concerned, on an important issue (Thomas & Clements, 1994).

AMSC is an umbrella group under the Federation of Australian Scientific and Technological Societies (FASTS), a political lobby group for science in Australia. Many aspects of FASTS' policy were adopted by both political parties in the lead-up to the recent (March 1996) Federal election.

Another Case Study

The Australian Research Council (ARC) initiates and co-sponsors reviews of major disciplines, and the most recent of these was in the advanced mathematical sciences. A report of the working party has recently been finalised (National Committee for Mathematics, 1996). This report was prepared by a collaborative process that stands in sharp contrast to the top-down consultative processes which were the cause of so much concern in the national curriculum project. It will be instructive to consider the process, the way in which mathematics educators were able to be involved, and the implications for mathematics education research that arose from this important and timely Review.

ARC discipline reviews are undertaken in collaboration with stakeholders, and the intention is to identify relatively long-term goals (10 to 15 years) for the relevant discipline. In the case of the Mathematical Sciences Review, the National Committee for Mathematics (NCM) of the Australian Academy of Science was the overall coordinating body. The NCM appointed a Working Party as executive body for the Review, and an Advisory Council helped develop the terms of reference and provided feedback on draft findings and recommendations. The Executive Officer and editor was Dr Barton, President of the Australian and New Zealand Industrial and Applied Mathematics group of the Australian Mathematical Society. Apart from the ARC funding provided to enable the Review to go ahead, there was no government involvement—the whole process of the Review and the production of the final report were controlled and managed by key people in the mathematical sciences community.

One of the writers of the present chapter (Thomas) is the current AMSC President and as such was a member of the NCM. The current President of the MERGA, Professor Leder, also contributed as a member of the Advisory Council. The process clearly took advantage of links which had been established between the various components of the mathematical sciences community in Australia. Consensus was reached, and although the issues primarily affected only some sectors of the AMSC community—in this case, the advanced mathematical sciences sectors—they were understood and supported by those who were less directly affected. At the launch of the report J. Thomas (1996) noted that, through the networks of the Council and the National Committee, the Review was able to give a more complete picture and—most importantly—was a consensus document.

In addition to indirectly supporting mathematics education research by drawing attention to the importance of advanced mathematical sciences in Australia, the report identified a number of areas where further work was clearly needed. Participation by MERGA members in the review process has assisted MERGA to develop its knowledge and strengthen its profile within the mathematical sciences community. The final report of the Review contained sections relating to school mathematics, gender, language, university teaching, and the need for increased participation in mathematics at all levels. Clearly, these are all important parts of MERGA's agenda, and should also be of interest to mathematics teachers.

In February 1996 the AMSC and the NCM, with support from the Australian Academy of Science, jointly sponsored a symposium to highlight one of the more important findings of the Review—the pervasiveness and importance of advanced mathematical sciences in all aspects of Australian life (National Committee for Mathematics, 1996).

On March 2, 1996, there was a change of Federal Government in Australia, and in the build-up to the election AMSC succeeded in getting the two major contending parties (Labor and the Coalition) to provide details of the level of support for their parties would be prepared to give to the mathematical sciences (Cook, 1996; Hill, 1996b), The *Campus Review*, a weekly university newspaper distributed throughout Australian and New Zealand, subsequently published an article under the heading of "The Mathematics Electoral Lobby" (Osmond, 1996). Specific support for the mathematical sciences was made part of the agendas of both main parties during political campaigns immediately before a national election. This was surely a remarkably successful outcome.

The national curriculum project clearly showed how a poor process can generate unhelpful squabbling and tension, and the Strategic Review showed how cooperation can generate compromise and mutual understanding. Constituent members of AMSC have collectively honed their political skills, but the revolution which would produce a cohesive mathematical sciences community is far from complete. The fact that AAMT has recently departed from the AMSC fold testifies to that fact.

As Jong (1995) has argued, in the process of discussing the difficulties that women still face in attempting to be good parents yet exercise their creativity to the full in their public lives: "Incomplete revolutions produce passionate and angry feelings" (p. 56). We should expect there to be more passion and anger before some of the political issues relating to the mathematical sciences are resolved.

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PART 2

ISSUES IN LEARNING AND INSTRUCTION

Constructivist Research On Teaching And Learning Mathematics

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Introduction

Whether one views constructivism as a philosophy (Ernest, 1993 p.87), a theory of knowledge (Bettencourt, 1993 p.39) or a theory of knowing (von Glasersfeld, 1993 p.23) it remains the most pervasive epistemology in mathematics education in the 1990s. Departing from the neo-Piagetian mainstream twenty years ago, constructivism has continued on a distinct path of development (Cobern, 1993 p.51) evolving into a broad theory that has acquired unique strength and power over alternative sets of narrow theories. The transmission model for instruction, that "knowledge can be transferred intact from the mind of the teacher to the mind of the learner" (Bodner, 1986 p.874), is no longer considered viable by mathematics practitioners and researchers alike and constructivism has become the referent for a considerable amount of mathematics education research.

Geelan (in press) identified the following six "forms of constructivism" which are representative of many different emphases within constructivist theory: (a) personal constructivism (e.g., Kelly, 1955; Piaget, 1972), (b) radical constructivism (e.g., von Glasersfeld, 1989), (c) social constructivism (e.g., Solomon, 1987; Renshaw, 1992; Solomon, 1994a), (d) social constructionism (e.g., Gergen, 1995), (e) critical constructivism (e.g., Taylor, in press; Taylor & Campbell-Williams, 1993), and (f) contextual constructivism (e.g., Cobern, 1993). Good (1993) suggests: "With the many versions of constructivism currently in use, we should be aware that one person's version is likely to differ from another person's version" (p.1015). He also points out the importance of trying hard to say clearly what we mean and to identify our biases up front.

Australasian reporting on constructivist research in mathematics education would appear to date from around the mid-1980s. This assumption is based upon the fact that the term *constructivism* does not appear anywhere in the proceedings of the 5th International Congress on Mathematical Education (ICME5) held in Adelaide, Australia in 1984, although some papers presented by American researchers at that Congress (e.g., Carpenter, 1984) did make passing reference to constructivist principles.

The elements of constructivism which have been used by teachers and educators for centuries refers to a theory of knowledge and learning. A new terminology, however, has been created in recent years, and this has been accompanied by a variety of interpretations which have been placed on the theory in mathematics education; for instance, the initial embrace of radical constructivism widened to include social constructivism and socioculturalism.

The authors of this chapter believe that a strength in the approach of much of the research reviewed here lies in the mix-and-match of different perspectives on constructivism in Australasian research. Our constructivist approaches have not always been new (consider the student-centred teaching approach of the Montessori method), but they do highlight our growth in the ways we have come to apply constructivist theory. An example of this is that we now understand that active learning includes interactive learning (Steffe & Wiegel, 1992, p.448) and that the social construction of knowledge occurs within groups of individuals and is concurrent with the construction of knowledge by the individual (Simon & Schifter, 1991, p.310).

Because there were many different aspects of constructivism considered in this chapter, our approach was to divide the research of the last four years into sections related to: (a) constructivist models for teaching and learning mathematics, (b) teacher beliefs and change, (c) constructivist approaches to assessment, (d) critical responses to the constructivist canon, (e) future research trends.

Constructivist Models for Teaching and Learning Mathematics

A constructivist environment is determined by the beliefs and the techniques that arise from the beliefs of the teacher. All learning situations can be constructivist, even lecturing (Mousley, 1992). It is these forms of teaching which become compatible with constructivist ideas that truly reflect the influence of this teaching approach in the modern mathematics classrooms (Cobb, Wood, & Yackel, 1990 p.146). A single model or definition of a constructivist learning environment does not exist as models and definitions arise from the actions of those who believe in the tenets of constructivism and who pursue those principles in their classrooms. "No simple rules for pedagogical practice emerge from a constructivist view of learning" (Driver, Asoko, Leach, Mortimer, & Scott, 1994, p. 11). Although constructivism has the potential to inform changes in mathematics teaching, it offers no particular vision of how mathematics should be taught (Simon, 1995, p. 114). A constructivist model requires a subtle shift in perspective for the individual who stands in front of the classroom. As the learning becomes more student-centred the teacher takes on the role of a facilitator, working to unite curriculum and student in a way meaningful for the learner (Bodner, 1986, p.877; Cobern, 1993, p. 51). Teachers' roles begin to shift from "telling and describing" to "listening and questioning" and "probing for understanding," although the shift occurs at varying rates and to varying extents among teachers (Maher & Alston, 1990, p. 161).

Pateman (1992) makes use of his experiences with the Hawaii Geometry Learning Project in making a case for a new paradigm in mathematics education. It is based on the epistemology of radical constructivism, a growing understanding of the importance of social interaction in the creation of knowledge, the acceptance of a new philosophy of mathematics known as "quasi-empiricism," and the use of social theory/ideology of critical theory. The latter provides a different perspective for teachers and mathematics educators for investigating and thinking about the processes of research and curriculum development through involvement in collaborative research.

Fetherstonhaugh's doctoral study (1993) involving two Year 9 classes utilised a constructivist learning model based on Kelly's (1955) personal construct psychology. A comparison of the treatment class with a class taught the same content in a traditional manner indicated that students from the constructivist classroom learnt the content as well as those taught traditionally. They translated formal school science into personal knowledge without any loss of scientific acceptability and became cognitively more complex individuals. Teaching style influenced student learning outcomes significantly.

FitzSimons & Sullivan (1992) outlined the philosophy, the constraints, and the practice of one mathematics program in a vocational post-secondary education course, and the researchers' attempts to present this course by way of hands-on experiences, negotiating meaning through interaction with others and through justifying their answers to problems. The results of the study indicated that these approaches led students to develop a positive appreciation of mathematics. A later study by FitzSimons (1993) concerned the positive effects a mathematics classroom conducted along constructivist lines had upon the life of one woman returning to study after leaving school at the age of 15. The successful approach involved boosting her selfconfidence and improving her subject knowledge .

Malone and Taylor's edited book (1993) on the proceedings of the Constructivist Topic Group which they organised at the 7th International Congress on Mathematical Education (ICME7) in Quebec in 1992 contains papers from 10 Australian researchers in this field of study. Also, their chapter in the formal proceedings of ICME7 (Malone & Taylor, 1994) summarised all the research reported in their Topic Group. Mildren's contribution to the first of these proceedings was the report of a study of 10-, 11-, and 12-year-old children (Mildren, 1993). It describes how a constructivist classroom environment provides the opportunity for metacognitive development through allowing students to construct mathematical thought actively and to reflect on their thinking. He concluded that the teacher's role should include the creation of a broad environment within which children are able to explore mathematical contexts and construct knowledge and understanding as they interact with their peers, their teacher, and these contexts. The teacher's role also requires dialogue with children so that classroom discourse is two-way, with the teacher listening to what children are saying. The constructivist environment created in this fashion provides the opportunity for metacognitive development because it allows the individual to actively construct mathematical thought and to reflect on his/her thinking.

Elliot and Hall (1993) described two studies involving the mathematical learning of "at risk" preschoolers in computer-supported interactive environments. Their investigations explored the effectiveness of a metacognitive teaching approach which provided a valuable scaffolding for the development of cognitive competence in mathematics. Results indicated that children who participated in mathematical activities within these contexts performed significantly better in tests of mathematical competence than did children engaged in more regular instructional approaches. Importantly, their approach encouraged planning, reflection and evaluation as children constructed their own knowledge.

A study by Green (1992), based on his doctoral research, recommended the teaching of instructional strategies along with examples and problems for teaching specific mathematical topics. His approach was to select carefully examples and problems which lend themselves to a constructivist epistemology incorporating the principles of building upon students' prior knowledge, group work, negotiation, consensus and the provision of time-in-class to allow students to reflect. He reported a successful intervention with a significant improvement in gain scores among his experimental group.

Mailes (1995) in his Masters research set out to assess the effectiveness of the Australian Maths Challenge program in enhancing student problem solving. Using the Maths Challenge as a framework, an enrichment program was set up using a constructivist approach to teaching for an experimental group of students. Quantitative results indicated that when compared with a control group which had received standard teaching, the experimental group improved their performance on a criterion test. Qualitative results, however, derived from the use of a questionnaire on attitudes to problem solving linked to student interviews, provided little evidence of any change in attitudes due to the enrichment program.

A study by Mousley (1992) explored the notion of constructivism by attempting to determine how a number of teachers and their students would interpret and develop a given activity, namely how to construct a cardboard shape and then to construct another twice as big. She referred to the usefulness of this activity in examining the notion that knowledge originates from activity with objects. She believed that many teachers have interpreted this claim by thinking that the provision of concrete materials or of hands-on activities is all that is required to relate mathematical concepts to the real world of the student. The results reinforced the notion that teachers must be aware of the need to regard objects not so much as concrete materials, but rather as vehicles for developing mental constructs. In another study by the same researcher (Mousley, 1993), classroom techniques and pedagogical ideas of an earlychildhood teacher were examined. The teacher's practice was discussed in terms of current educational theories—constructivism, problem-posing pedagogy, and the inclusive curriculum—leading to the conclusion that less teacher controlled and less formal organisation can lead to valuable learning for all stake holders.

Pearn and Hunting (1995) described their Mathematics Intervention Program which commenced in 1993, and which was designed to assist those children in Years 1 and 2 at risk of not coping with the mathematics curriculum. The program involved teachers interviewing all children in Years 1 and 2 in a school, making instant appraisals of the child's needs, providing the appropriate activities and then using tasks based on the five counting stages developed by Steffe, von Glasersfeld, Richards and Cobb (1983) to fulfil these needs. After 20 weeks in the program all children had improved their verbal counting skills and progressed one counting stage.

In an article on their New Zealand project entitled Understanding Learning and Teaching, Alton-Lee, Nuthall and Patrick (1993) explain how they explored 6th grade children's public and private experiences recorded by means of individual microphones during lessons. The sociocultural context of the classroom was found to have a profound effect on the way the curriculum was translated into the child's personal beliefs and knowledge. Children's class, race, and gender were found to have a powerful and often contradictory effects on the way a child makes sense of the curriculum.

Nuthall (in press) has produced a most detailed overview of student thinking and learning in the classroom which points out and describes the radical changes in conceptions about the phenomena that have taken place since the mid 1980s. His collection of recent studies of student experiences in classrooms in order to identify the common themes and issues is too extensive to review here. They covered studies that were primarily (a) psychological, (b) sociocultural, and (c) language or sociolinguistic in orientation. His message was that the study of single variables and the production of simple rules for improving teaching are bound to fail because of the enormous complexity of student experiences in the classroom. His solution lies in the development of new methods of research.

An awareness of constructivism and child centred learning and teaching approaches led Burnett (1993) to the implementation of instructional games into a classroom of children in their first year of formal schooling. The researcher's goal was to develop children's mathematical concepts by using language as a bridge between children's informal mathematical knowledge and abstract mathematical concepts. The success of the study was attributed to the discussion that stems from the games, the ability of the teacher to individualise instruction, and the social context in which the games were played.

Working within a constructivist paradigm does not imply a particular style of teaching, rather it implies adopting a style or process that facilitates the students in their construction of knowledge. Social exchanges in groups which formed part of Ireland's (in press) examination of his classroom practices is one such example. More Vygotskian and socioculturalist than constructivist, Ireland's study set out to test the rigour of the theoretical perspectives by adapting them into his personal pedagogic practices. According to Ireland, a Vygotskian perspective is compatible with constructivist pedagogic goals, for Vygotsky sees mediation (by various means) as the link between knowledge and learning, and such a view makes it possible for constructivist theory to be transformed into practice founded on its tenants. Ireland's study viewed constructivism as a referent which enhances a collaborative peer-interactive learning environment such as that which he developed in his study. Constructivism was seen as new knowledge being based on what the learner already knew, so that by collaborating in their zones of proximal development peers supported each other based on what they knew. This developed a "classwide" zone of proximal development that helped the students build greater understanding of their mathematical knowledge. This knowledge represented the students "taken-as-shared" understanding of the knowledge within the greater mathematical community. It was a product of that classroom's social and cultural diversity.

In learning mathematics (in the context of coming to terms with the taken-as-shared mathematics culture), the students' classroom environment was the main part of their sociocultural experience within which that learning occurred. The measure of success of Ireland's collaborative peer-interactive classroom environment lay in the student's perceptions of their environment. By acknowledging the importance of the role of social interactions, Ireland lent towards social constructivism rather than radical constructivism. Awareness of cultural differences further moulded this perspective into that of socioculturalism. This 'paradigm shift' by Ireland seems to reflect the evolution of constructivism extremely well.

Brown's study (1995) of 18 sixth-grade girls provided evidence to support the efficacy of a model of collective "argumentation" - referring to any type of discourse in which the principal goal is to find collective solutions to interindividual problems of coordination - based on the concept of social cooperation in the classroom. The model was shown to have the potential to direct a group's logic of argumentation in such a way that collective thinking is facilitated, so enhancing the children's appropriation of mathematical knowledge and the development of higher mental functions.

Teacher Beliefs and Change

In recent years, a considerable amount of research has been reported world-wide on efforts to encourage both practicing and pre-service teachers to restructure their teaching in order to adopt the vision of the National Council of Teachers of Mathematics (NCTM) *Standards* documents (1989, 1991) from the United States, and the Australian Education Council's *A National Statement on Mathematics for Australian Schools* (1990). Recommendations related to curriculum content, expectations of student learning, and instructional environments. Australasian researchers have been keen participants in this research effort. Britt, Irwin, Ellis, and Ritchie, (1993) conducted a case study on a teacher who incorporated into her teaching practice new approaches and organisational strategies that were consistent with constructivism. These shifts in practice—using observations to help identify low spots in teaching, encouraging students to take more responsibility, extending the use of groupwork, and so on—resulted in a change of her beliefs about how students might best learn mathematics.

Another study on teacher change (Mayers, 1994) sought to determine whether modifications could be induced in primary student-teachers' beliefs about, and attitudes towards, mathematics and mathematics teaching through their participation in a mathematics education course which adopted constructivism as its theoretical framework. Scales which were administered before the start of the course and after its completion were used to measure changes in students' attitudes and beliefs. The results indicated that students demonstrated a significant shift towards a constructivist perspective and there was a significant reduction in mathematics anxiety. On the other hand, a study by van Zoest, Jones, and Thornton (1994) reported that pre-service teachers who were involved in a program designed to engender positive beliefs about a socioconstructivist classroom environment had difficulty translating these beliefs into long term practice.

In a recent study, Perry, Geoghegan, Owens, and Howe (1995) discussed the nature of cooperative learning and the experiences of teacher education students. These matters were debated and interpreted by focussing on contemporary perspectives and challenging traditional views of the roles of social interaction and communication in learning mathematics. In an analysis of students' learning (Owens, Perry, Conroy, Geoghegan, & Howe, 1994) showed that students' attitudes towards mathematics and themselves doing mathematics improved. Case studies showed how the classroom, compatible with a constructivist view of knowing, influenced students' attitudes and problem-solving heuristics.

Taylor (1992) reported on a collaborative action research study in a Year 11 mathematics classroom. The results of the study suggested that pedagogical reforms which are based largely on a cognitivist theory of constructivism are susceptible to being subsumed by a powerful technical rationalist ideology, and that the hegemonic nature of this ideology is capable of disempowering reform-minded teachers from realising the practical viability of their constructivist related ideals.

Klein (1995), in reporting on her study involving the implementation of a preservice teacher education course in mathematics, discussed her concerns about the pedagogical practices based on psychological notions of a constructivist approach which was used. She considered that she failed to appreciate the implications of the sociocultural and political constitution of the students and proposed that the learner be thought of as discursively constructed.

Constructivist Approaches to Assessment

The knowledge that learning takes place as a constructive process has far reaching implications for assessment. For instance, it shifts the focus of instruction away from the delivery of content to an attempt to understand learning from the student's perspective. Leder's edited book on assessment and the learning of mathematics contains articles from 11 Australasian researchers, several of whom have focussed their research on the constructivist-assessment link. Doig and Masters (1992a), for instance, referred to The Basic Skills Testing Program introduced into NSW government schools in 1989 when discussing assessment aspects of children's literacy and numeracy. This program, which is used to assess around 70 000 students per year, uses "maps" to determine and analyse results; to display students responses to assessment items for the teacher's benefit, and to follow-up student responses in one-to-one interviews. Through these means it is more likely that classroom activities are introduced in a sequence meaningful to students rather than in a "rational" sequence determined solely by teachers and curriculum developers. In a separate account, Doig and Masters (1992b) provided detailed information on the construction and use of these maps.

Burton (1992), in describing her model of the curriculum portrayed it as consisting of three dimensions: the syllabus, the pedagogy, and the assessment/evaluative effects. In focussing specifically on what Burton had to say about assessment, this researcher believed that assessment must only be rooted in the mathematics that learners are building and learning. A style of assessment that is consistent with the learning that is taking place must be used, and it is essential that this style be negotiated between teacher and students. Teachers need to experiment with novel ways of assessing mathematics and collecting information on how these ways effect the learning outcomes of students.

Conducting a case study of a single secondary mathematics classroom, Clarke (1992) postulated a relationship between assessment and mathematical performance to discuss assessment as a mechanism for the social construction of mathematical competence. The case study data was used to illustrate the role of assessment in the classroom—it should be constructive in the nature of the tasks it employs, constructive in its reconciliation with instruction, and constructive in that it must inform the actions of the school community. A study by Ritchie and Carr (1992) described what they referred to as a "constructivist critique" of mastery learning in mathematics. They argued that mastery learning programs do not empower learners to be assessors of their own mastery. They suggested that constructivist assessment techniques are more likely to develop a psychological sense of mastery in learners.

Critical Responses to the Constructivist Canon

While the researchers and writers referenced in this chapter have, to this point, all emphasised the important role that the constructivist approach to teaching and learning mathematics is playing in today's schools, not every member of the Australasian research fraternity is equally enthused with the manner in which the theory has been adopted, interpreted, and put into practice. Whilst not going as far as some overseas writers in suggesting that the constructivist program is now in a state of gradual decline (Solomon, 1994b), several Australasian researchers have expressed their concerns. Galbraith (1992) cautioned that "close scrutiny can reveal feet of clay that is demoralising to loyal disciples," and he raised a series of questions and issues that theorists and practitioners should consider. These included such matters as the utility of the epistemological foundations underlying constructivism in relation to the tensions generated in the context of practice, and the anomalies existing in selected areas of traditional research such as concept development in view of its conventional interpretation in relation to constructivist logic.

Other researchers, for example Ellerton and Clements (1992), have reported on weaknesses in various interpretations of constructivist theory, such as the downplaying of linguistic activity in cognitive development. Ellerton and Clements identified the positive features of radical constructivism's contribution to mathematics education as being characterised by: (a) its contribution in assisting learners to construct mathematical meanings themselves, (b) its advocacy of quality social interaction, and (c) its identification of principles for improving mathematics teaching and learning. They also identified a number of negative characteristics of the theory, for example: (a) the "missionary zeal" evident among many of its proponents, (b) a downplaying of the role of linguistic activity in abstract thought, (c) an oversimplification of the answer to the question of "what is mathematical knowledge"?

Klein (1993) reported on an action research project, the data from which demonstrated constructivism's vulnerability to censure for a perceived lack of recognition of the effects of culture and power inherent in all discursive practices. Students, after participation in constructivist practices, appeared to gain no enhancement of the epistemological messages concerning mathematics and mathematics teaching that they had possessed before the intervention. Klein's more recent study (1995), reported earlier in the section on teacher beliefs and change, expressed her concerns about implementing pedagogical practices based on constructivist tenets of students' active and autonomous construction of meaning in the supportive environment of a pre-service teacher education course in mathematics. Aboriginal and Torres Strait Islanders in particular found the study of mathematics threatening, and it seemed that the constructivist, problem-centred pedagogical practices were particularly difficult as these did not "gel" with culturally-based understandings of the manner in which mathematics should be taught to these people.

One difficulty facing the constructivist teacher is that "running with the student's idea" and "tying knowledge to well-established structures" might well be too difficult a task to achieve (Arcavi & Schoenfeld, 1992, p. 331). The teacher does not want to appear to be betraying the student-developed ideas but must eventually have the class in agreement with the world view, that is taken-as-shared view of mathematical knowledge.

Future Research Trends

Constructivism can offer some useable tenets and theoretical foundations for everyday classroom use but the theory was never designed to be everything that philosophy and pedagogy has demanded. It is a theory of knowing and of learning, not a theory of teaching and instruction. "If research and theory building are divorced from teachers' re-structuring of their classroom activity, the socially-maintained conceptual and discursive frameworks by which 'teaching' and 'mathematics' are constituted as traditional practices will continue to inhibit the implementation of essentially different ways of working" (Mousley, 1992, p. 3). Future research will need to see a constructivist model applied to real classrooms. "If constructivism is to mean anything, it must mean that the theorist is irrevocably involved in life, in social interaction, in learning—in the very things the theory purports to explain. This being so, there is no meta-theoretical perspective, no 'outside' from which to understand the activities of teaching, learning and research" (Geelan, in press, p.13). Steier (1995) suggested that teaching, learning, and research must merge to become collaborative social learning. From a constructivist viewpoint, research becomes learning.

Another path lies in exploring ways of integrating various constructivist and neo-constructivist perspectives in mathematics education. Cobb (1994) argued that neither an individual student's mathematical activity nor the classroom microculture can be adequately accounted for without considering the other, and that by coordinating constructivist and socioculturalist perspectives we can explore the view "that learning is a process of both self-organisation and a process of acculturation that occurs while participating in cultural practices, frequently while interacting with others" (p.15).

Taylor (in press) is currently engaged in research based upon what he refers to as a "critical constructivist" viewpoint which appears to be moving in this direction. This form of constructivism recognises that the social reality of the mathematics classroom is governed by powerful myths that restrain the discursive practices of both teachers and students and compromises the reformist ideals of individual teacher and student empowerment. Taylor identified cultural myths such as "cold reason," "hard control," and "the teacher as Robinson Crusoe" which act in concert to create and conserve a highly coherent and seemingly natural social reality. The power of critical constructivism, he believes, lies in its weaving together of theories of social groupings (cultural anthropology), theories of valuing (axiology) and theories of knowing (epistemology) to provide a framework within which teachers can consider the origins and efficacy of the beliefs that shape their classroom roles as agents of enculturation.

Taylor (in press) has urged mathematics educators to become concerned with research on how they might enable teachers to construct alternative cultures of schooling and school mathematics in which students' lifeworlds, the ethics of classroom discursive practices, and the moral agency of the teacher are central to planning and conducting mathematical learning activities. He identifies this need as paramount in cross-cultural situations, particularly non-Western developing countries where the hegemony of Western curricula is at its strongest in driving a wedge between indigenous students and their "natal cultures" (Waldrip & Taylor, in press).

Conclusion

It was suggested at the beginning of this chapter that important changes to our understanding of student thinking and learning in the classroom have resulted from research into constructivist approaches to teaching and learning mathematics. The research and theory described in this chapter should make the nature of this change apparent. What is clear from the research reviewed here is that the focus has been largely on the primary grades (Confrey, 1994, p.6). Perhaps this is because the constructivist approach is so much more at odds with common secondary and post-secondary practices, at least in Australia. Elementary or primary schooling is oriented towards the individuals or students as people and so is more accepting of the student-centred approach which constructivism extols. Secondary and post-secondary practices are much more strongly linked to their subject and are, consequently, a more formidable challenge to adapt to constructivist practices. Perhaps we will witness a shift towards upper-school research in the years to come. Whether or not this occurs, it is clear that constructivist theory in its many forms is the subject of considerable interest in Australasia at the present time, and it appears set to continue in this fashion for some time to come.

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Mathematical Cognition and Instruction

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This chapter reports on Australasian research, since 1992, that has addressed mathematical learning and teaching from the perspective of human cognition, skill acquisition, and performance. We review research conducted within this general area but keep the focus on studies that attempted to develop explanations about mathematical cognition and instruction.

Given that theories of cognition and instruction can be applied within a range of contexts to examine students' learning, some of the studies reviewed here might appear in other chapters of this book under a different emphasis. Our focus here is on studies that highlight cognitive elements underlying mathematical learning and teaching, and on studies that provide interpretations that are grounded in cognitive theories. A number of these studies have analysed the cognitive demands imposed by (a) the structure of mathematical analogs and how they are used, (b) the solution strategies adopted by students, (c) the nature of the problem representations preferred and constructed by students, and (d) the type of errors committed during the course of mathematical problem solving. Studies that deal with the above areas are reviewed under the general theme of mathematical cognition.

The second part of the review considers studies in which the investigators explored students' knowledge construction and mathematical reasoning in both formal and informal learning contexts. We were also concerned with studies exploring the interaction between students' existing mathematical knowledge and their accessing of, and reasoning with, this knowledge. In the final section of this review, we devote our attention to studies that have used understandings gained from analyses of mathematical thinking in order to develop and assess various instructional formats.

Mathematical Cognition

Since 1992, a major concern of researchers in the Australasian region has been to develop explanations about why students experience difficulties in performing mathematical tasks, at all levels of schooling. This concern has led to a series of investigations in which researchers analyse the cognitive load imposed by various mathematical representations and its impact on students mathematical development. A major assumption in these studies is that the potential for mathematics learning is closely related to processing capacity, and that one can attribute some of students' learning difficulties to mathematical situations that impose too high a cognitive load.

Boulton-Lewis and Halford (1992) provide an important starting point for the examination of processing loads and mathematical concept development. Their structure-mapping theory of cognitive development is predicated on the assumption that the cognizing of a new mathematical concept entails the construction of a mental model that reflects the structure of the concept, and that the structure of the concrete representations used by teachers and students must mirror the structure of the concept to be understood. Their theory identifies four levels of structure mapping: element mapping, relational mapping, system mapping and multiple system mapping. The authors provide an illustration of these levels by analysing the mappings that must be made between the analog representation of a two-digit number, its number name, and its written symbol. Boulton-Lewis and Halford argue that all these different mappings impose a processing load, and unless each is well learned and automatised, the total processing load will be too great for the child and will interfere with his or her understanding of place value. English and Halford's (1995) book, *Mathematics education: Models and processes*, modifies and extends the structure mapping framework and presents a number of principles for learning by analogy. These principles are applied to a more comprehensive analysis of mathematical analogs (concrete and abstract) and to an analysis of students' learning in other mathematical domains.

Using the structure-mapping theory, Boulton-Lewis (1993a, 1993b) assessed the processing load imposed by the analogs and strategies used by teachers and children in dealing with the subtraction operation. A sample of 29 children, aged between 5 and 8 years old, and three teachers participated in the study. The teachers were required to identify those aspects of subtraction that they intended to teach and the representations they intended to use. Children were interviewed individually before and after the instruction on a range of subtraction items to determine their use of representations and strategies. The children's representations were coded according to the type of analog used (e.g., counters, Multibase Arithmetic Blocks base 10, fingers) and how they were used. The strategies employed by the children included forward counting, backward counting, and recall of number facts. Analyses of the children's representations and strategies showed that in Year 1, the largest number of responses involved the use of analogs without overt counting. Among the Year 2 students, the most frequently used strategy also involved the use of analogs without counting. However, in both these year groups, many children made use of the analogs incorrectly. Students from Year 3 used the recall strategy most often compared to the analog strategy. These results suggest that children either prefer to use strategies that involve concrete representation or when they do invoke a concrete mode of representation, they tend to make mistakes. Such behaviour could be due to the fact that many students do not understand the relationship between concrete analogs and symbolic representations of the tasks.

Boulton-Lewis argued that students' difficulties with new representations and strategies could be attributed to the increase in processing load caused by students having to attend to items of information which have not been understood well in the first place. The students in her study did not appear to use concrete representations to construct mental models of given mathematical situations. Teachers' responses revealed that the representations and strategies they chose had the potential to increase processing load, which could have contributed to the children's incorrect responses. It is recommended that when teachers use representations that are assumed to be beneficial to student learning, then they should use them in a manner that makes the links between the concrete materials and mathematical concepts more explicit.

The issue of the interaction between representations, strategies, and information processing capacity is further addressed in two other studies involving subtraction and addition tasks. Boulton-Lewis and Tait (1994) investigated the representations and strategies for addition tasks employed by a sample of 55 children in Years 1, 2,and 3. Children's responses were analysed in terms of three strategy categories: material (counting using analogs), verbal (counting without analogs) and mental (recall of memory facts). Results of this study showed that, in general, children preferred to use verbal and mental strategies indicating a disinclination to use analogs unless they could not perform a given task in any other way. The authors found that, despite teacher intervention to expose students to useful forms of representations requiring analogical reasoning, children continued to use other modes of representations and strategies. These results are similar to those reported by Boulton-Lewis (1993a, 1993b) involving subtraction tasks. The issue of how concrete analogs are used to enhance the development of mental models of mathematical concepts is an important one, and clearly further research is needed here. In particular, attention needs to be focussed on how we can assist children in constructing meaningful links between different mathematical representations.

Students' construction of mathematical problem representations and strategy use was investigated from an alternative point of view by Collis, Watson, and Campbell (1993). Their major concern was to examine the use of intuitive ikonic processes during problem solving as opposed to the commonly utilised concrete mode of thinking. A sample of students was given two sets of problems to solve. The first set was expected to attract a concrete symbolic approach and the second set, intuitive ikonic thinking. On the basis of the results, the authors concluded that the nature of the problem plays a major role in determining the type of strategy one adopts. For instance, students consistently applied concrete-symbolic strategies to concrete-symbolic problems, and ikonic strategies for ikonic problems. Students were also found to switch to an ikonic mode of analysis when the concrete approach failed to yield the answer. The authors reported that students who scored high on vividness of imagery used the ikonic approach more frequently. It appears that teachers intending to foster flexible problem solving approaches need to facilitate the ikonic mode of problem representation. The authors did not provide an explanation as to why students prefer to use the concrete symbolic approach. However, this question was addressed more directly in the following study.

In this study Campbell, Collis, and Watson (1993) investigated the interaction between concrete symbolic and ikonic functioning by studying two different aspects of visual processing associated with mathematical problem solving. Visual processing skills were provided in two modes: (a) use of mathematically based visual images and diagrams, and (b) incidental visual imagery associated with problem stories. Four groups of Year 10 students (n=50) were assigned to the various groups representing high and low concrete symbolic ability and high and low ikonic ability (vividness of visual imagery). Students were interviewed and their responses to four problems were recorded. Two of the problems required the use of diagrams or visual images while the other two could be solved by the use of concrete symbolic processes. Analysis of the students' responses to three of the problems led the authors to conclude that success in both novel and school-based problems could be associated with a concrete symbolic mode of processing rather than to individual differences in ikonic processing abilities. Again, as reported in the previous study, the students appeared to prefer to use a concrete symbolic processing mode. Also, it was pointed out that while the mathematically-based visual images and diagrams used in the study have ikonic elements, their production is driven more by students' concrete symbolic understanding of the structure of the problem. Incidental visual imagery associated with problem stories was argued to be related to individual differences in ikonic ability, and to be unrelated to ability in concrete symbolic processing.

Campbell et al. (1993) suggested that although incidental, non-mathematical visual imagery could contribute to the motivational aspects of problem solving, and that it has little value, if any, in the development of a concrete symbolic mode of thinking. These are important results but one is left wondering whether the two modes of processing are fundamentally different or whether they have common elements. The authors pointed out that, although one could detect mathematically-based visual imagery in ikonic representation, this mode of representation does not appear to play any critical role in understanding the structure of the problem. If that is the case, we need to examine the nature and role of ikonic representation in mathematical problem solving more closely. Such studies also have the potential to throw new light on the issue of the relationship between concrete symbolic and ikonic mode of thinking; clearly, further investigation is needed in this area.

The notion of problem representation and how it could constrain processing of information was the focus of the study reported by Stillman (1994). In this study, the author used the technique of mapping to categorise students' solution strategies and problem representations. This technique involves the analysis of mathematical responses to reveal their underlying structure and errors. Analysis of the solution transcripts revealed that students experienced difficulties in applying known facts and formulae in constructing a viable model of a given problem. Stillman (1994) argued that students could not make the connection between the various bits of information located in the problem representation. The results of this study show that the utilisation of prior knowledge during the construction of problem representation is a complex activity, and that research needs to focus on the processes and the quality of representations generated by students. Fine-grained data on these aspects of problem representations has the potential to improve our understanding of why students fail to connect individual items of information located in their representations.

Examining mathematical problem-solving difficulties from the perspective of processing load was also the focus of Ayres' (1995) study. He argued that many of the slips (careless errors) made by students during performance could be attributed to working memory difficulties, the suggestion being there is a positive correlation between increase in memory load and the number of slips. In this study, Ayres tested the hypothesis that if the use of calculators would reduce the load on working memory then students using calculators would show a decrease in the number of errors that can be described as slips. A comparison was made between the performance of students with and without calculator access on bracket expansion tasks. An analysis of their errors showed that the use of calculators did not appreciably reduce the number of slip-type errors made. The author found that many of the errors made by students were slips rather than bugs which are errors caused by the use of incorrect algorithms. It was thus inferred that the use of calculators did not have any appreciable effect on the reduction of memory load. Although the author pointed out the need to exercise caution in making generalisations on the basis of this pilot study, this would seem an important starting point in our efforts to advance a more cognitively defensible argument in favour of or against the use of calculators in the solution of linear algebraic problems.

In sum, the studies reviewed in this section showed that there is a great deal of research interest in the exploration of cognitive issues involved in mathematical learning. Specifically, researchers have focussed on the examination of relationship between the nature and use of concrete materials and their effectiveness in developing mathematical understanding. This issue has also received support in reforms advocated by major curriculum documents. In general, the findings of studies reported in this section indicate that concrete material, if used appropriately, could enhance mathematical understanding and problem solving. On the other hand, instructional approaches that do not examine the cognitive load implications of utilising concrete representations could impede the development of important mathematical concepts. A number of these studies also have implications for theoretical developments which needs greater articulation and investigation, particularly in the areas of mapping and problem representation.

Mathematical Knowledge Construction and Reasoning

In this section we review studies addressing students' knowledge construction and access in the areas of algebra and geometry, and studies focusing on the mathematical reasoning of primary school students in the problem-solving domain.

Studies on the nature of expert performance in selected domains have provided considerable impetus for researchers who are interested in the examination of mathematical performance. Such studies have contributed to an emerging consensus that there are three factors identified with expert performance: the extent and quality of the domain knowledge base, activation of executive or control processes, and the dispositional state of the individual. Mathematics educators have shown a great deal of interest in students' construction of mathematical knowledge, with a number of studies focussing on how students access and manipulate their knowledge during task performance.

In an analysis of the interaction between cognitive processes and knowledge base during mathematical problem solving, Sweller and Low (1992) presented convincing evidence in support of the argument that, in addition to teaching mathematical rules, instruction should find ways of helping students acquire a large number of schemas. Schemas are defined as clusters of organised mathematical knowledge considered to facilitate problem decomposition and classification. The authors draw on their research on geometry and algebra problem solving in order to provide recommendations on how teachers could design integrated worked examples that would have the potential to reduce cognitive load and thus enhance mathematical learning.

In a major study addressing geometric knowledge and the use of this knowledge during problem solving, Lawson and Chinnappan (1994) focused on the generative activity of a sample of Year 11 students (n=10) as they attempted to solve a series of four plane geometry problems. The investigators provided an estimate of the quantity of geometric knowledge that students were able to recall, and observed the use of this and related knowledge during the solution process. A comparison of the performance of high- and low-achieving students showed that the high-achieving students not only had more knowledge to retrieve than their low-achieving peers but more importantly these students made more use of knowledge that was accessed in order to produce new information. This event, referred to as *generative activity* seemed to contribute to the superior solution attempts of the high achievers. It was also noted that high-achieving students showed a greater tendency to monitor and direct the flow of new information that was generated in light of information provided in the problem than their low-achieving counterparts. Taken together, these results suggest that accessing of prior knowledge and the controlled use of that knowledge during the course of the problem solution are important ingredients in a productive solution attempt.

The issue of controlled and directed generation of information was also investigated in a study conducted by Cai (1994). The study compared the metacognitive behaviours of two subjects having high level mathematical experience with those of two subjects who had low mathematical experience. Mathematical experience refers to the level at which one has studied mathematics, and the extent of one's experience with higher mathematics. Examination of students' solution protocols revealed that, unlike the low-experience subjects, the high-experience subjects engaged in self-regulation. It appears that processing activities that can be identified with self regulation have the potential to help the solver generate, organise, and manipulate information in the problem space.

Chinnappan and Lawson (1994, 1995) hypothesised that access and flexible use of knowledge could also be the consequence of the organisational quality of students' geometrical knowledge base. On the basis of this assumption, the investigators developed a model of geometry knowledge organisation by identifying the different levels at which that knowledge could be extended. Five different levels of knowledge extension were hypothesised in this study: basic, forms, rules, application, and elaboration. The extension of knowledge at each level is argued to be dependent upon knowledge items constructed at the previous levels, with the basic level providing the initial knowledge building blocks. The sensitivity of this model was assessed by comparing the levels of knowledge extension exhibited by a sample of high school students. The results of this analysis showed that students who were classified as high achievers tended to exhibit geometry knowledge that was better organised particularly at the higher levels of knowledge extension than the low-achievers.

In another study, Chinnappan and English (1995) explored the potential relationship between the quality of students' prior mathematical knowledge and how this knowledge is accessed and used in solving high-school geometry problems. A sample of 30 students was asked to solve a trigonometry problem. The solution path adopted by each of the students was analysed in order to identify the types of schemas that were accessed during the solution attempt. The results of this analysis showed that the high-achieving group not only retrieved a greater variety of such schemas than their low-achieving group, but they also activated more than four times the number of schemas than their peers. The high-achieving group also used the schemas in a manner that suggested that the students were aware of the relationship between the various components of the problem's structure, something that was lacking in the efforts of their low-achieving peers. It was concluded that these high achievers displayed deep processing of information in contrast to their low-achieving counterparts who displayed surface processing. That is, the latter group attended to discrete and unrelated items of information and retrieved information at random without understanding its relevance to the problem at hand.

A number of studies examining students' mathematical reasoning processes have been conducted by English (e.g., English, in press a, b; English & Halford, 1995). One set of studies addressed the combinatorial reasoning processes of 4 to 12 year-old children (English, 1992a, 1992b, 1993a, 1993b, 1994; English & Halford, 1995). The children were individually administered sets of hands-on problems involving two-dimensional (X x Y) and three-dimensional (X x Y x Z) combinations. An analysis of the children's responses showed they could apply sophisticated reasoning processes to transform their initial, inefficient trial-and-error procedures into efficient combinatorial strategies. Children's use of metacognitive processes assisted in this strategy construction, which reflected their increased understanding of the combinatorial domain. The importance of children's metacognitive processes was evident in cases involving problem failure in the face of sophisticated domain knowledge.

In another study, English (English, 1993b, submitted; English & Halford, 1995) explored 9-12 year-olds' reasoning processes in solving novel mathematical problems of deduction. Case studies showed that children's success depended largely on how they interpreted the premise information (i.e., nature of the mental models they constructed) and how they handled the complexity of the relations involved in problem solution. Also of importance was whether children considered alternate possible models during the solution process, whether they checked and validated their models, and whether they made effective use of external representations. Among the findings were the complex reasoning processes shown by children in Grades 4 and 5, compared to their counterparts in Grades 6 and 7. The latter relied on external representations (e.g., table, matrix), which they tended to use in a proceduralised manner with little application of important reasoning processes. The study results highlight the importance of actively encouraging children's continued development of mathematical reasoning across the primary grades. In particular, we need to ensure that children do not rely solely on specific skills and procedures at the expense of important reasoning processes.

The algebraic reasoning of senior secondary students was the focus of another of English's studies (English & Sharry, 1996). She presented a theory of the development of algebraic abstraction which extend Sfard's (1994a, 1994b) and Mason's (1989) ideas on the learner's progress from operational or process oriented thinking to the abstract or structural perspective. The theory incorporates a process of analogical reasoning to account for the means by which the learner might construct expressions of generality and subsequently manipulate them as mathematical objects. Such reasoning entails similarity comparisons in which a mapping is made between the corresponding relational properties of algebraic examples. English applied the theory to an analysis of secondary school students' approaches to classifying a set of complex equations. A student who appeared capable of algebraic abstraction within the domain of the task was contrasted with two students who were at a "pseudostructural" stage, where their focus on syntactic surface structures prevented them from forming the relational mappings needed for the construction of generalised models of the equation types.

To summarise, studies that examined ideas about knowledge construction and mathematical reasoning draw our attention to the importance of structure of the mathematical knowledge base and the effect of this structure on students' reasoning processes. The findings reported here suggest that knowledge base that is well structured would aid students' ability at reasoning with given information.

Cognitive Research and Mathematics Instruction

A number of studies have looked at cognitive issues related to classroom instruction. Chinnappan and Lawson (1992) explored instructional methods that focus on the activation and use of previously learned mathematical knowledge. In their study, a sample of Year 10 students (n=27) was trained in the use of two strategies: generation and management. Generation training emphasised searching memory to access information that could be related to the problem. Management training, on the other hand, directed students' attention to planning and controlling the accessing of information. The two modes of training were provided via a series of worked examples highlighting the various aspects of generation and management. These worked examples were presented to the students in an instruction booklet. The effects of the two instructional strategies were assessed by asking the students to solve a series of training and transfer items. The training items were similar in structure to those provided in the instruction booklet, whereas the transfer items were different from the items that appeared in the instruction booklet.

The results showed that although instruction in the generation of information appeared to have helped the treatment group, the effect was not statistically significant on either the training or transfer items. Training in the use of management strategies, however, improved the near and far transfer performance of both the high- and low-achieving students. Taken together, these results suggested that mathematics instruction that requires students to focus on production of information is, in itself, not sufficient for the solution of problems. The investigators' aim in the instruction of management strategy was to direct students' thinking so that any information that was produced was applied in a way that would have optimised the chances of arriving at the problem goal.

The efficacy of generation training should not be underestimated on the basis of the above study, as the effect of the training was assessed in the context of solving problems. It would be interesting to replicate the experiment to examine the effect of generation training on tasks that are not necessarily problematic. Further, we have some evidence (Fong, 1994) in support of potential benefits of instruction on knowledge activation and use. Thus, we suggest that the instructional effect of knowledge generation strategy be investigated in future studies in a range of contexts and tasks.

In order to investigate the effects of specific learning experiences on spatial thinking, Owens (1992a) employed an experimental matched-group design in which subjects from the same class and spatial-ability cluster were randomly allocated to one of three groups: Group 1 consisted of students who worked individually on two-dimensional spatial problems, Group 2 students worked on the same spatial problems but in small collaborative groups, and Group 3 students (the control group) worked on a series of number problems in small groups. Over a six-week period, the treatment groups completed a series of activities using materials such as the seven-piece tangram, pattern blocks, pentomino shapes, and match sticks. While analysis of student performances in two of the posttests indicated no significant effects of instruction, the treatment groups together did show significant gains on the two-dimensional retention test. One could question why the learning experiences helped in knowledge retention and not in the solution of tasks in both the posttests. It is possible that the instruction could have facilitated the construction of schemas that were not sufficiently developed to be useful for the categorisation of problems (Sweller, 1988). In general we suggest that the results could be explained in from the perspectives of cognitive load and structural alignment (Gentner, 1983). The author (Owens, 1992a), however, emphasised the time required for constructs to develop and then carried out a qualitative study (1994, 1995) of the students' learning in class and test processes (1992b) and noted the importance of selective attention (influenced by schemata, visual imagery, and classroom interactions) in students' responsiveness and problem solving (1994).

Bobis, Sweller, and Cooper (1992, 1993, 1994) examined knowledge-driven performance in mathematics tasks and focused on the nature of students' knowledge structures. They investigated the general assertion that students learned more useful mathematical schemas when they were presented problems with non-specific goals because such problems prevent students from using means-ends analysis and direct attention to states within the problem space. The forward-looking approach, according to cognitive-load theory, reduces extraneous load, thereby freeing resources that allow students to focus on features of the problem-solving activity which facilitate the acquisition of schemas. The authors conducted two experiments to test this hypothesis. In Experiment 1, students viewed the completed geometrical model of a paper triangle before and while studying the instructions. The completed model was a product of folding a paper disc to form a special triangular shape. In Experiment 2, students were unaware of the completed model. Results of the two experiments indicated that the performance of students who were unaware of the completed model's appearance was superior to that of students who examined a completed model and who were instructed to study the completed model. It was suggested that the display of the completed model of the geometric task encouraged students to work backwards from the goal. On the other hand, in the case of students who were not shown the completed model, the reduced-goal specificity facilitated learning a geometrical paper-folding task. The authors concluded that the presence of the model increased the cognitive processing load by encouraging students to use a means-end strategy.

Continued interest in the use of cognitive load theory to investigate instructional effect is evident in the work reported by Atweh, Boulton-Lewis, and Cooper (1994) in the domain of algebra. A class of Year 9 students underwent instruction on the use of concrete materials to represent algebraic expressions and equations. The effects of the various representations were then assessed via a set of linear equations. Analysis of interview protocols revealed that all the students were able to infer that letters in the expression stood for unknown numbers. However, only one of the participants used the word variable to denote the letters. More importantly, only four of the students were able to accurately represent the linear expressions with cups and counters as they had been taught. When asked to use the given material, only one student was able to use the representations that were taught in the class in order to generate a solution for the first equation. In general, students experienced great difficulty in using the material to represent and then solve the linear equations. The authors suggest that the use of concrete material to model linear equations could have added to the cognitive demands on students' processing. Hence it is suggested that if teachers do wish to use concrete and pictorial representations, they must make the connection explicit between the material model and mental model.

Boulton-Lewis, Cooper, Atweh, Pillay, Wilss, and Mutch (1995) replicated the above study with a different sample consisting of 21 Year 8 students. The results of this study were similar in that most students did not use materials to solve problems involving linear equations. Again, Boulton-Lewis and her colleagues suggest that instruction using concrete materials to represent a variable in a linear expression increases processing load. The notion of a variable and its development via the use of multiple representations is an important issue but was not addressed in this study.

Quinlan's (1992, 1994) findings would suggest otherwise. He conducted a similar experiment with instruction in the use of algebraic representations with Year 7 students. The primary aim of the study was to develop students' understanding of algebraic generalisations, including the distributive law. Two classes were involved in this experiment: one class was asked to use arithmetic examples leading to generalisations, while the other class was instructed in the use of concrete objects. Analysis of students' performances in pre-and post-test items led the author to conclude that students who followed the second approach involving the use of concrete objects performed significantly better than their peers who made use of arithmetic examples. It was found that students who used the objects tended to engage in more discussion and exploration compared to those who did not. These results suggest that concrete analogs provide a more suitable model for the development of conceptual understanding of the distributive law than the traditional method involving the use of written examples. The results of this study appear to challenge the findings of Atweh et al (1994). However the Quinlan study was directed more at the developmental aspects of one concept (distributive law) within the general area of algebra. This is in contrast to the Atweh et al (1994) study in which the major interest was the exploration of the effect of use of concrete material on the representation and solution of a set of linear equations.

While the previous studies have focussed on the importance of examining problem representations and attendant load, they did not examine the role of thinking processes more directly. The study by Fong (1994) used the Information Processing Taxonomy Model to address this issue. He developed and evaluated the efficacy of a teaching model which could facilitate mathematical thinking. The model consists of what the author considers to be two major features of cognition, namely, primary and secondary production of information, together with two types of information that are available during the solution process. In this report the author described three types of worksheets that were constructed for use in the teaching experiment. The first set of worksheets was designed to provide information about the extent to which students have mastered basic skills in the area of ratio and proportions. The second set of worksheets required students to produce all possible information about the broad domain of ratio and proportions, and the third set of worksheets contained a series of problems on ratio and proportions. The author argued that this method of training shifts the burden of thinking to the students as students are actively involved in the generation of information, a theme that was addressed more directly by Lawson and Chinnappan (1994).

Adopting a somewhat similar approach to that of Fong (1994), Taplin (1992, 1994a, 1994b) reported the results of a study in which a sample of 54 practising primary school teachers attempted to teach a series of problem-solving strategies to their students from Grades 2 to Grade 6. Collectively, these strategies formed a model of problem solving which was developed from an earlier study of problem-solving behaviour of students who were classified as perseverers. The teachers were given four non-routine number problems and a list of effective strategies that could be used with each of these problems. The teachers worked with individual students and kept a record of their solutions and the nature of the intervention given. The results showed that the percentage of students who needed prompting in the use of the suggested strategies decreased across the four problems. It was also noted that there was a corresponding increase in the independent use of the strategies that the teachers taught. In addition, there was evidence to support the claim that across the problems less assistance was needed for the students to think of using a new strategy. In the main, the participating teachers reported satisfaction with the trialling of the teaching model. Experiments of this nature do contribute to the available literature on mathematics instruction. Further research could consider whether this model would be equally effective in the solution of novel problems in other areas of school mathematics.

Another study dealing with instructional issues in the problem domain was conducted by English (in press c). Her focus was on developing third-grade children's problem-posing abilities in both formal (symbolic) and informal (e.g., use of photographs) computational contexts. Children with different patterns of achievement in number and novel problem solving participated in a 12-week problem-posing program. Among the findings were the difficulties all children experienced in posing a range of problems within formal contexts, even after participating in the program. While the children displayed greater diversity within informal contexts, they still showed an overall preference for basic problem types. Children with different achievement profiles displayed different patterns of response to the informal activities; these patterns were reflected in the balance of structural and computational complexity shown in the problems they created.

Collectively, the results of the studies reported in this section show that researchers are actively engaged in exploring students' and teachers' mathematical cognition. It is of significance that these studies provide a cognitively-based rationale for the development and assessment of materials and teaching approaches. We would argue that, with this knowledge, mathematics educators and instructional designers are in a far better position to justify their policies and programs. Classroom mathematics teachers, on the other hand, need to exercise caution in implementing any of the above instructional interventions, as the expected learning outcomes could vary from one lesson to another. Nevertheless, the research reported here does provide a valuable body of knowledge as to why certain teaching approaches produce or fail to produce the desired learning outcomes. We are also better informed about how to fine tune some of the instructional procedures in ways that might foster better mathematical understanding.

Conclusions

How students acquire meaningful and useful mathematical knowledge, how they reason with this knowledge, and how we might facilitate knowledge construction and reasoning in the classroom are issues that will continue to challenge the mathematics education research community. The studies using knowledge structures as units of analysis have provided important insights here. However we need more research examining the nature of students' knowledge structures, how these are developed, how and why they are accessed (or not accessed) during problem solution, and how we can foster effective knowledge construction. Likewise, studies of students' mathematical reasoning have suggested that children have quite sophisticated reasoning processes at their disposal, however these do require nurturing throughout the school years if students are to develop the mathematical power we desire.

The studies reviewed here have also covered practical and theoretical grounds on the types of instructional formats that might facilitate (or hinder) knowledge acquisition and effective reasoning. We are less clear, however, on why such instruction aids or hinders in this development. Research is needed to shed more light on the mechanisms underlying teacher-student interactions in the classroom. This type of research would provide us with data about activities that might foster students' interest in mathematics and help them experiment with the mathematical information.

A significant number of research studies reported here have attempted to deal with the issue of analog use to facilitate mathematical understanding. This trend is a welcome one and is consistent with the spirit of major mathematics curriculum documents that encourage us to explore ways of enriching the range of representations used and generated by students. The conflicting findings on the effectiveness of these representations, however, point to the need for further research in the area. Indeed, there remain many unanswered questions regarding students' mathematical cognition and while the studies reported here have only scratched the surface, they do provide a valuable beginning for Australasian research into the new century.

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Advanced Mathematical Thinking

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Over the last decade, there has been an increasing amount of research on how students learn the more advanced mathematics encountered in the upper secondary and tertiary years (see, e.g., Tall, 1991). Research in this area in Australasia has focussed on: (a) the secondary-tertiary interface, with particular interest on students' beliefs and attitudes; (b) students' understanding of more advanced mathematical concepts; the impact of technology on such concepts; (c) the implications of research for the teaching and learning of more advanced concepts; and, (d) ways in which such research extends existing theoretical frameworks.

Secondary-Tertiary Interface

Research conducted in this area has investigated student preparedness for university, beliefs and attitudes about learning mathematics, and language factors.

Student Preparedness

A general concern has arisen at the rote, manipulative way students often approach mathematics in the upper secondary and undergraduate tertiary years. Gates, Robinson, and Blest (1995) claimed that Australian universities are increasingly accepting students with insufficient backgrounds in mathematics. They argued that university entrance is focused too often on quantitative, examination-based assessment and that a high score in a final examination does not necessarily reflect understanding, but may be achieved with highly developed skills in memorisation of procedural knowledge. The research of Morton, Reilly, and Forbes (1993) looked at factors which promote success in standard examinations. Their study considered results in two nationwide (bursary) mathematics examinations in New Zealand in calculus and statistics. They found that the more public mathematics examination students attempted, the better they performed, and that students from single sex and/or private schools did significantly better in the examinations than those from co-educational/state schools.

Concerns about students entering university are not restricted to Australia. Pongboriboon (1992) described how many students at a university in Thailand experienced difficulties in coping with first-year mathematics courses because they lacked desirable knowledge and skills. Even though all appropriate content areas appeared to have been studied in high school, it was felt by the teachers, students, and university lecturers involved in the study that time constraints resulted in crucial concepts receiving insufficient emphasis at school. Added to this was the "successful" survival method adopted by students for examinations, of merely manipulating mathematics in a superficial way, rather than trying to gain any depth of understanding. The end result was that university lecturers mistakenly assumed students understood material in the school curriculum and students found they could not keep pace with the speed of presentation at university. Pongboriboon concluded that high schools need to prepare students for the more autonomous style required in university lectures and that university assessment needs to be widened beyond examinations.

Beliefs And Attitudes

Crawford, Gordon, Nicholas, and Prosser (1993, 1994a, 1994b, 1995) described how the normative, external assessment at the end of high school, with its accent on competitiveness, resulted in the majority of students entering university with a fragmented view of mathematics. At a major university, a sample of 300 first-year mathematics students were asked about their conceptions of mathematics and how they personally learnt it. Of these students, 77% regarded mathematics as being comprised of numbers, rules, and formulae with maybe some applications to problems. The approach to learning for 82% was either rote memorisation, or drill and practice. Further, the research showed that the minority who had a more cohesive view of mathematics had not been rewarded by the assessment system with higher grades. FitzSimons (1994) conducted a similar study with 75 students beginning an Associate Diploma of Applied Science at a College for Technical and Further Education (TAFE). She found a higher proportion of students (97%) than Crawford et al. with a fragmented view of mathematics. Also surprisingly high was the number of students with positive attitudes to mathematics who considered that they did not have the ability to cope with advanced mathematics. FitzSimons concluded that TAFE students' beliefs about mathematics differ significantly from those students entering university. In particular, TAFE students generally believe that they have not achieved a high enough degree of mathematical understanding in secondary school.

Frid (1992, 1994) classified students' attitude to learning calculus in terms of their "sources of conviction". This term refers to beliefs about what constitutes mathematical truth and validity. She identified three categories: collectors, technicians, and connectors. Collectors exhibit sources of conviction which are external in that they reside in a set of disjointed rules and procedures presented by teachers or textbooks. Collectors tend to do poorly in calculus. Technicians exhibit a mixture of external and internal sources of conviction. Like collectors, their knowledge is based on rules and procedures, but they view these rules and procedures logically and can skilfully apply them. Connectors exhibit internal sources of conviction and can interpret calculus for themselves. They are good technicians, but they also display a stronger sense of personal understanding of calculus concepts. From 17 calculus students interviewed, only 5 were classed as connectors, while 8 were seen as collectors. Frid concluded that instruction methods need to encourage students to have more personal involvement in the construction of their calculus concepts.

Language Factors

The role of language in advanced mathematical thinking has also been investigated. Miller (1992) reported on a study where Year 11 students were asked to write definitions for 20 mathematics vocabulary words. The mean number correct was 13 for urban students and 10 for rural students. Miller argued that because methods of mathematical communication become more formal and symbolic in the senior years, more opportunities should be provided for students to describe their mathematical thinking both orally and in writing.

Frid (1994) found that 17 first-year calculus students made less use of symbols to represent, explain, or justify ideas than technical or everyday language. Given that about 75% of responses to skill-based problems were correct, she concluded that students' ability to perform standard symbolic operations does not translate into using symbols to convey ideas. Also, everyday language and technical language tend to be integrated when describing mathematical situations (Frid, 1993). For example, when they are describing a graph, such terms as *flat, just stops, jumps* were integrated with technical terms, such as, *slope, tangent* and *derivative*. Unfortunately, the integration was not always appropriate in relation to the accepted mathematical terms. One student believed a graph was discontinuous because it did not continue with the same defining equation throughout its domain. Frid concluded that classroom constructed interpretations need to be developed to meet those of the mathematics community at large. This assists students to form multiple representations of ideas, express relationships within and among representation systems, and formulate generalisations.

In summary, research at the high school-tertiary interface has shown the need for a more conceptual and less procedural approach to the teaching and learning of advanced mathematics. Such an approach should address the technical and everyday language used to describe the ideas involved. Overall, the need for more insights into how people develop advanced mathematical concepts and how a learning environment to foster such development can be established is indicated.

Advanced Mathematical Concepts

The thrust of research on advanced mathematical concepts in Australasia has centred largely on calculus and associated concepts. Again, there seems to be a general concern at the procedural approach taken by many students learning calculus. Ryan (1992) described this as "a rush to the rule". She showed that even when dynamic, interactive computer software was employed to introduce the derivative from first principles, students often appeared to miss the subtleties of the activities and exhibited the same rush to rule as students who followed a traditional first principles approach. She concluded that the introduction of "the rule" too soon may be a distraction for students exploring the underlying processes.

Research has shown difficulties associated with concepts embedded within calculus, such as, limits, tangents, and rate of change. Furina (1994) conducted a preliminary investigation into how five students calculated limits. She found that the students operated as if algebraic manipulations produced real or firm evidence for calculating a limit, while numerical calculations on a spreadsheet provided only tentative evidence that the limit found was the correct one. In reality, numerical methods tended to produce an answer while algebraic techniques alone often led to a false or no conclusion. She concluded that the two techniques need to be more integrated. Ryan (1991) showed deficiencies in many students' concept of slope in that their concept image was very firmly fixed to the context of straight lines. For example, in a question where a distance-time graph was given, and students were asked to determine the maximum speed, approximately 20% of students inappropriately chose the section of the graph that looked most like a straight line. As she pointed out, the concept of slope needs to be extended to include the idea of a curve being "locally straight". The suggestion is that often students' understanding of gradient and tangent is too primitive at the time they usually start learning calculus. The tangent image is usually restricted to that of a tangent to a circle - it touches but does not cut. Further, the idea of the slope of a tangent to a curve being a measure of instantaneous rate of change is not a spontaneous intuition (Ryan, 1992). Results of an investigation with students who had just finished secondary school (White, 1990; White & Mitchelmore, 1992) supported beliefs that rate of change is poorly understood. In an item, which was a (supposedly) routine rate of change question, many responses indicated no link at all between derivative and instantaneous rate of change. Other responses indicated some notion of average rate rather than instantaneous rate and tried to find the average decrease in one second and incorporate this into the solution for the instantaneous rate of change. The idea of a secant with its average rate of change and of a tangent with its instantaneous rate of change relationship appears to be missed by many calculus students.

The concept of function is fundamental to calculus. However, Gates et al. (1995) reported that the responses of 110 first-year university students suggested that studying calculus has an insignificant effect on understanding function notation and that perhaps there is often no perceived connection between the two. On the basis of 200 junior high-school students' responses to items involving tables of values, Ryan (1994) observed that in the early stages of dealing with functions, many students appeared to be operating pre-algebraically in the sense that they dealt with only one instance of a pattern at a time and did not generalise for all pairings (x,y). Students at this stage do not appear to see a table of values as establishing a relationship between two variables. At a higher level, Arnold (1992a) surveyed 383 students in their last year of high school or first year of university across four states of Australia. When asked to select from the six images for a function proposed by Barnes (1988), the algebraic formula was the most popular image, followed by a graph or curve. Very few respondents selected a table of values or a mapping. However, when asked to describe a function in their own words, the most common response was some type of rule or relationship. From students' who could distinguish between relations which were functions and relations that were not functions, 46% used the algebraic form only (for each x there is a unique y), 29% the geometric form only (the "vertical line" test) and 25% both forms. The third group were the smallest and significantly more successful at solving a range of unfamiliar function items (Arnold, 1992b). Overall, the need to integrate the different ways of conceiving functions appears crucial to being able to use them.

At a more fundamental level, White and Mitchelmore (1992, 1993, 1996) showed how an inadequate concept of a variable stifles any attempt to solve standard calculus items. In items where the independent variable was not clearly visible, students were either totally confused or focussed on any available visible symbols without thought of whether or not they were differentiating with respect to the appropriate independent variable. Another example of students preoccupation with visible symbols (dubbed the x , y syndrome) occurred in an item which asked for the area of the largest rectangle with its lower base on the x -axis and upper vertices on the curve of a parabola. Instead of using an expression for the area of the rectangle, many students inappropriately solved the derivative of the parabolic function equal to zero, because this function had a visible x and y . Their view of the rule for maximising, using calculus, was based superficially on the symbolic form $\frac{dy}{dx}$. There was confusion also when students were confronted with a variable expression to be calculated for a specific value. The confusion was that the variable expression and specific value were seen as totally separate. Some students even gave two answers. Generally, a manipulation focus on rules that can be applied only to visible symbols is totally destructive. However, such a manipulation focus appears to be a prevalent attitude.

In summary, the development of advanced mathematical concepts involved in calculus is hindered by a symbol orientated focus on rules which is not integrated with the underlying processes. Further mention is made regarding their development using technology in the next section.

The Impact of Technology

References have already been made to computer-assisted learning about calculus and functions. The other major impact of technology arose with the introduction of graphics calculators. The effect these have on instruction and learning is still being evaluated. Steele (1995) conducted an experimental study that involved 180 Year 11 students studying graph-sketching. Daily use of the calculator was found to lead to greater improvement on graphing items than less frequent use. An emphasis on scaling led to a greater ability to deal with potentially misleading questions where the finite view of the function provided initially by the calculator omitted important features of the graph. Steele advocated the use of graphics calculators because, even though using them takes more time, student difficulties are confronted, more readily overcome and general understanding is promoted.

Boers and Jones (1992a) also found students were generally favourable to the introduction of graphics calculators. They liked the fact that problems were now presented and analysed both algebraically and graphically. Despite the price, 98% had bought a calculator after 10 weeks. By the end of the second semester, 57% of students agreed that they would recommend the calculator to other students learning mathematics, while only 12% disagreed.

Britton (1994) had a more mixed reaction to the introduction of graphics calculators for use once a week in tutorials during a first-year university mathematics course. Early in the course, only 42 of the 148 students involved gave clearly favourable comments to a questionnaire about using the calculators. Observation by the researcher indicated that later in the course the students seemed to accept the calculators although they saw them as a useful tool for doing things rather than as an aid to building their understanding.

Bradley, Kemp, and Kissane (1994) also explored students views on graphic calculators. They found that students favoured the use of graphics calculators in a first-year calculus course although not all students supported their introduction. Describing the same course, Kissane, Kemp, and Bradley (1995) fitted student comments about graphics calculators to six metaphors (tool, laboratory, teaching aid, curriculum device, cheating device, and status symbol) for technology that had been proposed in earlier reports. The data provided legitimacy to the proposed six metaphors; moreover it suggested a seventh, namely, nuisance. Eleven students, from over seventy, who replied to a questionnaire, complained that graphics calculators were time consuming, added to the load of what had to be learnt, and were inaccurate and inefficient for measurement. Kissane et al. concluded that the different orientations associated with the different metaphors could well lead to mismatches between teachers and students similar to the relational-instrumental mismatches described by Skemp (1989).

Apart from student attitudes to using graphics calculators, the question of their role in assessment has been addressed. Kissane et al. (1995) raised questions of equity concerning access, and the varying quality of the calculators themselves. They observed that the solution to forbid the use of graphics calculators appears to solve the problem, but may increase inequity because any ban can be applied only to formal assessment. Better resourced individuals and schools will still benefit from the learning potential of graphics calculators. The authors concluded that equity issues are important and that there is no quick solution. Rather, there is a need for appropriate bodies to focus attention on the important issues surrounding assessment. Boers and Jones (1992b) investigated the use of graphics calculators in a calculus examination. They found the fear that graphics calculators would deskill students is misplaced. Instead, a much greater problem is that students did not utilise the calculators to anywhere near their full potential under examination conditions. In addition, students had difficulty integrating algebraically and graphically derived knowledge. For example, in questions that required finding the limit of a function at a certain point, without asking specifically for a graphical response, little evidence was found that the graphics calculator had been used. Further investigation (Boers & Jones, 1993) showed gender differences in performance on the calculus examination with females out performing males. However, against expectation, it was found that the males had been more successful when the integration of graphical and algebraic information was required, while the females had done better on the purely algebraic questions.

In summary, teachers and students alike generally seem favourably disposed to the introduction of graphic calculators in the mathematics curriculum. However, there is still much work to be done to make the best use of the technology especially with regards to assessment.

Curriculum Perspectives

As with concept-based research, studies into curriculum issues at upper secondary and early tertiary level have focussed primarily on calculus. White (1995, p.167) cited Grimison as stating:

> Educators across the world are currently examining the traditional pride of place which calculus has assumed in the senior secondary school and first and second year college and university programs.

Grimison went on to point out that in New South Wales differential calculus was introduced as a small topic in the Honours course of the Leaving Certificate in 1911. In 1944, it was included into the Pass course. In 1953, the amount of time devoted to both differential and integral calculus was vastly increased.

> Little justification on pedagogical grounds seemed ever to have occurred as to why this branch of mathematics was deemed suitable for inclusion in the senior course for an increasing number of students.

The major thrust of curriculum reform in calculus has been to move it from being a set of routine calculations, which are seen by students as pointless and only undertaken to achieve university entrance, to a subject that is interesting in its own right.

In 1988 the Australian Government set up a program to encourage the increased participation of young women in mathematics and science. This program sponsored a curriculum development project in calculus together with an evaluation study across four states and territories (Barnes, 1995a, 1995b). It resulted in the development of a more "user friendly" course in calculus with more emphasis on graphical interpretations, real world problems, and historical aspects of the subject, thus encouraging co-operative learning. An awareness of equity across schools was also considered. Hence, options were included to support ways to investigate rates of change which did not rely on the availability of advanced technology. Cavanagh (1996) followed up this development in calculus with his own action research project. He found that Barnes' approach not only made calculus more relevant and interesting for his students, but their general understanding of calculus was enhanced. Further, the students' ability to perform standard differential calculus techniques was not adversely affected. Accordingly, Cavanagh advocated introducing calculus via a more concept-based approach rather than the traditional first principles model.

In summary, there is currently a strong move to change the way calculus is taught in high school. These changes are being (or will soon be) introduced into some Australian high school curricula and are being supported in New South Wales by some more concept-based questions in external examinations and assessment tasks.

Theoretical Perspectives

Research into student understanding has led to developments in theoretical perspectives on formal mathematical thinking. These perspectives have focussed on the nature of the abstraction process, the theory of van Hiele, and the SOLO Taxonomy.

Abstraction

Bastick (1993) described the abstraction process as being viewed quite differently by mathematicians (generalisation from particular contexts to context independence) and non-mathematicians (complexity associated with increasing hierarchical levels of mathematics). He argued that any development in mathematics understanding does not follow a logical hierarchical development laid out in a curriculum nor does it follow the abstraction process from particular contexts to context independence. Rather, development in mathematics understanding depends on internal affect-laden contexts which he claimed are essential organisers of intuitive understanding of mathematics at all levels. Mitchelmore and White (1995) observed how everyday usage of the term *abstract* leads to a conflict in which abstract mathematics (in the context independent sense) is seen to be both easier and more difficult than more context dependent mathematics. A literature review undertaken to identify the source of this conflict revealed that the term *abstraction* may be used to denote either a process or a product, and that *abstract* may mean either general or apart. It was then argued that, whereas mathematics is *abstract-general* as a result of abstraction processes, mathematics teaching often leads to *abstract-apart* ideas as a result of dealing only with the products of abstraction processes. The initial conflict is resolved by distinguishing uses of mathematics within levels of abstraction (where abstract-apart ideas are adequate) from uses between levels (where abstract-general ideas are essential). One example cited involved students' difficulties with rates of change items reported in White and Mitchelmore (1992, 1996) and earlier in this chapter. The conclusion drawn was that correctly symbolising derivatives in complex situations requires an abstract-general concept of a variable, whereas symbolising in situations which have few variables can be achieved with an abstract-apart concept. In the former case, several letters must be given meaning in order to identify the independent and dependent variables and to distinguish between constants, variables, and specific values of variables. With only two variables, there is virtually only one derivative to choose and so the symbols can be viewed apart from their meaning. It was suggested that greater interest in abstraction as a process would be beneficial to both the theory and practice of mathematics education.

The Theory of Van Hiele

The van Hiele theory (van Hiele, 1986) and, in particular, its notion of the existence of a series of five hierarchical levels in geometrical understanding have continued to be investigated throughout the 1990s. A general review of this work can be found in the chapter on Space, Geometry, Measurement, and Visualisation in this volume. However, two studies are particularly relevant to the focus of this current chapter as they are directed at van Hiele's second highest level, namely, Level 4. At this level, thinking is described as including being able to: (a) understand the role of deduction; (b) to construct deductive proof; and (c) to apply necessary and sufficient conditions. Such skills represent advanced mathematical thinking.

Lawrie (1993, 1994) focussed her investigation on a detailed analysis of first-year university students' responses to a test devised by Mayberry (1981). The test was developed for Mayberry's doctoral work and involves a validation process by several well known mathematics educators, including van Hiele. However, using a similar, but very much larger sample, Lawrie was able to identify a number of inconsistencies and problems associated with the test. The main issues identified in her study, associated with Level 4, were: (a) visual prompts can reduce the level of mental demand of a question; (b) the notion of "constructing as proof" as an indication of Level 4 is too simplistic as certain proofs are achievable at van Hiele's Level 3; (c) the application of necessary and sufficient conditions is very difficult and is able to be used by only the most competent students; (d) more attention needs to be given to the application, in a non-prompted sense, of relationships such as congruency and similarity as an important defining principle for Level 4.

The second study that considered the van Hiele Theory was very different from that described above. It was directed at concepts that lay outside of the secondary school curriculum and could be considered at the upper end of the Level 4 spectrum. This study, by White and Pegg (1994), investigated indicators of formal thinking in a series of geometry problems. A sample of 20 third-year tertiary students was given test items that restricted the information that was available to them for use in the proof. Some test items were administered before and after a course on the development of non-Euclidean geometry. Before the course, the students were not able to deal adequately with restrictions to the knowledge they were allowed to assume. After the course, students appeared to cope well with the notion of restrictions to assumed knowledge, but this accommodation did not translate into the writing of acceptable proofs. The conclusions drawn were: (a) writing proofs where there are restrictions to assumed knowledge clearly involves a sophisticated form of deductive reasoning; and, (b) an indicator of advanced formal thinking is the ability to develop a proof where cases, which take into account different relationships (as opposed to properties) between necessary and sufficient conditions, are accommodated. In terms of Level 4 thinking, the hypothesis offered was that there is a hierarchy of growth in understanding geometry. This can be described in broad terms as: leading from the level of being able to consider various cases that involve the interaction of different properties of mathematical objects to the level of considering of various cases in which the interaction involves different relationships. Further, White and Pegg stated that it is this latter feature that accounts for most of the jump in cognitive load experienced by students when dealing with questions at this level.

While these studies were founded on the ideas of van Hiele, other studies have much in common with the general spirit of this work. This has arisen through the synthesis of many of the ideas in the van Hiele theory with those associated with the SOLO Taxonomy of Biggs and Collis (Pegg, 1992a). As a result, Level 4 thinking, in van Hiele's terms, can be re-interpreted by a number of levels within the formal mode described within the SOLO Taxonomy. The value in this approach is that it opens up other aspects of mathematics for exploration while, at the same time, not negating the considerable empirical basis already established in geometry.

The SOLO Taxonomy

The SOLO Taxonomy was developed by Biggs and Collis in the late 1970s and recent developments can be found in several articles (e.g., Biggs & Collis, 1991; Pegg, 1992b). Several investigations applying this framework to notions of advanced thinking have involved two aspects, namely, (a) broad descriptions of thinking associated with the formal mode, and (b) the identification of cycles of levels within a mode. Before discussing the studies it is appropriate to provide a brief background so that the research undertaken can be seen in context.

The formal mode is hypothesised to be the fourth mode of thinking to become available to humans as they mature. It emerges from the concrete symbolic mode but does not subsume it or any of the other previous modes (Biggs & Collis, 1991). Thinking in the formal mode can be described as the ability to go beyond particular circumstances and allow consideration of other possibilities. It is about generating hypotheses and speaking in terms of a general case. There no longer needs to be a real-world referent for the student.

During the 1980s it was proposed that there existed a cycle of levels, referred to as unistructural, multistructural, and relational which recur in each mode (Biggs & Collis, 1982). Despite this assertion, little if any research was undertaken to identify specific characteristics of these levels in the formal mode. It is clear why this was the case. There are problems with identifying an appropriate sample, and developing questions that would elicit a range of responses. Further, the concepts involved are complex and demands levels of abstraction that may only have been attained by researchers on a limited number of topics.

In the early 1990s both Campbell, Watson, and Collis (1992) and Pegg (1992b) independently identified more than one cycle of levels within the concrete symbolic mode. This feature has since been identified in areas outside of mathematics, for example, Levins and Pegg(1993) found cycles in the concrete symbolic mode in students' understanding of plant growth concepts. The key point is that this finding allowed for the tracking of growth in understanding within a mode over a much greater range of a given concept and allowed for a more consistent and detailed analysis of students' responses.

The studies reported below have combined these aspects, that is, they have focussed their attention on ideas that reflect the descriptions of the formal mode and they have been sensitive to the possibility that there are cycles of levels in the formal mode. This was done by considering the possibility that most students in the senior secondary years and the first year of university study can, at best, respond at levels within the first cycle or at the unistructural level in the second cycle of the formal mode. The relevant investigations were carried out by Pegg and Woolley in geometry and by Coady and Pegg in algebra.

Pegg and Woolley (1994) summarised the findings of a study which involved 55 senior-secondary students who undertook a series of deductive geometric questions. The questions could be solved by recourse to either congruency ideas or necessary and sufficient conditions. There were two main aspects to the findings. First, a clear difference was identified between responses coded as formal and those coded as concrete symbolic. The latter, ignoring a deductive approach, relied instead on a form of natural logic. The formal responses all attempted to sequence a set of arguments while at the same time relationships, such as congruency, were used spontaneously as a tool to solve the problem. Second, general descriptors were developed for the levels, unistructural, multistructural, and relational, in the formal mode to describe the variety in students' responses. They progressed from an acceptance of the need for the use of congruency arguments, to being unable to control all elements in the question while applying congruency concepts, to having an overview of the demands of the question and being able to prove the result. The use of necessary and sufficient conditions was coded as the first level (unistructural) in the second cycle.

Coady (1995a) undertook an extensive study, as part of her doctorate, into first-year university students' understanding of several algebra topics. In particular, the focus of the work was on the meaning and use students made of the notion of a variable in topics that include proportionality, variable substitution, function notation, algebraic fractions, and inequalities. This work has its roots in the ideas of Collis (1975) and Küchemann (1981). Coady and Pegg (1993) addressed this earlier work explicitly by investigating three of the most difficult Küchemann items. They found that of the 278 students tested at the beginning of the academic year only 9% of the students were correct on all three items and 35 % had all three items wrong. This study also suggested further research ideas about higher-order functioning which were addressed in the thesis.

Some aspects of Coady's doctoral work have been reported in a series of papers (Coady & Pegg, 1994; Coady, 1995a, 1995b; Pegg & Coady, 1993). In general, these papers focussed on particular topics and associated issues. They detail empirical evidence of how more than a hundred students, undertaking a mathematics-based program, performed on specially constructed items which (a) addressed key concepts in each topic, (b) were within the range of difficulty and experience of the students; and, (c) were not seen to be traditional, that is, from a textbook. Coady was able to categorise the responses in terms of the SOLO Taxonomy and identify characteristics of performance within the first cycle of the formal mode. Two main features stand out from this work. First, there was general agreement between these findings in algebra and those discussed above in geometry. Second, an integrative framework was identified which assisted with both the coding of responses and the identification of student problems. This "template" can be described briefly by four points. They are: (a) obtain an *overview* of the question including all requirements, (b) *focus* upon the critical aspects including the constraints or conditions needed to be addressed, (c) *note*, in some form, the ramifications of these critical aspects, and (d) *integrate* the critical aspects to the question and its subsequent solution.

Conclusion

Research in the area of advanced mathematical thinking is relatively new in the Australasian region, and there has been little work reported in this area before 1990. Nevertheless, as this review has demonstrated the picture is changing and a number of researchers are entering the field. Further, several research areas are opening up. While research in this area has unique problems, as identified earlier, these can be addressed. In so doing, there is the potential to offer teachers and students, at this level, strong practical advice based on theoretical perspectives. Such a possibility should allow able students to advance further or, perhaps, understand ideas more clearly. It may even open the door for those students who wish to be challenged by such concepts and have never been able to enter the game. The need is there and so are the challenges - much can be done.

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Research in Mathematical Problem Solving

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Introduction

Research in mathematical problem solving in Australia in the first half of the 1990s has been conducted against a backdrop which has continued to emphasise the role of problem solving in teaching and learning mathematics. The emphasis is evident in a number of spheres of mathematics education in Australia, such as the literature on learning and teaching, the formulation of national and state curriculum policy, professional mathematics pedagogy, assessment practices, the production and availability of suitable teaching resources, and the professional development of mathematics teachers.

The literature on learning and teaching in mathematics demonstrates an increased stress on the significance of constructivism as an approach to understanding the process of learning of mathematics and as a guide to teachers as facilitators of learning. Because of the central role of problem situations in constructivist theories, considerable attention has been paid to the notion of problem solving as an essential activity for learning mathematics. A scan of Australian professional journals of mathematics pedagogy reveals that teachers have been reporting frequently on their use of problem solving and investigation in classes, and have been encouraging their peers to do likewise (Horne & Supple, 1992; Mousley & Rice, 1993; Beesey & Rasmussen, 1994; Queensland Association of Mathematics Teachers, 1995).

In the formulation of national and state curriculum policy, problem solving and related activities have been highlighted as key activities in the mathematics classroom. *A National Statement on Mathematics for Australian Schools* (Australian Education Council, 1990), a key document which has helped shape the development of national and state policies on mathematics curriculum, has included problem solving as a major goal in teaching mathematics: "Students should develop their capacity to use mathematics in solving problems individually and collaboratively" (p.12). Subsequent documents (Australian Education Council, 1994a, 1994b) provided school systems with a framework for (a) the implementation of the National Statement, (b) development of local curricula and (c) a common language for the assessment and reporting of students' achievement in mathematics. The Mathematics Profile (AEC, 1994a) includes problem solving strategies as a substrand in the "Working Mathematically" strand of the curriculum along with other substrands such as investigating, conjecturing, applying, verifying, using mathematical language, and working in context.

The theme of problem solving in mathematics has permeated the production and marketing of teaching resources suitable for supporting a problem-based approach to mathematics teaching. Inspection of catalogues of materials available from professional associations such as the Australian Association of Mathematics Teachers and the National Council of Teachers reveals a plethora of available materials (print, audio-visual, manipulative, and software) which are suitable for problem solving, investigations, projects, and puzzles in the classroom. The expansion of these services by professional associations reflects the growing acceptance and use of such materials by teachers.

Following on the increased emphasis on problem solving and investigations in teaching mathematics has come a broadening of assessment practices. This can be seen at both the local school level and the system level (Money & Stephens, 1995). It is recognised that traditional methods of assessment are not appropriate for circumstances where students have engaged themselves in extended tasks or have been encouraged to be creative in their methods of solving real world problems (Clarke, Clarke, & Lovitt, 1990). Not all developments in alternative methods of assessment have been successful or received wide acceptance amongst teachers, and the debate on appropriate ways of assessment and specific techniques across large populations of students is ongoing (Davis & Waywood, 1992; McCrae, 1995).

Finally, professional development of mathematics teachers has been a growing area in Australia and problem solving has been a particular topic covered significantly by participants. Professional packages and programs have been developed to assist teachers implement problem solving techniques as recommended in state and national curricula (O'Connor, Griffiths, Downton & Hammond, 1993; Australian Association of Mathematics Teachers (1994).

The chapter is organised into the following sections:

1. cognitive processes, including studies of logical and intuitive processes, visualisation and spatial thinking, and language factors;
2. metacognitive strategies, including many studies of management strategies used by students during problem solving;
3. pedagogy, including teachers' usage of "good" questions and other pedagogical approaches;
4. gender differences in problem solving;
5. special client groups, concerning pre-schoolers, non-English speakers, and Aboriginal students;
6. technology, including studies on the use of LOGO and HyperCard for problem solving;
7. attitudes and beliefs of students and student teachers, and
8. contexts and applications.

It was clear that one area to have received greater attention in the past four years was that of metacognitive strategies, and ways of assisting students to manage their problem solving processes.

Cognitive Processes

The research on cognitive processes has covered both routine and non-routine problems, mainly in the number or spatial domains. Three main themes have emerged from the research. The first of these is the role of visual imagery, which has been explored by Mulligan (1992), Owens (1992), Collis, Watson and Campbell (1993), Dawe and Anderson (1993), Owens and Clements (1993), Owens (1994), and Thomas, Mulligan, and Goldin (1994). A second theme has been the strategies and processes used by problem solvers over a range of age-groups and levels of expertise (Clements and Ellerton, 1992; Davis, 1992; Groves, 1992; Gerot, 1993). Two groups of writers (Mulligan, 1992; Collis, Watson, & Campbell, 1993; Thomas, Mulligan, & Goldin, 1994) have explored students' use of intuitive processes. Problem-type is a further variable which has been investigated by Ayres (1993) and Collis, Watson and Campbell (1993).

Collis, Watson, and Campbell (1993) used the SOLO Taxonomy as a basis to explore the interface between logical and intuitive processes used by 16 Year 9 and 10 students on five mathematics problems. The problems were selected because it was possible to solve each of them by either school-taught (concrete-symbolic) or intuitive (ikonic) methods or by some combination of these. One set, however, was more likely to favour responses of a concrete-symbolic nature and the other responses of an ikonic nature. The students were given a battery of tests of visual imagery and were interviewed individually on their responses to the problem tasks. A think-board was used to monitor the occurrences of the following processes: (a) recognition of a previously-used concrete-symbolic structure, (b) recognition of an appropriate concrete-symbolic procedure, (c) use of a concrete-symbolic diagram, (d) reporting of visual images, (e) the use of real-world experience, and (f) the occurrence of an "Aha" experience. The latter three of these processes represent ikonic responses. It was found that students consistently applied concrete-symbolic strategies to concrete-symbolic problems and ikonic strategies to ikonic problems. There was more evidence of students switching between modes when solving concrete-symbolic problems than ikonic. Vividness of imagery was found to influence the frequency of ikonic strategies, whereas controllability of imagery or level of creativity did not. This study has implications for the selection of appropriate tasks for the assessment of mathematical problem solving. It also implies the need for teachers of Year 9 and 10 level to take into account the significance of the ikonic mode when designing teaching and assessment rather than, according to tradition, paying the most attention to non-visual methods.

Mulligan (1992) also explored children's intuitive models for problem solving—in this case, in a two-year study of Year 2 and 3 students' responses to multiplication and division word problems. Her study indicated that it was possible for young children to solve a variety of multiplication and division problems without having received instruction in these concepts. Mulligan identified three basic levels of strategy use for both multiplication and division (p.33): (a) strategies based on direct modelling and counting using counters or fingers; (b) strategies based on counting, addition, and subtraction without direct modelling; and (c) strategies based on known or derived addition and multiplication facts. The preferred intuitive methods were repeated addition for multiplication, and "building up" for division. The implication of this study is that teachers could pay more attention to linking children's intuitive strategies to formal teaching of multiplication and division. In another number-related study, Thomas, Mulligan, and Goldin (1994) interpreted children's internal imagistic representations of the numbers 1-100. These were interpreted with respect to developing theoretical models for mathematical learning and problem solving. The analysis was based on Goldin's model for problem-solving competency structures.

The role of visual imagery and spatial thinking in problem solving has been under investigation in recent times. Owens (1992) demonstrated that, for lower primary students, experience with two-dimensional activities improved their performance on spatial tasks compared with those who had experience with number puzzles. Owens and Clements (1993) generated a model of learning that synthesised observations made during a large study in which primary students were engaged in spatio-mathematical tasks. Data in the research pointed towards the process of visualising as important in the sense that it influenced the idiosyncratic construction of meaning. Owens (1994a) conducted a qualitative study of problem solving in mid-primary school students and made an interpretation of how children learn through spatial problem solving. Such learning involves a continuous interaction between the person's cognitive processing and the context of the learning experience. The context of these activities influences the person and, in turn, the person's responsiveness affects the immediate social and physical environment. The details of the processing were investigated in the study and this led to the emergence of the importance of responsiveness and selective attention in problem solving.

Owens (1994b) studied how visualisation and other thinking processes affected spatial problem solving in Years 2 and 4 primary school students as they solved two-dimensional problems over a six week period. Analysis of a large amount of both qualitative and quantitative data resulted in the following conclusions: (a) imagery became more dynamic and abstract as the students developed concepts; (b) images "summarised" concepts, events, and sequences of actions; (c) visual imagery contributed to the formation of conceptual representations and personal mental models; (d) visual imagery played an important role in assisting students to establish the meaning of a problem, in channelling the problem-solving approaches of the students, and in influencing students' cognitive constructions; and (d) although visual images are of various kinds and are used in various ways, imagistic processes are, nevertheless, extremely important in the problem-solving process.

Lowrie (1994) investigated the impact that visual and non-visual learning approaches have on the mathematical problem-solving performance of primary school children. He found that their preference for solving mathematical problems visually or nonvisually could not be attributed to learning style alone, but to other factors such as problem type, complexity, novelty, solver understanding and success. Competent children usually used visual methods after studying the problem for several minutes, while the less competent children spent only about 2-3 minutes studying a problem before providing a solution, usually in a nonvisual form. Lowrie urged teachers to ensure a balanced program in which visual and nonvisual learning approaches are given equal emphasis and to expose children to alternate solutions to mathematical problems.

Work by Clements and Ellerton (1992) has raised the question whether there is an over-emphasis on process skills with insufficient attention being given to the role of language factors in school mathematics. In a study of upper primary students in five countries using a Newman analysis of written tasks, a large percentage of the errors in mathematical problem solving occurred at the reading, comprehension, or transformation stages, that is, before the students had applied process skills.

In a case study in problem solving in advanced mathematics, Davis (1992) presented a counter-example to the notion that good problem solving skills can be equated with the automation of domain specific rules. Although such automation is important, it is not sufficient. The study showed that false starts and detours that occurred prior to a solution, along with the interactive catalytic effect of a group in problem solving.

In a study of processes and strategies of third and fourth graders tackling a real world problem amenable to division Groves (1992) found that children overwhelmingly chose mental calculation as their calculating device. They predominantly used repeated addition or subtraction rather than division, the latter was always used in conjunction with a calculator. The study confirms the importance of attaching meaning when using calculators, but also the need to develop skills such as estimation and approximation and a strong intuitive understanding of aspects of the number system such as decimals.

The question of why goal-free problems can facilitate learning was investigated by Ayres (1993). It was found that novices solving two-move mathematics problems which require the calculation of a subgoal and a goal make significantly more errors on the subgoal stage. The increase in errors at the subgoal stage is called the stage effect and is considered to occur because of increased cognitive load at the subgoal stage during application of a backward working heuristic such as means-ends analysis. This experiment found that presentation of goal-free problems, which automatically reduces the use of means-ends analysis, prevents the stage effect from occurring. In contrast, the same problem set presented in a conventional fashion generated a strong stage effect. A comparison between the groups also found that the goal-free group was superior at finding the correct solution path through the problem space. It was concluded that presentation of goal-free, two-move problems would enhance learning in comparison to conventional problems because there is both a reduced cognitive load and an automatically simplified path to solution.

The role of explanation in mathematics was investigated by Gerot (1993). It was argued that as a result of how mathematics is presented, especially in textbooks, explanation is highly implicit. Students are usually given mathematical problems and are told how to do them, but the whys - the underlying principles and connections - are understated. Thus students are left to infer these from examples, visual representations and manipulations, with the result that mathematics makes little sense to many learners. A strategy for determining at what points explanation breaks down is proposed by the author.

The research by Dawe and Anderson (1993) on visual imagery and metacognition in problem solving, brings together current interests of the co-investigators. Anderson has been investigating the metacognitive strategies used by junior secondary students, to solve non-routine problems in mathematics. Dawe has been studying the use of visualisation by teacher trainees, in the comprehension and solution of higher order non routine problems in mathematics. The project examined the introspections of teachers and students as they solved non- routine problems, and the implications for cooperative classroom learning. The increasing use of computer graphics, and graphics calculators in schools and tertiary institutions, has reawakened interest in the specific role of visualisation in the problem solving process. Of particular interest is the interaction of visual imagery with metacognitive strategies in non-routine problem solving. The data and its analysis will provide an in-depth qualitative case study of the role of verbal and non-verbal mental activity in comprehending and solving non- routine problems.

The research on cognitive processing has suggested some practical implications for classroom teachers. Perhaps the most consistent finding was that of the importance of visualisation to the problem solving process. Another was the need for teachers to pay greater attention to encouraging the use of intuitive strategies, even with older students. The research also recommended that teachers should acknowledge the importance of factors such as the context of the learning experience, and the student's ability to interpret the problem and understand the underlying principles.

Metcognitive Strategies

Metacognitive strategies have been given considerable attention. The research has covered a range of age-groups (from primary-aged children to adults) and a diversity of themes. Included amongst these themes are: the existence and use of different types of knowledge (e.g., Siemon, 1993; Stillman, 1993; Goos, 1994; Kaur & Blane, 1994); and differences in the quality and quantity of successful and not-so successful problem solvers (Geiger, 1993; Lawson & Chinnappan, 1994; Lowrie, 1994; Stillman, 1994; Taplin, 1994a). Some research has also investigated the effects of instruction on improving metacognitive processing (Chinnapan, 1994; Owens, Perry, Conroy, Geoghegan, & Howe, 1994; Taplin, 1994b).

English (1993) investigated the strategies that 7- to 12-year-old children apply to the solution of combinatorial problems. The focus was on novel problem-solving tasks which required the children to develop their own strategies, rather than relying on set procedures. Her findings suggested that the children were able to employ a series of increasingly sophisticated problem-solving strategies.

Goos (1994) explored the metacognitive strategies used by two senior secondary students working together on mechanics problems. Specifically, she explored two questions: why do students fail to use their existing knowledge in attempting to obtain problem solutions, and why do they persist with a particular chosen problem-solving strategy regardless of whether it leads to the desired goal. Qualitative data were collected from observation of the two students working co-operatively on a set of problems, and from retrospective interviews with the subjects. She found that collaborative interactions had a significant effect on the success of the problem-solving outcomes, but only if students have a mutual respect for each other's perspective on the task, and there is an equal distribution of both knowledge and power (p.162).

Cai (1994) also explored the metacognitive behaviour of a small group of postgraduate students—two with a high level of mathematical experience and two with a low level—within the four problem-solving processes of orientation, organisation, execution, and verification. Cai collected protocols of concurrent verbalisation during the subjects' attempts to solve a three-dimensional geometry problem. Retrospective reports were also collected. The high-experience and low-experience subjects were compared on length of time taken to solve the problem, length of protocols, solution strategies and knowledge of mathematical concepts. This study suggested that metacognitive awareness does have some bearing on problem-solving success, and that a higher level of mathematical experience implies a higher rate of metacognitive behaviours.

Taplin (1994a) was concerned with the management of strategies used to solve non-routine number problems. She used task-analysis maps to monitor the patterns of strategy use by problem solvers in the upper primary and upper secondary schools. The analysis indicated that successful problem solvers were more inclined than others to be flexible in their willingness to change to a new strategy if the old one was not working. They were also more able to explore different approaches with the same information, rather than constantly looking for new cues within the problem. Taplin identified a strategy sequence commonly used by the successful problem solvers, which involved selecting a strategy, refining its use if necessary, and recognising when it would be more appropriate to leave that and try a different approach. Taplin (1994b) described a small-scale experiment which explored whether the management of problem-solving strategies could be improved when problem solvers applied the strategy sequence. Descriptive analysis suggested that its use did enhance children's management of problem solving strategies and their likelihood of success.

Patterns of problem-solving behaviour of Year 11 students were investigated by Geiger (1993). The students received instruction in the use of heuristic strategies, control behaviours, and the effects of beliefs on problem-solving performance over the course of a school year. When students attempted problems which required application of recently learned mathematical ideas, Geiger found patterns within the categories of engagement, planning and monitoring, use of heuristic strategies and verification skills, knowledge of facts and procedures and beliefs. Successful and poor problem solvers differed in their degree of use of executive skills such as using heuristic strategies, monitoring, and verifying. A noteworthy result concerned the importance of a student's knowledge base for an initial identification of the problem structure, the implementation of routine procedures, and the selection of the most appropriate solution method based on this initial classification. Student's belief systems were manifested in their level of persistence, by their default behaviours, and by the lack of sensibility in some of their solutions. Geiger proposed a Student Response Feedback Framework as an instrument for teacher use in problem solving with secondary school students.

In another study with Year 11 students, Stillman (1993) investigated the relationships between mathematical and cognitive processing and metacognitive activities during problem solving. Data collected over a two year period indicated that students have a store of metacognitive knowledge which can influence their problem-solving activities during orientation, organisation, execution, and verification phases. In the orientation phase of problem solving many students jumped headlong into an attempt at the solution without pausing to reflect and appeared to be concerned more with the mechanical aspect of getting a solution than with taking time to plan, to monitor their thinking, and to verify their solution strategies. Highly capable problem solvers differed from others in both quality and quantity on (amongst other things) the following characteristics: (a) a clearer focus of the problem which enhanced their representation of the problem, (b) the ability to generalise mathematical material during orientation and execution phases, and (c) flexibility in thinking which allowed them to switch mental operations and solution methods while in the execution phase of problem solving.

Owens, Perry, Conroy, Geoghegan, and Howe (1994) investigated the affective processes which emerged during a study of an adult mathematics class that was exposed to a cyclical teaching model involving participation in the phases of experiencing, discussing, generalising, and applying. Paired groups engaged in the phases as they were involved in a series of mathematical problems with many opportunities for discussion. Whole-class student-led sharing sessions allowed students to continue their collaborative construction of meaning and solutions. For many of the students, this cooperative, problem-centred approach facilitated their mathematics learning and developed in them a confidence in their own abilities to, at least, get started on mathematical problems. There were links between problem heuristics and the development of either positive or negative affect. Being able to solve problems and having a sense of achievement in learning new materials and having a sense of ownership of their learning were seen by the students as the major reasons for changing attitudes and beliefs. Results of the study suggested that affective states are not as clear-cut as researchers like Goldin (1988) suggest. Students with affects such as fear and anxiety about mathematical problem-solving exhibited a range of affective states relating to areas such as teaching primary school mathematics which was associated with mathematical problem-solving. Students who had high levels of risk-taking, high verbal analytical skills, or positive experiences with teaching primary mathematics were able to link knowledge and attitudes and feel more positive about mathematical problem-solving despite their initial negative feelings. Statistically significant increases were obtained in attitudes to mathematics (which were closely linked with the course), and in perceptions of attitudes to mathematics of past experiences, in beliefs about mathematics, and beliefs about mathematics learning. However, increases were not significant in beliefs about mathematics teaching. It was suggested that students' attitudes towards mathematics may have already been changed positively by their experiences in previous mathematics curriculum subjects. The authors discuss the value of the collaborative construction of the social norms within the learning community but also point out the difficulties of establishing such a classroom with students possessing a great range of mathematical backgrounds and attitudes and beliefs.

In a study of geometry problem solving with high-achieving and low-achieving Year 11 students, Lawson and Chinnappan (1994) identified five major categories of processing events that are involved in solution attempts: identification, management, generation, self-assessment, and error. Analysis of students think-aloud protocols for these events revealed that on the more difficult problems, the high-achieving students activated significantly greater numbers of identification, management, and generation events than their low-achieving peers. These successful problem solvers demonstrated a high level of mental processing activity, used longer sequences of generative activity, showed a greater tendency to notice and correct errors, and showed a high degree of metacognitive awareness.

Chinnappan (1994) examined the effect of instruction in a knowledge-generating strategy (Domain Knowledge Access Strategy) on the activation of the five processing categories identified by Lawson and Chinnappan (1994) with particular focus on two of these categories, management and generation. It was found that this instruction with ten Year 10 students, which focussed on identifying given information, analysing the problem for clues and using these clues to retrieve related information from memory, resulted in a significant effect on activation of generation events and led to significant improvement in outcome in the treatment group over the control. It did not however produce the expected significant gains in the use of management events which were seen as important in the facilitation of knowledge access and use. The author suggested that explicit instructions on the value of access-related general strategies could lead to enhanced knowledge retrieval, elaboration, and exploitation in trigonometry problem solving.

Kaur and Blane (1994) investigated the strategies used by students in Years 5 to 8 from a range of Singapore schools when solving mathematical problems to determine (a) whether there was a time lag between students being able to perform mathematical calculations and their ability to use the same mathematical skills to solve problems; (b) whether most students have only a limited range of problem-solving strategies or heuristics from which to select; and (c) whether there is a developmental sequence through which more sophisticated strategies become available to the student. Results confirmed that there is a time lag for most school students between their acquisition of appropriate mathematical skills and knowledge and their ability to apply this to solving problems. Interview data revealed that unsuccessful problem solvers did not arrive at the solution or they obtained an incorrect solution for a variety of reasons. These included lack of comprehension of the question posed; lack of schema and strategic knowledge; inability to translate the problem into a mathematical form; inappropriate strategy used; computational errors; lack of mathematical knowledge; carelessness; and incorrect interpretations of the conditions of the problem. There were a few basic problem solving strategies which most students tended to use across all year levels, namely, arithmetic manipulation and guess and check, the latter strategy being used by a significant number of students across all year levels. For all problems there were students who relied on the three strategies: logical reasoning, modelling, and number manipulation to find a solution; it was apparent that some strategies were specific to particular problems.

A meta-model of children's approaches to problem solving by Siemon (1992) categorised metacognitive knowledge into two orthogonal dimensions, conceptual knowledge (metacognitive knowledge and cognitive goals) and procedural knowledge (metacognitive experiences and cognitive actions), with two levels of ability for each - low and high. Siemon claimed that failure to solve a problem may be due to weakness in either area. In line with her model, Siemon (1993) conducted a year-long teaching experiment explored the possibility of changing Year 4 students approaches to mathematical problem solving. A metacognitive question and answer technique was used to negotiate meaning, explore problem representation, discuss possible solution strategies, and reflect on the problem-solving enterprise. Analysis of the transcript data, classroom observations and children's work samples revealed that while those with most to gain and nothing to lose demonstrated the greatest shift in approach, each child's approach was successfully challenged to some extent.

The common message recurring throughout the research reported in this section is that metacognitive processes do contribute to problem-solving success and that instruction in these processes seems to have positive effects in enhancing problem-solving performance. Several writers have also reported the value of collaborative interactions.

Pedagogy

Two studies appear in the pedagogy category. Sullivan and Clarke (1992) reported a cross-age and cross-cultural exploration of the role of "good" questions, while Mousley (1993) explored the interplay of a number of pedagogical issues in an early childhood, problem-posing classroom.

Sullivan and Clarke (1992) investigated several aspects of students' responses to what they described as "good" questions. These were questions which require more than simply recall of information, allow the pupils to learn from the act of completing the task, and are open (that is, there are multiple possible correct answers). Their sample group consisted of students from years 6 and 10 in Melbourne schools, year 10 in a school in Penang, and first year primary teacher education students in Melbourne. They found that it was possible for students to give multiple answers to these questions although many students, particularly those in year 6, did not do so without prompting. They also found that the use of the questions could promote learning, and that the questions require some degree of productive thinking beyond recall of facts and procedures. The outcomes of the study warranted further investigation into the implementation of "good" questions into the classroom setting. One such further study, reported by Sullivan, Bourke, and Scott (1995), was an investigation of classroom activity based on the use of open-ended tasks in a grade six class. It was found that open-ended questions were suitable for students who are confident and for those lacking in confidence. Students showed improvement in tests of closed items as well as open-ended items.

In a study of children constructing number concepts in a problem-posing classroom, Mousley (1993) noted that the idea of radical constructivism, which puts children's interests and activities at the centre of the curriculum, is worrying to teachers who are trained through their own schooling experiences and formal teacher education to plan activities in order to meet learning objectives. So, when children work on problems of their own creation, teachers feel concerns about controlling and evaluating the development of knowledge. Through examination of the classroom techniques and pedagogical ideas of one young early childhood teacher, Mousley advocated the view that less formal organisation can result in valuable learning for children, teachers, and colleagues. The teacher's practice in an infant classroom was discussed in terms of three current educational theories: constructivism, problem posing pedagogy, and inclusive curriculum.

Both of these studies raise important questions for further investigation into the nature of appropriate problem-solving experiences, and their implementation in the classroom.

Gender

There were two reports of investigations into gender differences, one at secondary level (Low and Over, 1993), and the other at primary (Leeson, 1995). The former of these focused on algebraic word problems, and the latter used items from the Australian Primary Schools Mathematics Competition.

Low and Over (1993) asked 10th-grade boys and girls to classify algebraic word problems in terms of whether or not there was sufficient, missing or irrelevant information for solution. The girls in all three ability levels studied were less successful than the boys in identifying (a) irrelevant information in the word problems, and (b) problems which had missing information. However, the girls were as accurate as the boys in classifying problems containing necessary and sufficient information for their solution. In an additional study with boys and girls in Grade 11, Low and Over compared performance in solving algebraic word problems containing sufficient information and irrelevant information. The main result was that girls who performed as well as boys in solving problems with sufficient information had less success than boys on problems with irrelevant information. They argued for instruction in the acquisition of schematic knowledge of algebraic word problems for all students. They claimed that practice in text editing which focuses on students identifying necessary and sufficient information for problem solution should facilitate schema acquisition.

Leeson (1995) looked at performance on non-routine problem solving as one aspect of an analysis of gender differences in the Australian Primary Schools Mathematics Competition. He reported that boys scored significantly better than girls on 29 of the 44 problem-solving items, that there was no significant difference on 12 of the items, and that girls scored significantly better than boys on 3 items.

Both of these studies indicate that there are still certain types of tasks on which gender differences occur. Both report a tendency for these to be in favour of boys, in certain circumstances. These findings suggest that, even in the 1990s, there is still a need to understand more about the nature of gender differences.

Special Groups

Research in this category covers students of different ages (preschoolers and kindergarten, primary, secondary, and tertiary), and different cultural backgrounds and languages (Aboriginal Australian, Taiwanese, and white Australian).

The experiences of two boys were traced during their first formal mathematics experiences at pre-school (Leder, 1992) to ascertain how they interacted with their teacher during these sessions and the ways they worked with unfamiliar mathematical problems. Supplementary data were gathered on the children's home background, personal characteristics, and their home learning environment. This allowed comparisons to be made between the learning climate at home and in the more formal pre-school setting. There were similarities in the home backgrounds of the two boys, yet the learning climate fostered by the parents showed subtle differences in parental expectations on each boy, with one (William) being encouraged to be independent while the other (Jonathan) was treated more passively. The quality of the teacher's interaction with the two boys differed with a greater number of high cognitive level questions being asked of William. In contrast, Jonathan was given more directives. William was a conserver of volume and could solve mathematical problems presented in a play setting whereas Jonathan could not. Leder concluded that there can be marked differences in learning environments even within the same formal teaching setting, and that teachers play a significant role in creating some of the differences.

Davis and Pepper (1992) hypothesised that preschoolers who were classed as good counters would be significantly better than either developing or poor counters at a task which required them to re-distribute 12 crackers to 3 dolls after the 12 crackers had been previously distributed to 2 dolls. They found that there was little difference between poor, developing, or good counters who could solve the problem in four or fewer steps. They concluded from the variety of the children's strategies that the re-distribution task was a genuine non-routine problem for these children. Further research was called for to understand more about the factors which might relate to the development of children's ability to count and to solve this type of re-distribution problem.

Galligan (1995) conducted a pilot study with five Mandarin-speaking Taiwanese and five English-speaking tertiary students to measure the time it took them to understand and complete word problems in their native tongues. Three tests were developed to measure students' ability to do mathematical word problems; the first consisted of 16 questions with no vocabulary (e.g., $1.3x - 5 = 3$), the second consisted of 15 contextually free problems (e.g., If $t + 5 = s$, which is the bigger number, s or t?), and the third consisted of 21 contextual problems (e.g., Ben is taller than George; Dave is shorter than George - Who is tallest?). On the first test the mean number correct was similar, but the Taiwanese students took about one third the time that the English-speaking students took. On the second test the mean number correct was similar with the Tiawanese students taking slightly less mean times than the English speakers to understand the problems and to complete the test. On the third test the Taiwanese speakers' score was significantly better than their English speaking counterparts and their mean times to understand the problems and to complete the test were also less. Galligan concluded that the regular structure and minimum level of redundancy of the Chinese language compared with English may have advantaged the Chinese-speaking students in problem solving.

Elliott (1993) examined the impact of scaffolded mathematics instruction on the learning outcomes of 30 high and low achieving children aged between 5 and 6 years, enrolled in the kindergarten class of a state primary school in a suburb of a large Australian city. She compared the effectiveness of a metacognitively-rich teaching approach with that of a "best practice" approach within the context of regular classroom teaching. Children from both levels who participated in the metacognitive groups recorded significantly greater improvement in measures of mathematics achievement than did those who participated in the best-practice sessions. Elliott concluded that scaffolding of metacognitive knowledge and strategies by focussing on planning, organising, and monitoring appeared to be useful in assisting the children to become more conscious and reflective in constructing their own knowledge. In the scaffolded context children consciously focussed on both task content and their understanding of task demands, as well as progress toward achieving gaols.

In a study of problem solving difficulties of Aboriginal students, Smith (1993) noted that their underachievement in mathematics is widely acknowledged and appears to be more extreme in the crucial area of mathematical problem solving. His research conducted on Aboriginal students in the Northern Territory presents evidence that such difficulties do exist and are more noticeable among rural rather than urban Aboriginal students. In particular, the evidence indicates that problems involving the concept of difference provide even greater difficulty.

Findings from these studies suggest that there are social and cultural context variables which influence the development of students' problem-solving ability in different ways. Researchers, teachers, and teacher educators need to be cognisant of these variables as they work with their respective client groups.

Technology

Two of the three studies reported here deal with the software package LOGO and children in the early years of primary school. The third study involved preservice teachers who were involved with a computer based tutorial to improve problem-solving strategies.

Yelland (1992) conducted a study of young children problem solving with LOGO, and analysed the strategies and interactions of gender pairs. The purpose of the study was to examine the performance of young children as they worked on LOGO tasks individually and in one of three gender pairs (girls, boys and boy/girls). Specifically, the studies sought to explore the strategies and interactions of the children, and to describe and discuss the extent and nature of differences in performance. The research consisted of three pilot studies, a final trial of tasks, and the main study, with a total of 108 children participating. The tasks contained in the research design used both the floor and screen versions of LOGO and consisted of preliminary tasks, a path and copy tasks. The results revealed that, initially, the girl pairs made more moves and took more time to complete the floor turtle path task than either the boy pairs or the boy/girl pairs. The girl pairs tended to be more accurate than the other gender pairs in their attempts to copy the four task items. When the children attempted the same tasks using the screen version of LOGO, there were no significant differences between the gender groups on the same performance parameters. Furthermore, the results from the research indicate that qualitatively different levels of performance, based on the processes that are deployed in the problem-solving context, are evident. An analysis of the interactions of the pairs of children revealed that, in the floor path task, where significant differences in performance were found, the girls asked for, and received, more information in the form of explanations and specific propositions for moves than either the boy or boy/girl pairs. The boy and boy/girl pairs made more moves without consultation with their partner and the boy/girl pairs disagreed with proposals made by their partner less frequently than the other gender pairs. The study highlights the importance of considering both the type of task and performance over a reasonable period of time when describing the nature and extent of young children's learning when solving problems in a novel environment. It also provides teachers with important information regarding the differences in strategies and interactions of young children working collaboratively in the LOGO environment, which may assist in their planning and organisation of learning experiences with LOGO.

Williams (1994) also explored the use of LOGO, in this case, the use of the LOGO robot and Polya's Four-Step Problem Solving model to encourage the development of thinking and social skills. The class of Year 3 students was given mathematical tasks which explored some mathematical principles of measurement, direction, angles and degrees. Her results suggested an improvement in problem skills.

James and Taplin (1994a, 1994b) and Taplin and James (1994) reported on the research and development of a HyperCard driven tutorial designed to improve problem-solving strategies. The program was originally designed to be used by pre-service teachers, but is appropriate for use by other target groups as well.

Given the availability of software packages such as LOGO there appears to be a pressing need for more research focussed on exploring the potential of this software in enhancing problem-solving skills at all levels.

Attitudes and Beliefs

The studies reviewed in this section examine a number of variables associated with students' affective characteristics and beliefs about mathematics and problem solving. Another study by Owens et al. (1995) was reviewed in the first section.

Swinson and Shield (1994) investigated student teachers' beliefs about mathematics teaching and learning through their responses to different tasks, one of which was a discussion of their attempts at problem solving. The students' comments on their problem solving experiences suggested that they believed the product and not the process was the important part of solving a problem, and that the use of diagrams aided visualisation and assisted with finding solutions. Swinson and Shield suggested that determining students' prior beliefs about problem solving and other aspects of mathematics is an important component of developing programs to cultivate beliefs and understanding.

Stacey and del Beato (1992) investigated sources of certainty and uncertainty in students during mathematical problem solving. Their study suggested that over-confidence explains why students generally are reluctant to check their work. Students with wrong methods that gave easy arithmetic were almost as certain that their answers were correct as students with the correct method. Uncertainty arose from students making a generalisation and from making straightforward calculations. Students were more likely to check their calculations more than their reasoning. Further, groups tended to choose a simple wrong solution even when a correct solution had been proposed.

A study by Forgasz and Romeril (1993) of Year 7 students' problem choice showed that perceptions of problem difficulty are linked to mastery of content areas. Students selected different questions in tests and regular class conditions. It was found that choice of problems was associated with confidence, risk taking, intrinsic factors for the students' choice, and challenge was a motivating factor to only a very few students. The time spent on problems varied greatly among students; high achievers spent more time on solving the problems.

While no studies relating to attitudes and beliefs were reported in the previous review (1998-1991) the few studies reviewed here suggest that researchers are acknowledging the significance of these variables in students' learning of mathematical problem solving. Currently, there are some classroom studies being conducted which focus on instructional interventions aimed at enhancing students' metacognition and problem solving. A number of these are looking at students' beliefs about and attitudes toward problem solving and this seems a fruitful area for future research.

Contexts and Applications

A *National Statement on Mathematics for Australian Schools* (1991) has as one of it's strands "choosing and using mathematics" which includes 'mathematical modelling' and 'applying mathematics'. The two studies reviewed here deal with modelling and students' learning about quadratics from realistic problem contexts.

Afamasaga-Fuata'i (1993) conducted a case study to investigate students' conceptualisations of quadratics. The methodology involved interviews between a researcher and students solving quadratic contextual problems. A multi-representational computer program, *Function Probe*, was available as a tool to aid students in their problem solving. The results showed that using realistic situations as problem contexts invited multiplicity of interpretations and methods for defining quadratic functional relationships. By reasoning empirically from the problem context, students conceptualised quadratic relationships iteratively and in terms of summation - in contrast to the more common view of quadratics as a product of two linear variations. Further, requiring students to verify and justify their strategies by cross referencing between multiple representations of functional relationships and problem context led them to construct viable schemes to characterise quadratics in terms of rate of change, dimensionality, and symmetry.

In a study of problems and projects for the teaching of modelling, Carr (1993) noted that mathematics students have traditionally been shown "other people's models", but when learning to be modellers themselves, they need interesting problems likely to motivate thorough discussion and involvement. Hence the whole modelling process requires careful thought and has many phases. The study showed that students working in small groups, on extended investigations or projects, lead to effective learning of all aspects of the modelling process. Carr asserted that assessment should be designed to give the students credit for every phase of the modelling process. He discussed some materials widely used in Australia, and described ways of encouraging teachers to share with others their successes in teaching modelling.

It seems that some notice has been taken of Groves' (1989) comment that "modelling should be viewed as a natural part of using mathematics to solve real problems" (p. 16) and that some research has been directed in this area. However, the importance of modelling is not reflected in the number of studies done in this region over the past four years. It is hoped that teacher/researchers will be encouraged to conduct investigations in this area.

Conclusion

The large number of studies referred to in this chapter is an indication of the extent to which problem solving in school mathematics has been investigated formally in Australasia in the period 1992 to 1995. The great bulk of the research has been in the areas of cognitive and metacognitive processes. Key findings in cognitive processes included the importance of visualisation and intuitive strategies in the problem solving process. Research into metacognitive strategies highlights the importance of students being able to manage the problem solving process effectively and the value of group interactions. Studies in the pedagogy category have confirmed the role of open tasks and less formal organisation of the curriculum for the learning and teaching of mathematics.

The studies in gender issues and special client groups show that these issues still need to be addressed in Australasia in the 1990s, especially in relation to the role of social and cultural context variables in the development of problem-solving ability. A number of benefits have been shown in the studies reported on the role of technology in problem solving in the mathematics classroom, more research is required to reveal what precisely are the benefits and what ways it can be incorporated effectively into classroom practice at all levels.

Finally, the importance for the problem solving of the areas of attitudes and beliefs, and applications has been demonstrated by research in the four-year period. However, in the light of this importance, it is hoped that such research will grow in the next four years.

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Researching Language Factors in Mathematics Education: The Australasian Contribution

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In the last four-yearly review of mathematics education research activities in Australasia (Atweh & Watson, 1992), it was claimed that over a period of 20 years or more the volume and quality of the research carried out by Australasian mathematics educators into language factors in mathematics learning was second to none (Ellerton & Clarkson, 1992). In this chapter it will be shown that during the period 1992-1995 the quantity and quality of Australasian research in the field has been maintained.

Australasian Research 1992-1995: Major Contributions

The Mathematics-Language-Classroom Interface

In the last four-yearly review (Ellerton & Clarkson, 1992) it was pointed out that the mathematics/language interface has many dimensions, and that there was a major need to develop a framework in which the various dimensions were identified, and the relationships between the dimensions explored.

Figure 1 shows the framework put forward by Ellerton (1989). This framework reflects the complexity of the multi-faceted relationships between mathematics, language, the mathematics classroom, and the underlying cultural context. The framework was used in a series of monographs on language factors in mathematics learning published by Deakin University in 1991 (see, e.g., Clarkson, 1991). A variation on the framework is being used by Batchelor College, in the Northern Territory, in action research programs involving Aboriginal teacher education students (Batchelor College, 1991). It has been found that the framework suggests "a number of connections which ... facilitate discussion in a multicultural context," and helps "clarify and guide our thinking" (Jones, Kershaw, & Sparrow, 1995, pp. 20-21).

Figure 1 can be regarded as a cross-section of a three-dimensional depiction of relationships. Culture is all-pervading, and communication within culture is of fundamental importance for education. Mathematics is one form of communication, and what takes place in mathematics classrooms is influenced by culture, modes of communication, mathematics, and curriculum theory. Furthermore, classroom and other discourses in mathematics education are influenced by natural language, psycho- and sociolinguistic forces, and the extent and form of the engagement of learners in problem-solving activities. As with all frameworks or models, the representation is imperfect. However, Figure 1 does provide a framework for reflection on the various forms and ways in which language factors influence the teaching and learning of mathematics.

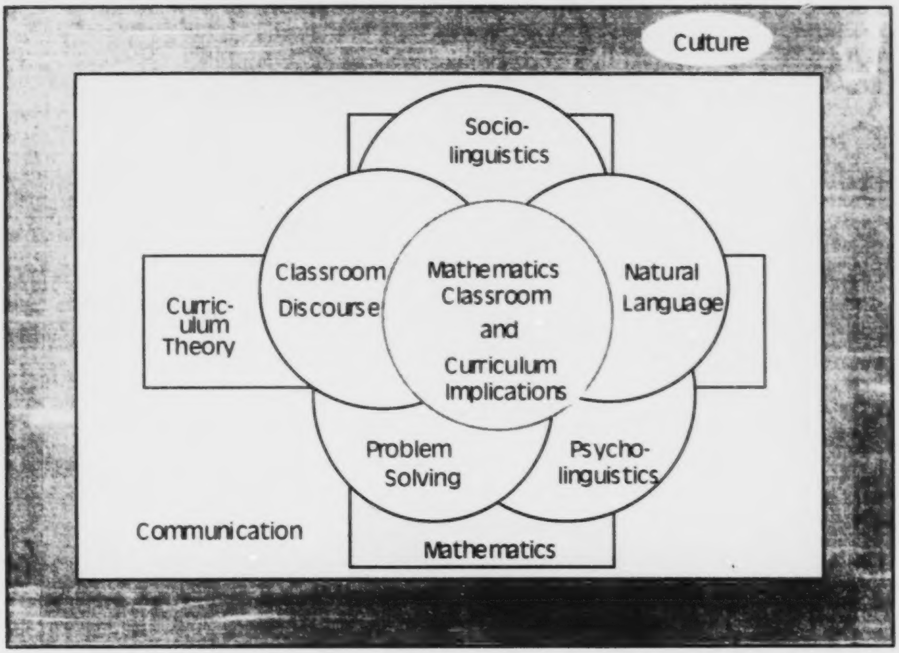
Culture
Socio-linguistics
Classroom Discourse
Mathematics Classroom and Curriculum Implications
Natural Language
Curric-ulum Theory
Problem Solving
Psycho-linguistics
Communication
Mathematics

Figure 1. The interface between mathematics and language

Culture, Language, and Mathematics Education

Figure 1 suggest that all aspects of mathematics education necessarily have cultural foundations. Although the definitions of, and relationships between, the components of how culture relates to the other facets identified in the framework illustrated in Figure 1 are open to debate, the theorising of these relationships could be an important vehicle for unifying and directing future mathematics education research, mathematics curriculum development, and the teaching and learning of mathematics.

Language and Garma

Jones et al. (1995) have drawn attention to the essential differences between *western* mathematics as represented in the curriculum of the Northern Territory

Department of Education, and *Garma* mathematics—*Garma* is a Yolngu word for an open public ceremonial ground and is used in education as a metaphor for the public negotiation of mathematical knowledge, necessarily incorporating, like a ceremony, a variety of different perspectives. Jones et al. quote extensively from Batchelor College documents in which it is made clear that language and culture are vitally important aspects of Garma mathematics. According to Jones et al.:

> Fundamentally different world views such as those just described translate equally disparate language structure and function and vice versa. ... English is not structured for the purpose, whereas relevant Aboriginal dialects are. In a very real sense culture is language. (p. 19)

Jones et al. (1995) refer extensively to the work of Christie (1994) and Harris (1990) in elaborating the view that although western mathematical knowledge is essentially antithetical to Garma mathematics, there are aspects of contemporary thinking about western education, such as cooperative group learning, which also have an important place in a Garma approach to mathematics education.

The importance of locating education theory and practice within cultural frameworks was highlighted in the following statement by Harris (1990):

> The nature and degree of the difference between Aboriginal and European culture is so great that the only honest conclusion we can arrive at is that they are largely incompatible. The two cultures are antithetic—consisting of more opposites than similarities. They are warring against each other at their foundations. Recognising and accepting the truth of the term incompatible was for me ... the point of theoretical liberation and the starting point for a more effective educational theory to be applied in Aboriginal schools. This degree of difference is so great that it is harder to find what they have in common in cultural terms than it is to see the differences. (p. 9)

If Harris' view is accepted, and if in fact language is intimately related to culture, then there would appear to be massive implications for those responsible for curriculum development in general, and for mathematics curriculum development in particular.

As Jones, Kershaw, and Sparrow (1995) have commented, the process whereby Aboriginal and other non-western groups will have the right for their children to experience a curriculum which links with their personal worlds rather than being in opposition to it, is underway. Essentially "cultural reaffirmation has translated into a decolonising process, particularly with respect to education" (p. 2). There can be little doubt that western mathematics, together with its associated language and symbol forms, represents a highly specialised way of thinking, and to a certain extent that explains Béchervaise's (1992) description of mathematics as a foreign language.

Yet, the reality is that most Aboriginal children—indeed most Australian children—are required to attend school and learn a western style of mathematics. Learning the language and symbol systems of western mathematics is an essential component of the enculturation (some might say indoctrination) process. This was strongly brought out in a keynote address by Harris to the Mathematics Education

Research Group of Australasia in 1992. For example, Harris (1992), in talking about *school shapes*, stated that she had found it to be quite nonsensical to ask Warlpiri children to give her a square, or to show her a circle, or a triangle, because these shapes were not meaningful to them.

One of the challenges facing mathematics education researchers is to explore further links between language, culture and mathematics education, and to make sure that their findings influence curriculum decisions. It is our view that this aspect was not given enough attention in *A National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991) or in *Mathematics—A Curriculum Profile for Australian Schools* (Curriculum Corporation, 1994a).

Research Linking Language and Indigenous Counting Systems

In his doctoral study, Lean (1992) carefully documented details of over 1000 indigenous counting systems in Oceania, Micronesia, and Polynesia, with special attention being paid to the 750 counting systems of Papua New Guinea. One thing Lean's (1992) thesis made clear is that culture, language, and indigenous mathematical forms are intimately linked. Each different language has a different number system.

Even when two different language groups have number systems which would appear to have the same structure (e.g., base 5), Lean's (1992, 1995) fine grain analyses demonstrated conclusively that the indigenous counting systems were qualitatively different in important ways. Some systems (e.g., body part systems in the western Highlands of Papua New Guinea) did not have names for numbers and, for example, *two* in one system was unlikely to mean exactly the same thing as *two* in another. Many systems did not have generic terms for what western writers have called cardinal and ordinal numbers.

Bishop (1993) argued that the fact that mathematics is not language-free has profound implications. After arguing that the original language base of western mathematics is probably an amalgam of Indian-Greek-Arabic-Latin influences, he added that from this base Italian, Spanish, French, German, and English developed their mathematics language repertoires over the past 400 years. At present, English is probably the principal medium for international mathematics research developments, and yet the indigenous languages of many groups in Australia, Papua New Guinea, Oceania, rural Africa, and tribal groups of most nations in Southeast Asia, have language registers which are distant from those of English, or for that matter, any of the other languages mentioned above.

Because there is such a strong link between culture and language, mathematics curricula imposed on minority groups can be regarded as representing a form of cultural imperialism. Although it is certainly true that not all languages express western mathematical concepts in an efficient way, it is equally true that many of the concepts in the indigenous languages and cultures would not be easily expressed in English or in other languages spoken by large numbers of people. As Bishop (1993) has stated "If countries and societies within countries are to engage in the process of cultural reconstruction then the language element in relation to informal, non-formal, and formal mathematics education is critical" (p. 20).

The "Asian Perspectives on Mathematics Education" Project

Within the context of different cultural groups living within Australasia, there are a large number of different languages and counting systems. These include not only all of the indigenous languages and counting systems of Papua New Guinea, but also those of the Aboriginal and Torres Strait Islanders as well as of the Maoris. The English language is the dominant language, but many other European and Asian languages are spoken. Since the second World War, immigrants from numerous nations have made their homes in Australasia, and many children in schools are from non-English speaking backgrounds. In Papua New Guinea schools, where the language of instruction is mainly English, English is often the third or fourth language of learners (see Clarkson, 1992a, 1992b, 1992c; Marshall, Gilmour, & Lewis, 1992).

Further reference will be made to research into the effects of bilingualism and multilingualism on mathematics learning later in this review.

For educators, the implications of the fact that many children attempting to learn western forms of mathematics in schools in the Asia-Pacific region have very different linguistic and numerical heritages are just beginning to be explored. Throughout the period under review Bell, who directed the Australian Government-funded Asian Perspectives on Mathematics Education Project, made notable contributions in this regard, by his Project's publication of six volumes on materials which drew attention to relationships between mathematics and a number of languages commonly spoken within the Asia-Pacific region. Separate volumes and accompanying videotapes have been published for Chinese, Indonesian, Japanese, Korean, Thai and Vietnamese languages and contexts (see for example, Bell, 1993a).

In addition, the Asian Perspectives on Mathematics Education Project published two volumes of research articles on mathematics education in the Asia-Pacific region (Bell, 1993b, 1995). Many of the articles in these volumes explored the influence of linguistic, cultural, and social variables on mathematics learning in the region (see for example, Barton, Begg, Butel, & Ellis, 1995; Clarkson & Kaleva, 1994; Minato, 1995).

Classroom Discourse

Wittgenstein and Sociolinguistics

During the 1990s studies emanating from the Centre of Mathematics and Science education at Queensland University of Technology have emphasised the need to view discourses in mathematics classrooms from social and linguistic perspectives (see Atweh, Cooper, & Kanes, 1992). Atweh (1993) pointed out that the work was conceived from within the "social critical sociology perspective" (p. 61). It adopted ethnographic methods of data collection, including interviews and classroom observations, and used functional theory of language as the basis for analysis.

Another significant set of studies, also in the domain of sociolinguistics, has emanated from the Institute for the Learning of Mathematics and Language, at Griffith University. The theoretical underpinnings of these studies have been described by Kanes (1992, 1993), who has used the later Wittgenstein concepts of *language-game*, *form of life*, and *family resemblance* to explore learning and teaching interactions in school classrooms. The central sociolinguistic tenet on which Kanes' work has been based is that mathematical concepts have no meaning outside the social context in which they operate, and that therefore mathematics is mediated through language, rather than through thinking. The writings of functional linguists, and especially Halliday, are drawn upon, and special attention is paid to genres of discourse.

Data from Year 9 classes in two private schools in Brisbane were analysed, and relationships between *exterior practice, theory*, and *internal practice* examined. Kanes (1992) defined these terms as follows:

Exterior practices: The domain of practice. What we understand about what we are doing when we teach, learn or perform in a mathematical way.

Theory: ... What we understand about our practice as teaching, learning or mathematics.

Internal practices: ... What we are actually doing when engaged in the activities of exterior practice or theory. (p. 156)

According to Kanes (1992), the language games of mathematics classrooms have a generative function. "In their activity in practice, larger meanings and purposes, forms of life, are generated and these, ... constitute in various ways the theoretical elements of learning, teaching and knowledge" (p. 356).

The idea that mathematical knowledge is not more than a "multiplicity of self-referring practices or activities" is of course linked to contemporary relativist theories of knowledge. It is challenging to reflect on the possibility that the well-known rule for the addition of fractions [$a/b + c/d = (ad + bc)/bd$] is not true a priori, but rather is a consequence of the language that some (though not all) humans have developed to describe particular situations (Clements & Del Campo, 1990). Discourse in classrooms where the addition of fractions is being taught, therefore, is often reduced to the teacher trying to get students to accept a particular language game which is by no means natural.

Kanes and Nisbet (1995) contrasted transmission and meaning-centred models of learning which derive from different epistemological stances, and called for further research along the same lines. Such research typically takes extracts of classroom discourse and seeks to identify the real purposes behind student and teacher behaviour, and indeed to lay bare elements of the hidden curriculum.

Atweh (1993) used a similar sociolinguistic framework to that developed by Kanes to analyse the differences between two Year 9 classrooms. He found that although both were teaching the same topic (linear functions), the rules of the game were very different in the two classes. In one class, the teacher presented mathematics as meaningful activity related to the real world, and in the other, as a set of rules to manipulate symbols. One of the teachers presented mathematics in a way that would be useful for students proceeding onto higher studies in mathematics, whereas the other was merely helping students to cope with the topics laid down. The general atmosphere in the two classrooms was quite different, as were the structures of the languages used. Clearly, the classrooms represented two different cultures.

Other Studies on Classroom Discourse

In separate studies, Frid (1993, 1994, 1995) and Frid and Malone (1995) investigated, from a social perspective, the impressions of mathematics and of mathematics learning of primary and secondary students (in North America and Australia). Frid found that many of the students had a notion of appropriate symbolic procedures, but lacked knowledge of what the symbols represented. Thus, for example, when asked to find the derivative of a quotient, a student wrote "You take the derivative of the numerator times the denominator, and you minus the numerator times the derivative of the denominator. And you put it, but you have to get the derivative of these" (Frid, 1993, p. 31). Students responding to questions of this type typically got the correct answer (or something close to it), but complained that the symbols and what they did with them had no meaning.

A form of language game was being played, and continued throughout the students' learning of calculus. According to Frid (1993), the teacher's role was one of being both a facilitator and a mediator: a facilitator in the sense of providing both questions and examples, and a mediator in the sense of being a liaison between students' language interpretation and related constructed meanings and those of the mathematics community at large. Frid concluded that her interviews with students had revealed that there was a need to "shift away from the traditional 'teacher-as-expert' mode of communication" in mathematics classrooms.

Quite clearly within the Australasian mathematics education research community there has been a heightening awareness of the effects of social and cultural factors on mathematics learning (see, e.g., Beesey, 1995; Davis, Jones, & Pearn, 1994; FitzSimons, 1995; Goos, Galbraith & Renshaw, 1994; Macmillan, 1995; Pearn, 1994). This is, in fact, an international trend (see Bishop, 1992; 1993). Bishop (1992), in a comparative study, showed that the cultural perspective challenges in fundamental ways "the instrumental 'arrow' of intended, to implemented, to attained" (p. 185). According to Bishop (1992):

> The cultural perspective requires us to culturalise the curriculum at each of the levels, and demonstrates that no aspect of mathematics teaching can be culturally neutral. The cultural 'messages' in the educational enterprise are created and manifested by people. People create the national and local curriculum statements, people write the books and computer programs, people bring their cultural histories into the classroom, and people interpret and reconstruct the various messages. (p. 185)

Bishop concluded that this "cultural perspective reaffirms the centrality of people in the educational enterprise, and demonstrates that mathematical knowledge is constructed, interpreted and shaped by people who can exercise choices" (pp. 186–187).

We would add that one of the most fundamental aspects of all cultures is language, and that it is amazing how so many mathematics education researchers have paid little more than lip service to the centrality of language factors in all aspects of mathematics education (Mousley, Clements & Ellerton, 1992).

Expanding the Modes of Communication

"Literacy for the New Millennium"

The last four-yearly review of research (Ellerton & Clarkson, 1992) drew attention to the large amount of positive interaction which had occurred between Australasian teachers of language arts and mathematics, and between language arts and mathematics education researchers. Mathematics educators had learned much from the ideas of language educators such as Graves and Cambourne. The idea of adapting language arts education principles in mathematics education settings is still common (see, e.g., Bickmore-Brand, 1993), but increasingly it is being recognised that Australian mathematics educators, with their long history of research in language aspects of education, have something to contribute to language arts education.

In 1993 the Australian Reading Association invited the Australian Association of Mathematics Teachers to organise special pre-conference sessions on mathematics education, as a preliminary to the first International Conference on Literacy for the New Millennium. The pre-conference sessions attracted a large number of papers from a wide range of mathematics education researchers, and a compilation of 22 papers was published jointly by the Australian Council for Education Research and the Australian Association of Mathematics Teachers under the title *Communicating Mathematics: Perspectives from Classroom Practice and Current Research* (Stephens, Waywood, Clarke & Izard, 1993).

Space does not permit each of the 22 papers to be reviewed here, although reference will be made to some in later sections of this chapter. In this book the emphasis was clearly on research carried out by Australian and New Zealand mathematics education researchers into how the communication of mathematical concepts, principles and skills might be enhanced in informal, non-formal, and formal education settings.

The papers in the book were grouped into five main categories:

1. Communicating mathematical meaning (comprising papers by Cooper, Atweh, Baturo & Smith; Dawe; Frid; and MacGregor);

2. Fostering good classroom practice in mathematical communication (comprising papers by Bell; Bickmore-Brand; Griffiths & Clyne; Williams; and Watson);

3. Students writing in mathematics (comprising papers by Malone & Miller; Stephens; Waters & Montgomery; and Waywood);

4. Assessing mathematical communication (comprising papers by Clarke; Izard & Haines; Meiklejohn & Hedley; and Southwell); and

5. Culture and linguistic background (comprising papers by Begg; Clarkson & Thomas; Kepert; and Nathan, Trinick, Tobin, & Barton).

Linking Theory and Practice: Beyond "Process Maths"

Many publications during the period under review testified to a continuation of the tradition that research into the role of language factors in mathematics learning should both influence and be influenced by teaching practice in schools throughout Australasia. Perhaps the most significant of these, from a research perspective, were two jointly-written books and a series of jointly-written articles by Rachel Griffiths and Margaret Clyne.

Griffiths and Clyne's two latest books, titled *Language in the mathematics classroom* (1994a) and *Maths makes sense* (1994b), have continued the authors' longstanding efforts to help teachers expand the modes of communication in mathematics classrooms. In particular, drawing on the work of Del Campo and Clements (1987), they offer a wide range of activities which would engage students in actively and expressively constructing mathematical meanings through speaking, writing, creating, drawing, and physically moving and doing. Throughout their writings (in addition to the two above-mentioned books, see Griffiths and Clyne, 1993a, 1993b, 1994c, 1995a, 1995b) they stress the need for teachers to plan, develop, and implement mathematics programs in which there is a balance between receptive and expressive communication.

In most of the books and articles cited above Griffiths and Clyne show how children's stories can provide story-shell frameworks, and therefore context, for young children's mathematical investigations. Griffiths and Clyne (1989) themselves are the authors of *Puffin Mathspack*, a pack of 20 different children's books.

Since the mid-1980s, when the Key Group project in Victoria was successful in encouraging many practising teachers to work together collaboratively in action research projects aimed at linking practice with mathematics education research (Robinson, 1989), it has been common for teachers of mathematics to work together while investigating how language factors influence their students' learning. This tradition is represented in the book by Lilburn and Rawson (1993), two primary teachers, in which many of the main themes of mathematics education research over the past decade were adopted. Lilburn and Rawson also provided a wide range of possible classroom activities, each of which is introduced with a small section on how the activity might link with the overall primary curriculum, and in particular with the mathematics curriculum.

Many practising teachers of mathematics have written articles which indicate that they have made conscious attempts to adopt the role of "teachers as researchers" (Downton & Montgomery, 1995; Ferguson & Montgomery, 1994; Horne, 1995). In particular, there have been many instances of teachers attempting not only to expand the modes of expressive communication in their own mathematics classrooms, but also to encourage other teachers in their schools to do likewise (see, e.g., Burnett, 1992; D. Clarke, 1995; M. Clarke, 1995; Costello, 1995; Costello & Day, 1995; Hawton & Watts, 1993; Hollingsworth & Wilson, 1993; Mildren & Collins, 1992; Moya, 1993, 1994; Murphy, Joyce, Mancell, Stojanovski, & Arvanitis, 1994; Nulty & Skoss, 1995; Waters & Montgomery, 1994; Watts, 1992; Williams, 1992).

Writing to Learn Mathematics

Writing-to-learn-mathematics approaches have been increasingly used around the world over the last decade. Ellerton and Clements (1992) identified five forms of *writing mathematics* which are regularly used in some contemporary mathematics classrooms, and proposed a research agenda to evaluate the effectiveness of five different forms: (a) process/conference approaches, (b) problem posing, (c) journal writing, (d) interactive monitoring of children's learning of mathematics, and (e) investigative mathematics project reports. There are, of course, other manifestations of writing mathematics (such as children writing letters to classmates, who have been away from school, about the mathematics they have missed, as originally proposed and used by Ellerton (1988), or projective essays in which students respond to appropriate and interesting stimuli).

With each of these forms of writing, Ellerton and Clements (1992) identified the following research issues.

Research issues related to effects on students. If students participate regularly in a particular form of writing mathematics, are they likely

1. to perform as well as students who do not participate in this form of writing mathematics on standard tests of mathematical skills, concepts and principles?

2. to link more readily their mathematical understandings with their personal worlds?

3. to become more efficient and effective at monitoring their own mathematical thinking so that they improve their own problem-solving and problem-posing performances?

4. to develop more positive affective (e.g., liking/disliking, confidence) responses to mathematics and mathematical situations? In particular, are students likely to develop feelings of ownership over mathematics they construct or write about?

5. to become more aware of their own abilities, attitudes and preferences in mathematics, and to be prepared to modify these in response to their own reflections?

6.*Research issues concerning teachers.* There are at least three issues relating to teachers.

7. From a teacher's point of view, is classroom time spent on writing-mathematics activities time wasted or time well spent from the point of view of developing students' mathematical understandings and positive attitudes?

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8. From a cost-benefit point of view, do students' mathematics scripts provide teachers with sufficiently valuable diagnostic and predictive data to justify the time taken for students to produce the scripts, and for teachers to read and comment on them, and to maintain pertinent records?

9. Certain ethical questions arise: for example, are teachers entitled to assess a student's journal entries?

A research issue for curriculum developers. From a curriculum developer's point of view, does the new genre of mathematical writing demanded by extended projects (such as the investigative projects in the Victorian Certificate of Education) resemble the kind of writing in which professional mathematicians engage, and if so, is this a strong justification for modifying curricula and assessment?

Developing a stronger theoretical base. The research agenda outlined above implicitly suggested that too much of the research in the burgeoning area of writing to learn mathematics was uncoordinated and did not address some of the hard questions which urgently needed to be answered. There have been several reports of studies which have attempted to provide the writing to learn mathematics approach with a stronger theoretical base—a necessary preliminary step, perhaps, towards the design and conduct of research which seriously addresses the issues raised.

One such report was that by Clarke, Waywood, and Stephens (1993), who re-analysed data associated with the journal writing of a longitudinal study, involving 500 secondary students. Perhaps the most significant contribution of the Clarke et al. (1993) paper was the four-column table in which they summarised their approach to describing and analysing student journal use.

The first column was headed Summary, and contained six entries which placed student behaviours in a sequence from simple "Able to regularly copy part or most of the board notes into the journal" to complex "Able to extrapolate from material presented in class, or in a text, and reshape it in terms of own learning needs."

The second column was headed Exemplification, and contained six entries (one for each of the behaviours in the Summary column) in which indicators of the behaviours were listed. The third column concerned the Questions Asked by Students, and the Fourth, Applications of the Mathematics which the students identified in their journals.

A different, more philosophical, perspective was provided by Waywood (1993), who offered an image of writing in mathematics as a process whereby the self is redefined. Waywood started his paper with a practical problem—why do students appear to have difficulty in constructing reports? In attempting to answer this question, he compared the distinction between the concepts of *story* and *study* with that of *narrative* and *reason*. He also referred to the history of the distinction between *language as lived* and *language as reflected on*, and discussed its relevance to the teaching of mathematics.

Waywood's theoretical stance is complemented by his large experience in encouraging students and staff in a school to engage in journal writing in mathematics, over a number of years (see Waywood, 1992). He recommended that students should be encouraged *to tell a story* simultaneously with *pursuing a study*. For him, an appropriate heuristic might be to integrate telling a story—a chronicle account, record, or journal—with doing a study (Waywood, 1993).

Shield (1995a, 1995b) and Shield and Swinson (1994) attempted to develop a structure for describing student writing in mathematics. Shield (1995a) reported a study in which he examined several hundred examples of student expository writing over a two-year period, and identified the following list of the types of elaborations produced by students. It was unlikely that all of them would be found in any one presentation—for example, the felt need to provide a diagrammatic representation would probably be related to the mathematical topic being discussed.

1. nucleus: definition or general statement of procedure;
2. goal: statement identifying the concept or procedure being explained;
3. demonstration: worked example of the concept or procedure elaborated with (a) symbolic representation, (b) verbal description, (c) diagrammatic representation, and (d) statement of convention;
4. justification: statement justifying parts of the procedure in terms of known ideas;
5. link: reference to prior knowledge or everyday experience;
6. practice: exercises to be attempted by the reader by modelling on the demonstration (Shield, 1995a, p. 38).

There are other distinctions in the writing genres which can be usefully made. Waters and Montgomery (1993), for example, distinguished between routine writing and recording—which includes writing to explain—and exploratory writing in which ideas are explored and new understandings developed. Waters and Montgomery included numerous examples of children's exploratory writing in mathematics, the children being aged between 5 and 12 years. They located their discussion in a constructivist framework, arguing that "through diary writing, writing to explain, mathematics logs, proposals, reports, resumés, and resultant discussion, students firstly develop the language necessary to describe their mathematics, and secondly they come to know the limits of the accepted body of mathematical knowledge" (p. 207). They claimed that, partly as a result of the mathematical writing aspect of their own mathematics programs, their students were "more mathematical and more literate mathematically" (p. 207).

Language and the Assessment of Mathematics Learning

In recent years the virtues of using a range of assessment procedures to gain different perspectives on students' receptive and expressive mathematical understandings have been widely advocated (see, e.g., Clarke, 1992, 1993).

Clarke and Jasper (1995) discussed constraints on authentic classroom assessment (e.g., time, teachers' subjectivities, and the difficulty of identifying individual student performance from work done within cooperative groups, or in projects undertaken outside the classroom). They then suggested strategies which might minimise these constraints: the annotated class list, practical tests, student constructed test items, student self-assessment, student mathematics journals, student mathematics portfolios, student investigative projects, student-generated solutions to challenging problems, and open-ended problems. A table in the Clarke and Jasper (1995) paper which provided an overview of constraints on classroom assessment has proved to be particularly useful in that it has established a framework for reflection on pertinent issues. The table named each of the strategies listed above, and then commented on constraints for each in regard to use of teacher time, authenticity, subjectivity and consequences.

Southwell (1993) presented and discussed data from journals written by University of Western Sydney mathematics education students. She also attempted to develop a method of evaluating journal entries, and concluded that in order to do this well, "the teacher needs to be aware of what is meant by understanding, habit and intelligent learning, procedural and conceptual learning, and surface and deep knowledge structures" (p. 234). These different ideas and terminologies are linked through "the process of reflection, which in turn involves association, integration, validation and appropriation" (p. 234).

According to Southwell (1993), examination of student writing needs to acknowledge the stage in reflection that the student has reached. Levels of writing were summarised, and the issue raised concerning whether assessment of student writing in mathematics will either improve student learning, and/or benefit the teacher.

In case one is tempted to be critical of the lack of precision in the definition of levels in this context, one needs to recognise that, between 1991 and 1993, there was a large, nationally-funded project aimed at establishing levels of behavioural response on the various strands of mathematics. The national Mathematics Profile (Curriculum Corporation, 1994a) which adopted an outcomes-based approach, attempted to provide a basis for classifying student behaviours across five content and one process strand into eight levels. One of the major aims of the developers of profiles was to standardise monitoring and reporting procedures across the nation. Although the Mathematics Profile documents drew sharp criticism from some mathematics educators (see, e.g., Ellerton & Clements, 1994), there can be no doubt that it encouraged teachers to use a wider range of assessment procedures than they had in the past.

A book of work samples and an accompanying guide to the Mathematics Profile (Curriculum Corporation, 1994b; Olssen, Adams, Grace, & Anderson, 1994) were replete with examples of students' writing, and detailed discussion on how each example could be associated with one of the profile levels. According to Meiklejohn and Hedley (1993), the underlying intent of the "nationally accepted standards ... is that students around the country will be compared and rated against a set of common and accepted benchmarks" (p. 252). It should be noted, however, that to state that the profile was nationally accepted was not in accord with the strong protests voiced by mathematicians and mathematics educators who were concerned about the quality of the mathematical thinking evident in the Profile documents, and the lack of an adequate research base for the levels proposed (Ellerton & Clements, 1994).

Meiklejohn and Hedley (1993), in discussing the application of the profiling concept in Queensland, stressed the emphasis which the Profile places on communication in mathematics. According to Meiklejohn and Hedley, Queensland mathematics curriculum documents stated categorically that any set of learning activities in mathematics must involve students in cooperative activities which involve oral discussion. The aim of such activities should be to communicate ideas, extend understanding through students having to clarify ideas, and build on a variety of social and cultural perspectives. Meiklejohn and Hedley reproduced Year 5 children's writing in response to a given mathematics problem in order to illustrate how the same question can generate various levels of responses.

Lessons From the Victorian Certificate of Education's Common Assessment Tasks

Since 1991, the Victorian Certificate of Education has incorporated a broader range of assessment styles for all mathematics subjects taken by senior secondary students in Years 11 and 12. Of interest in the context of this review is that in 1991 and 1992 two of the four Common Assessment Tasks (CATs) involved the preparation and submission of written reports over a period of time. CAT 1 was an investigative project which required the preparation of a 1500-word written report of a mathematical investigation into a set theme; CAT 2 required the submission of an individual response to one of four challenging problems (see Stephens, 1993, for further details).

Something of the dramatic effect on pedagogy that followed the introduction of the projects was captured in the book by Martin (1993a) and in the papers by Martin (1993b) and Mousley (1990). For example, during the weeks that students were preparing their CATs 1 and 2 reports there was an unprecedented demand on mathematics books in school, university, and public libraries. However, the main effect from a teaching/learning perspective was that students were required to produce polished versions of mathematical writing in which mathematical concepts, principles, and applications were explained, interpreted, and developed. In other words, language factors were accorded a more direct and different role in mathematics learning than ever before.

In 1993, the CAT 2 problem-solving task was dropped, largely as a result of public reaction to the controversies mentioned above. The University of Melbourne Department of Mathematics in particular was convinced that under the new VCE program, the mathematical standards of beginning tertiary students had declined sharply (Swedosh, 1995). Waywood (1993) was dismayed at the decision to drop the CAT 2 task, and claimed that "the connection between writing and learning has motivated many pedagogical practices and movements—writing across the curriculum, writing to learn, and writing in mathematics" (p. 162).

What is most pertinent to this review is the extent of the *ripple effect*, that is to say the effect the introduction of these forms of assessment at the highest level of secondary schooling had on school programs at lower levels. Research has shown that following the introduction of the new modes of assessment at the senior secondary level, teachers at junior levels have been more inclined to give students extended project work and extended problem-solving tasks (Blane, 1992; Clarke, Stephens & Wallbridge, 1993; Stephens, Clarke, & Pavlou, 1994).

Ellerton and Clements (1992) asked the following researchable questions with respect to CATs 1 and 2:

1. Is the genre of writing expected in these reports fundamentally different from that expected in school mathematics in the past?
2. Do students who satisfactorily complete an investigative report acquire research techniques that were never previously acquired in school mathematics?
3. Is the time spent on researching and writing an investigative report justified, given a criterion of satisfactory performance in subsequent tertiary mathematics courses?
4. Is the time spent on researching and writing an investigative report justified, given a criterion of being able to carry out independent research investigations in later life? (p. 157)

Although Questions 1 and 2 have not yet been researched, directly, the studies by Clarke, Stephens, and Wallbridge (1993) and by Stephens, Clarke, and Pavlou (1994) strongly suggest that both should be answered in the affirmative.

In relation to Question 3 (concerned with preparation of students for tertiary mathematics courses), Swedosh's (1995) doctoral study generated data which suggested that, from a tertiary perspective, time spent on CATs 1 and 2 was effectively wasted, and that the new VCE mathematics subjects did not prepare students adequately for tertiary mathematical study. However, the main dependent variable in this study was performance on a test based on content in the old (pre-VCE) courses and first-year tertiary mathematics courses which had remained constant for some years. From a wider perspective which took into account more than mere preparation for the existing tertiary mathematics courses, it could be argued that the research design biased the result of the study.

Question 4 will need to be researched in the future.

Gender-related differences in performance on the VCE CATs. Leder (1994a, 1994b) and Cox (1993, 1994) have produced statistical analyses showing that although for the 1992 VCE mathematics subjects, females tended to have higher mean assessments for CAT 1 and CAT 2, this was not usually the case with assessments for CAT 3 and CAT 4 (which were based on traditional pencil-and-paper, externally-set, timed examinations).

When the extended problem-solving CAT was dropped from the VCE Mathematics schedule in 1993, the pattern of 1992 results continued, with females tending to perform better on CAT 1 (extended project), and males better on the more traditional CATs 2 and 3 pencil-and-paper examinations (Cox (1994, 1995). As Leder (1994a) concluded, collectively the data "provide a very clear example of the way in which the format of assessment tasks can influence the construction of gender differences in mathematics performance" (p. 41).

Clearly, the introduction of the extended projects and problem-solving tasks in the original VCE Mathematics design generated data which revealed that certain forms of assessment for non-trivial mathematics are likely to produce interesting gender-related patterns of performance data. Language factors would appear to be a major contributing factor to the differences observed. Unfortunately, the issue of what is the *best* balance ultimately became a political matter, and the potential educational benefits (Waywood, 1993) of the extended problem-solving CAT—at all levels of education because of the strength of the ripple effect—have been lost.

Why Children Make Mistakes on Written Mathematics Tasks: Continuing the Newman Tradition

During the period under review the Newman technique (1983) for analysing errors made on written mathematical tasks, originally developed in the mid-1970s by the Australian language educator, Newman, was again widely used by mathematics education researchers, both in Australia and in Southeast Asia (Bulbert, 1995; Clarkson, 1992a, 1992b; Clements & Ellerton, 1992, in press; Ellerton & Clements, 1995; Forbes, 1995; Graham, 1995; Kaur, 1995; Sulaiman & Remorin, 1993).

Increasing Proportions of Language-based Errors

As with all of the earlier Newman studies, data which have been generated have pointed to the overwhelming importance of the reading, comprehension, and transformation stages of problem solving. In each of the 1992–1995 studies, between 45% and 80% of all errors analysed were classified as belonging to one of these three categories, and of the remaining error causes, between 10% and 20% were careless. In most studies process-skills errors accounted for between 5% and 15% of all classified errors.

Clements and Ellerton (in press) have carried out two large studies in which an extended form of the Newman interview procedure was used. Because of the modified approach, and the use of a correspondingly modified form of analysis, these studies have generated qualitatively different results from previous Newman studies.

For one of the studies, 116 Year 8 students, in 12 classes in 5 schools in New South Wales and Victoria answered a set of written questions which all the teachers of the students agreed that their students should not have had difficulty comprehending or solving—half of the questions were in multiple-choice form, and the other half in short-answer form. When Newman interviews were conducted (by five trained interviewers) it was found that 80% of errors first occurred at the reading, comprehension and transformation stages, and only 6% of errors first occurred at the process-skills stage.

This study with Year 8 students was different from previous Newman studies in that Newman interviews were conducted for all questions for all students in the sample—including those questions for which correct responses had been given. In fact, data obtained from the interviews revealed that for about one-fourth of their correct responses, students either did not understand or had only partial understanding of the concepts and skills which the questions were testing. In such cases, Newman error categories were also attached to these correct responses, but these are not accounted for in the above statement that 80% of errors occurred at or before the Transformation stage.

Another aspect of this study with Year 8 Australian students is of interest. It was found that different questions produced quite different error patterns. Thus, for example, for the following question, 40% of the errors were of the Process Skills variety, and only 15% were in the reading or comprehension or transformation categories:

> Ice-creams cost 85 cents each, and apples cost 45 cents each. How much altogether would 7 ice-creams and 5 apples cost?

By contrast, however, for the following question, only 6% of the errors were of the Process Skills variety, but 90% were placed in the reading or comprehension or transformation categories:

> Arrange the following fractions in order of size from smallest to largest:
> 1/3 1/4 2/5

The second Clements and Ellerton (in press) study involved a comparative investigation of errors made by 101 Year 7 students in two junior secondary schools in Malaysia and 61 students in Year 6 or Year 7 in two primary schools in Perth, Australia. The same pencil-and-paper test instrument was used, with the Malaysian version being in *Bahasa Melayu* and the Australian version in English. Every teacher of the pupils agreed that the questions were well worded, and were at an appropriate level of difficulty. Newman interviews were conducted by trained interviewers in the language of instruction for the children—*Bahasa Melayu* (in Malaysia) and English (in Perth). Altogether, about 1480 errors were analysed.

The patterns of error classifications in the two samples were similar, with between 60% and 70% being in either the comprehension or transformation categories, and 10% (for the Malaysian sample) and 15% (for the Australian sample) in the process skills category.

Bulbert (1995), Forbes (1995) and Graham (1995) also used Newman-type interviews to investigate correct and incorrect responses given by Year 8 students in New South Wales to multiple-choice and short-answer questions. They also found that over 70% of errors occurred at the reading, comprehension, or transformation stages, and that many correct responses were given by students who did not understand or who only partly understood the mathematical concepts with which the questions were primarily concerned.

Kaur's (1995) study involved 626 students from Years 5 to 8 in Singapore, with 139 of these being interviewed. The investigation, which combined Newman error analysis techniques with procedures from Ransley's (1979) problem-solving model, generated data which, because of the different error classifications, are difficult to compare with data from most other Newman studies. Clearly, though, Kaur also found that a high proportion of errors occurred at what Newman referred to as the Transformation stage (but Kaur referred to as "strategy selection"). Also, like Clements and Ellerton (in press), Kaur emphasised that different error patterns arose for different questions.

Implications for mathematics diagnosis and remediation. In all of the Newman studies conducted so far it has been found that error profiles for students obtaining the same total score on a test can be quite different. Thus, for example, two students might get 8 questions correct out of 24 on a test. One of the students could have made 12 comprehension errors and 4 transformation errors; the other 12 process-skills errors and 4 careless errors. In such a case it is evident that the two students need quite different kinds of remedial help. Such analysis strongly suggests that the common practice of streaming students purely on the basis of "number of questions answered correctly on a standardised test" would be difficult to defend on purely educational grounds.

Findings from Newman research raise the difficult issue of what educators can do to improve learners' comprehension of mathematical text or their ability to transform (that is to say, to identify an appropriate sequence of operations which will solve a given word problem). At present, relatively little research is being conducted on this, although MacGregor (1993a, 1993b, 1993c), and MacGregor and Stacey (1995) have provided advice for teachers on aspects of language which need to be taken into account in mathematics classrooms. The classroom implications of this issue should become a focus of mathematics education research over the next decade.

Comprehension

Vocabulary. One of the major findings from Newman research is that almost all children in Australasian schools can decode the text of standard mathematics questions—that is to say, they can read the words and symbols. However, the next step of comprehending what they have read—that is to say, making sense of the whole package of words and symbols—is much more problematic.

One of the factors influencing comprehension is knowledge and understanding of specialist vocabulary. Marshall, Gilmour, and Lewis (1992) carried out a study in which they tested 2111 Papua New Guinea students from secondary and tertiary levels on their understanding of the following 16 important mathematical words:

*component**
correspond
estimate
external
illustrate
*initial**
*limit**
*magnitude**
*maximum**
negative
proportion
*random**
*rate**
*sequence**
*standard**
substitute

Most of the Year 7 students did not have a firm understanding of any of the words as assessed by their recognition of (a) a synonymous expression without context, (b) whether a given word was used correctly in a sentence, (c) the meaning when a given word appeared in a science-context stem, and (d) the meaning when a given word appeared in a non science-context stem?

The researchers concluded that Papua New Guinea students experience mathematical difficulties "due to their limited lexical knowledge." They recommended that mathematics teachers attempt to teach the mathematical meaning of words to students directly, and that students not only be encouraged to engage in group discussions prior to solving mathematical problems, but also to draw pictorial representation of problems, and to estimate answers.

Miller (1993), and Malone and Miller (1993) reported an investigation into the understanding of standard mathematical vocabulary (e.g. *fraction*, *factor*, *diameter*, *divide*, and *quotient*) by 2371 students in Years 8 to 11 in rural and urban schools in Western Australia and the Northern Territory. In summary, the results indicated that students had a surprisingly poor grasp of key words and, in particular, only small proportions of students at each Year level could adequately express meanings of the five words listed above (fraction, etc.) in their own words. Females had a better grasp of vocabulary than males, and urban students demonstrated a better understanding than rural students. Non-Aboriginal students gave almost twice as many correct responses as Aboriginal students.

The seriousness of the situation revealed was suggested, perhaps, by the fact that only one-third of Year 11 students could demonstrate any understanding of the word *factor*. Very few students at any level demonstrated an understanding of the term *quotient*, but it could be argued that, in the 1990s, this is rarely used except by mathematicians, and then only in circumstances such as recalling the rule for the derivative of a quotient.

Nathan, Trinick, Tobin and Barton (1993) pointed out that, in the Maori language, mathematics vocabulary has not been well developed over the past 100 years, and has not kept pace with developments in technology. The construction of new words has been left to the Maori Language Commission which has a considerable resistance to the use of transliterations—that is to say, giving English words a Maori look and sound. A vocabulary booklet developed by the Maori Language Commission and mathematics teachers was published in 1991, and Nathan et al. (1993) reported that a major revision was underway. It is anticipated that the revised booklet will "speed up the integration of Maori mathematics vocabulary" (p. 298).

Semantic structure. It is disappointing, however, that in the last four years there have not been more Australasian studies into the effect of semantic structure on comprehension. Clements and Ellerton (in press), in their Newman study (mentioned earlier) included questions with various semantic structures; for example,): "Roslyn is 12 days older than Mary. If Roslyn's birthday is January 29, on what day is Mary's birthday?" (A question with which 74% of the Malaysian sample and 70% of the Australian had difficulty.) The researchers found that semantic structure influenced problem difficulty far more than vocabulary or the application of process skills. Unfortunately professional development support materials in Australia and new Zealand rarely draw attention to the importance of semantic structure.

MacGregor and Stacey (1995) showed that most students in Years 7 to 10 in 22 secondary schools in Melbourne, were able to read and comprehend almost all the parts of five questions which were related to real-life situations (instructions on the label of a medicine bottle; interpreting airline and train timetables; bank deposits and withdrawals; and words in a sentence). However, only a few of the questions involved anything other than the simplest of semantic structures. It is not surprising that the question which proved to be the most difficult was the one with the most complex semantic structure.

Other research reported by MacGregor and Stacey (1994) and Stacey (1994) indicated that many students who can read ordinary English often misinterpret information when meaning depends on semantic structure. Most students find it much easier to interpret correctly "A line is 2 cm longer than a match. Draw the line." than "A match is 2 cm longer than a line. Draw the line." Stacey (1994, p. 3) pointed out that the following wording for an algebra question would be likely to cause similar difficulty: "In a class there are six more boys than girls. If we write p for the number of girls and s for the number of boys, which of the following equations are correct?"

Christie (1995) drew attention to the cultural specificity of certain elements of semantic structure such as the polarised comparative terms *more than* and *less than*. There are only two studies, reported by Galligan (1995) and M. Thomas (1994), that refer to the wider psycholinguistic literature. Galligan, after identifying differences in the structures of Mandarin and English languages, reported a study in which the performance and time taken by five Chinese and five Australian business studies students attending the University of Southern Queensland were compared. The students answered questions on three tests, which were in their first language—one test involved pure calculations, another comprised context-free word problems, and the third word problems in context. Answers were investigated in a one-to-one, confidential interview situation.

Galligan (1995) found that the Chinese students made fewer errors and answered more quickly than the Australian students. She attributed the performance difference to structural differences in the two languages. Differences in syntactic and semantic structures, the use of connectives, sentence density, etc., were noticeable, and although the questions were translated as far as was possible so that they said the same thing in the same way, the Mandarin forms of the questions were not as complex as the English forms.

M. Thomas (1994) studied the effects of the wording of tasks which required a given sentence to be translated into an algebraic equivalent. Two examples of tasks were:

> A school has v girls and t boys. There are 10 more girls than boys. Write an equation relating v and t.
>
> A band makes four times as many singles as albums. It makes q albums and z singles. Write an equation relating q and z.

These questions were given to 75 Year 9 students at a selective school in New Zealand and 128 university teacher-education undergraduates.

It was found that 13% of the Year 9 students and 40% of the university students gave incorrect responses to the first of these questions, and 43% of the Year 9 students and 44% of the university students responded incorrectly to the second question. For the second question, M. Thomas (1994) identified semantic structure as the major factor underlying the difficulties encountered by students. Although semantic structure was not planned to be central to the studies by Outhred (1995), Mulligan (1995), and Mulligan and Mitchelmore (1995), in fact their findings drew attention to the importance of semantic structure for children's comprehension of mathematics.

Bilingualism and Mathematics Learning

The 1988 census in Australia reported that about 15% of Australia's population spoke a language other than English in their homes (Clarkson & Dawe, 1994). Some schools in large cities have particularly large concentrations of non-English Speaking Background (NESB) students. MacGregor (1993c) cited the case of an inner-city primary school in Melbourne where English was the first language for less than 5% of the children, and significant proportions of children spoke Vietnamese, Chinese or Turkish. At the same school there were children who spoke Greek, Nauruan, Serbian, Croatian, Khmer, Portuguese, Urdu, Macedonian, Fijian, Spanish, Laotian, Arabic, Polish, and Indonesian.

There are many classrooms in large cities in Australasia in which there are 10 or more different first languages represented among the children. In almost all of these classrooms, the language of instruction is English. Effectively, a form of what has become known as *immersion* occurs, where everything the learners do involves them in interaction with the symbols and sounds of a language which is their second or third (or sometimes fourth or fifth) language.

Research on Bilingualism

Research into the effects of bilingualism on mathematics education has been carried out for over a decade by Clarkson and Dawe who have both been strongly influenced by the linguistic theories of Cummins (1979, 1986). Cummins took it for granted that, with learners who speak two or more languages, there is clearly an interplay in the learning process between the language codes. This may assist or may detract from learning. If the use of more than one language by individual learners is an accepted part of classroom culture then this can produce a cognitive advantage once a threshold of competence in the two (or more than two) languages is reached. If, on the other hand, learners are made to feel guilty for employing any language other than the language of instruction, "then a subtractive environment with deficit cognitive outcomes results" (Clarkson & Thomas, 1993, p. 265).

Clarkson and Dawe (1994) are currently engaged in a joint longitudinal study of the mathematical problem-solving processes of 600 Italian, Arabic, or Vietnamese speakers who were in Year 4 in 1994 in schools in Sydney and Melbourne. In the research, Clarkson and Dawe are investigating the levels of competence which the students have in their different languages, including English, the extent to which individual students switch between languages during problem solving, and the effect of time and schooling on such code switching.

A series of studies into the effects of monolingualism, bilingualism and multilingualism on mathematics learning in Papua New Guinea have been reported by Clarkson (see Clarkson, 1992c; Clarkson, 1994; Clarkson & Kaleva, 1993). Analyses of data from these studies would appear to provide support for mathematics instruction in Australasia schools to be in their first language, at the same time as they are learning to speak English. In Queensland, in particular, the idea of immersion has been taken further, with English-speaking-background primary school children learning mathematics in another language as part of Languages other than English (LOTE) programs (Berthold, 1995). Support for strengthening the first language in schools is given by Thomas (1995) and the study of VCE results to be reviewed later in this chapter. Thomas (1995) also emphasised that first languages and not so-called economic languages like Japanese and Mandarin should be studied.

In New Zealand, bilingual education programs focus on the Maori language, and a framework for teaching mathematics bilingually to Forms 1 and 2 students has been developed (Nathan, Trinick, Tobin & Barton, 1993). This framework aims to integrate bilingual education and mathematics education by:

1. extracting mathematical ideas from within the Maori culture which support cultural themes;
2. providing opportunities for students to connect ideas within and across cultures, and within and outside the mathematics curriculum; and
3. emphasising Maori language and Maori structures in the organisation of mathematics programs. (p. 296)

However, teachers in bilingual classrooms in New Zealand have found that there is a large range of fluency in the Maori language.

As would be expected from Cummins' (1979) threshold hypothesis, students who are fluent in neither English nor Maori tend to experience difficulty with mathematics. This is exacerbated by the fact that adequate resources which reflect a Maori perspective are not available. Furthermore, there are relatively few specialist mathematics teachers who are fluent in the Maori language, and most if not all teachers in the bilingual program feel overwhelmed by the commitment expected of them (Nathan et al., 1993).

These problems notwithstanding, the latest mathematics curriculum documents published by the New Zealand Ministry of Education include a Maori version of the national curriculum (Ta Tahuhu o te Matauranga, 1994).

Non-English Speaking Background Students and Examinations

Leder, Rowley, and Brew (1995) analysed the results obtained by students for whom English is their first language, and by students for whom English was not their first language (ESL students) on the six assessment categories of the four Classroom Assessment Tasks (CATs) of the 1992 VCE—the investigative project, the challenging problem, the multiple-choice examination, and the extended-answer examination. For each subject ten letter grades were awarded (A+, A, B, ..., E), and these were converted to numerical values (10, 9, 8, ..., 1). This done, the data were analysed statistically, and it was found that on 23 of the 24 comparisons, ESL students had a higher mean result than non-ESL students.

Factor analysis carried out by Leder et al. (1995) generated a 2-factor structure, with CAT 1 and CAT 2 results loading heavily on one factor, and CAT 3 and CAT 4 on another. Other analyses, involving consideration of results on VCE English, made it clear that the first factor could reasonably be identified as a Language factor, and the second factor a Mathematics factor.

Perhaps the most interesting aspect of the Leder et al. (1995) results was that the ESL students actually outperformed non-ESL students on CAT 1 and CAT 2 Mathematics, despite the fact that these two CATs required extensive technical—but nonetheless creative—writing in the English language. Leder et al. commented that the generally large standard deviations of the ESL group suggested that "relatively high performance of some may mask the relatively poor performance of others within the ESL group" (p. 431). Be that as it may, this purely statistical finding does raise important questions, not the least of which is how can it be the case that, overall, ESL students can perform so well on language-based CATs?

One possible explanation arises from a line of thinking presented by Thomas (see Clarkson & Thomas, 1993; Thomas, 1995). Clarkson and Thomas (1993), in attempting to explain their observation that, of students studying tertiary mathematics in Victoria, NESB students born overseas were over-represented and NESB students born in Australia were under-represented, invoked Cummins's (1979) *threshold hypothesis*. They argued that it was likely that NESB students born overseas are likely to have better facility with their first language and English than NESB students born in Australia. The overall facility with language creates, perhaps, an overall cognitive structure which is better placed to deal with mathematics tasks, including language-based mathematics tasks.

Clarkson and Thomas' (1993) conjecture needs to be tested through systematic research. There are other likely explanations. For example, it could be argued that a significant proportion of NESB students born overseas come from high SES families, and it makes little sense to compare them with NESB students born in Australia.

Further, as Bishop and Larkin's (1994) review indicated, most mathematics teachers have neither the resources nor the training needed to provide adequate support for NESB students in mathematics classes. More than 90% of teachers surveyed stated that they definitely needed further professional development training for their work with NESB students. Although a few schools have mathematics and science staff with ESL training, these are still in a minority. "More importantly there are indications that the issues have not been addressed by whole faculties but only by particular staff, if at all" (p. 103).

Open-Ended Questions

Given the large amount of interest in open-ended questions among mathematics educators, teachers, and curriculum developers, it is surprising that during the period under review the amount of Australasian research into the effectiveness of open-ended tasks for learning mathematics was limited.

Reluctance to Give More than One Answer

Clarke, Sullivan, and Spandel (1992) investigated the issue of whether something could be done about the apparent reluctance of students in Years 6 to 10 to provide more than a single answer to such open-ended questions as "If a rectangle has a perimeter of 30 units, what might be its area?"

Clarke et al. (1992) found that this reluctance to give more than one answer was not confined to mathematics, but was also found in three other areas of the curriculum which were investigated (English, social studies, and science). Furthermore, in mathematics and English, the disinclination to give more than one response was evident at both Years 7 and 10 (but in social science and science, Year 10 students were more likely to give multiple responses than Year 7 students). Male and female students were equally disinclined to give multiple responses.

When students were explicitly requested to seek multiple solutions, it was found that there was a significant increase in the number of multiple responses in all four areas of the curriculum investigated. With mathematics, the effect of *cueing* was particularly noticeable at Year 10.

Two of the concluding comments in the Clarke et al. (1992) paper were of special interest. The first was: "It may not be realistic to expect students of any age to access recently-acquired skills in open-ended or problem-solving situations" (p. 220). The second sets the scene for this brief review of Australasian research into open-ended questions: "Substantial additional research is required if we are to understand the meanings constructed by students in responding to open-ended tasks sufficiently well to employ such tasks with success in mathematics classrooms" (p. 220).

Language, Thinking, and "Goal-Free" Problems

Owen and Sweller (1985) claimed that open-ended, or goal-free tasks can facilitate learning because the learner is not required to attend to directed goals and therefore has more processing capacity available to explore different aspects of the problem tasks.

Note that Sweller (1992), in repeating the claim that learning is enhanced by goal-free problems, cited several of his own studies carried out over a period of about ten years. He stated that the result had been obtained "over many experiments using geometry, trigonometry and kinematics problems" (p. 53). Sweller went on to say that goal-free problems require less time to solve than equivalent conventional problems, but result in "improved performance in subsequent test problems" (p. 53).

Over the past four years, however, there have been a few reports written by Australasian mathematics education researchers in which data provides support for Owen and Sweller's (1985) claims (see chapter in this review by Chinnappan and English). The findings of a study by Sullivan (1992), for example, would appear to be at odds with the Owen and Sweller conjecture. Sullivan compared the effects on

Grade 6 students, from two classes in a primary school in Melbourne who were given seven one-hour lessons on length, perimeter and area. The students were randomly allocated to an experimental and a control group. The program for the experimental group consisted totally of students responding to *good* questions of the open-ended variety. There was no direct teaching. The control group was taught the topic in a traditional way, the approach being based on a commonly used textbook.

Sullivan (1992) found that there was no statistically significant difference in the gain scores for the two groups. Very few multiple answers were given to the four good questions in the post-treatment instrument. Sullivan concluded that neither group developed the capacity to tackle open-ended questions. Further there were no statistically significant differences between the groups in gain scores on attitude.

Scott (1994) studied the problem-solving processes used by two Year 6 students who had been involved in open-ended mathematics classroom activities. Both of these students demonstrated performance improvements on open-ended tasks. Although Scott made it clear that she accepted the limitations of her study, she concluded that open-ended tasks provided opportunities for students (a) to give multiple responses; (b) to become involved in divergent thinking; (c) to communicate more freely with other students; (d) to explore applications of mathematics within given contexts; and (e) to assess their own, and other students' mathematical understandings.

Sullivan, Bourke, and Scott (1995) found that a statistically significant greater proportion of the 30 students in a Year 6 class responded correctly to item P3 below than to either item P1 or item P2:

P1: What is the perimeter of this shape?

[A diagram showing a rectangle 10 cm x 4 cm was given but the lengths of the side were not shown]

P2: Draw a rectangle with a perimeter of 8 cm.

P3: A shape has a perimeter of 16 cm. What might the shape look like?

Sullivan et al. (1995) claimed that one would have expected P1, P2, and P3 to have increased in difficulty in the order presented, and the fact that this did not occur was considered to be consistent with the findings of Owen and Sweller (1985).

The absence of interview data in the Sullivan et al. (1995) paper was unfortunate because such data might have thrown light on why the Year 6 students found P1 relatively difficult. We would suggest that, in fact, the wording of P1 is ambiguous. Another interesting result in the Sullivan et al. (1995) study was that, although a ten-week training study succeeded in getting many students to give multiple answers to P3, on a posttest given three months later, most students had reverted to giving just one answer.

More Research Needed on the Effects of Open-ended Questions

The balance of research and classroom experience seems to suggest that school children find it difficult to respond in divergent and creative ways to open-ended task situations. One can conjecture a number of explanations for this. First, the traditional culture associated with mathematics classrooms has, as an integral component, closed questions which can be solved quickly. Open-ended questions are unfamiliar, and therefore threatening to many students. Second, one of the reasons why they are threatening is because in order to respond adequately to an open-ended question, respondents need to locate the question within their cognitive structures and recognise links with other aspects of mathematics. They then need to pose questions of their own before they are in a position to give multiple solutions.

For example, for the open-ended question "If a rectangle has a perimeter of 30 units, what might be its area?" respondents seeking multiple solutions probably need to ask questions such as: "What side-lengths does the rectangle have?" "What side-lengths might the rectangle have?" "Can the area be different if the perimeter is the same?" "What happens to the area if you have a long, thin rectangle?" "What is the maximum possible area?" "What is the smallest possible area?" "Is a square a rectangle?"

Such questions arise almost automatically in the mathematically trained mind. But for young learners grappling with the concepts of perimeter and area, these questions are far from automatic—and yet multiple answers cannot be provided unless at least some of the questions (or other appropriate questions) are posed. Thus, despite Owen and Sweller's (1985) claim that open-ended questions can reduce cognitive load, for many learners the opposite could be the case. In fact, as Sullivan (1995) has noted, open-ended questions demand "a higher level of thinking than does the conventional question" (p. 177).

Cheeseman and Doig (1995) suggest that "open-ended questions have the potential to provide a basis for more effective, reflective teaching, where action is based on understanding students' thinking" (p. 490). They added, "It is intimidating to have the full range of this thinking exposed, but once done, is taking appropriate action not the real challenge of teaching?" We agree with these sentiments, but wonder what research has shown on the question, "What is appropriate action for the teacher?" Sullivan (1995) has provided some helpful advice to teachers in regard to that question. He has suggested that for any open-ended task teachers should plan and use associated *preliminary prompts* and *extension questions* to provide structure which will assist teachers and students to make the most of what are, potentially at least, educationally rich situations.

A research agenda needs to be developed and followed for this challenging development in mathematics education. We need to move beyond the rhetoric.

Cooperative Group Learning

Advocates of Cooperative Group Strategies which Facilitate Interaction

Both *A National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991) and *Mathematics in the New Zealand Curriculum* (New Zealand Ministry of Education, 1992) drew attention to the potential benefits which may accrue when students work together, cooperatively, on problems. Nevertheless, despite official advocacy of cooperative learning strategies, there has been a noticeable absence of Australasian research findings which provide support for what has been recommended.

A number of papers appeared during the last four years in which advice was given to teachers on how cooperative groups should be used for the teaching and learning of mathematics. Typically these reports have not presented data to support the writers' claims. For example, Bellingham (1994) reported on her use, as a teacher, of cooperative groups for mathematics problem solving in Grades K–2. Bellingham emphasised the importance of choosing the right activities for the right groups. She stated that she always chose tasks that the groups could complete without teacher assistance. She often spent the first 15 minutes of a mathematics group-work session introducing the task to the students, and then, after the group work, 15 minutes were dedicated to discussing issues which had emerged during the session.

Smith (1994) suggested a strategy by which traditional "practice" problems could be converted into "clues" and these were given to members of cooperative groups. Thus, for example, the typical question "What is the volume of a container with dimensions 30 cm x 20 cm x 50 cm?" was converted to a task in which the students were told they needed to find the height of a container from the following four clues: (a) The container holds 30 litres of water; (b) The base of the container is 600 cm^2; (c) The base of the container has a perimeter of 100 cm; and (d) The shape of the container is a rectangular prism. Smith went further and recommended that each individual should be given all of the clues first, and should attempt to solve the problem. They should then form pairs in which they discuss their answers, and reach an agreed-upon solution. The set of solutions should then be put forward for whole-class discussion.

Anderson (1994) is another to have stressed that cooperative group learning should not be seen as a strategy which replaces whole-class teaching. For Anderson, small group discussion is an appropriate strategy which can facilitate comprehension "as students attempt to explore, investigate and solve problems together" (p. 378).

More Critical Perspectives on Cooperative Group Learning

For many mathematics educators, the move towards cooperative group learning represents an article of faith—something which per se is desirable. Most teachers would support G. Thomas' (1994, p. 319) claim that the role of the teacher in creating an environment is one where the following considerations are accepted as important:

- language is an important part of mathematics learning
- talking with others can be beneficial in mathematics learning

The real issue is designing programs and organising learning experiences so that rich communication occurs regularly, and naturally. Mousley and Sullivan (1994) noted that teachers need to become conscious of their own actions, and of "the wider consequences of their teaching behaviours and their reasons for managing classrooms in certain ways" (p. 432). Perhaps this is why there is little evidence that teachers use cooperative group learning as much as might be expected from what is advocated in teaching journals.

Smith (1995) pointed out, despite the fact that Davidson (1990) and a number of other reviewers have claimed that academic achievement has been enhanced by cooperative group learning, Slavin (1987, 1990) has stated that greater relative achievement is only likely to occur if group rewards are given and there is individual accountability.

Higgins (1994) also referred to a statement by Bennett and Dunne (1992) that very little had been written about the kinds of task which are appropriate for cooperative group work. Bennett and Dunne (1992), for example, emphasised the need for cooperative activities to be located coherently within the normal mathematics program, rather than be included as add-on activities. This need to incorporate cooperative, problem-solving strategies has been an important aspect of Australasian mathematics curriculum development over the past decade (e.g., Lovitt & Clarke, 1988). Mousley (1993), for example, referred to "the current emphasis on using strategies such as cooperative group work to give students more opportunity to express individual understandings, to encourage the development of shared understandings, and to stimulate the negotiation of meanings are products of an emphasis on the construction of meaning in social settings" (p. 328). But as Mousley acknowledges, much of this is still at the level of rhetoric.

One of the few papers which critically reviewed the literature on cooperative group learning was that by Higgins (1994), who quoted an excerpt from Good, Mulyran and McCaslin (1992) that it was yet to be demonstrated "whether small group instruction influences the development of critical thinking, students' views of mathematics content and problem solving, students' ability to reason mathematically, and their developing social intelligence" (quoted in Higgins, 1994, p. 340). Higgins warned that small group instruction on its own is unlikely to make the discussion and practice of mathematics more meaningful, adding that it is not "simply a matter of providing less teacher support and encouraging more learner independence" (p. 339).

Interestingly, Higgins (1994) also questioned the dependence of many advocates of cooperative group learning on Piagetian cognitive conflict theory, pointing out that—in line with Vygotskian theory—there are many situations where mutual guidance and support would appear to be more desirable than attempting to establish cognitive conflict situations.

Ethnographic study carried out by Higgins (1994) involved the analysis of 113 videotaped lessons, each of approximately an hour in length, spread across eight classrooms of Year 5 and 6 children in New Zealand. In each classroom, there was a focus on a small group of children.

Key factors identified in the Higgins study were the identification and use of tasks likely to generate rich discussion about important mathematical ideas, and to encourage all children to participate by the use of multiple sub-tasks. Also, the role of the introduction to the task by the teachers was found to be important. In fact, Higgins stressed the need for those interested in cooperative small group activities to pay careful attention to the design of tasks, and to the role of the teacher so that effective patterns of interactions will be facilitated.

Gooding (1994) conducted a study involving 24 Grade 6 children who had had considerable experience with group work in mathematics. Gooding's research revealed that some students working in small groups of 4 were much more active—in the sense that they talked more and provide reasons for their actions—than others. Some students, in fact, make relatively few, if any, mathematically correct statements, and provide no reasoned explanations.

Gooding (1994) found that students involved in cooperative group work had a greater mean gain than students in control (conventional) settings. However, she concluded that teachers who actively involve their students in cooperative group settings would find it difficult to identify students who were not actively participating and who were not benefiting academically.

Gooding and Stacey (1993) identified the factors which facilitated and hindered learning in the groups involved in Gooding's research. Factors which were reported as facilitating learning were:

1. adequate background knowledge to participate in the discussion,
2. receiving appropriate explanations when required,
3. using the newly corrected ideas (e.g., to help others, or to place cards),
4. sharing sufficient prior knowledge so as not to be ignored or excluded from discussion (e.g., by others with better background knowledge).

Often, in Gooding's research, students who were not well equipped to participate in discussions were effectively excluded from most mathematical interchanges in the groups. Gooding and Stacey (1993) recommended that teachers "should avoid placing students with a wide range of prior knowledge together, at least for group tasks which are difficult and require conceptual reorganisation or problem solving" (p. 49. They further reported that in their research high and medium students worked together profitably, as did medium and low students. They advised against forming groups which included only students with low prior knowledge because in such cases there may not be sufficient knowledge in the group to make sense of the task (see Slavin, 1987, for similar comments).

There are other questions which have been raised but have not yet been adequately researched. For example, Clarkson (1991) noted that the way different cultures use language, and the status of cooperative ventures within traditional cultures, may enhance or inhibit small group work in mathematics classrooms. The implications of this for the planning and organisation of learning environments in mathematics classes are at present unknown.

Teacher Education and the Culture of Mathematics Classrooms

If, as research has suggested, teachers tend to embrace publicly the ideals of cooperative group learning without knowing how to put them into practice, then there is clearly a challenge for those developing teacher education programs to attempt to change this state of affairs. Perry, Geoghegan, Owens, and Howe (1995) accepted this challenge with teacher education students at the University of Western Sydney. Altogether, 37 teacher education students were involved in the study for 3 hours per week for 13 weeks. Careful attention was given to designing cooperative activities in which students worked together to develop, reach consensus, validate, and defend their solutions.

According to Perry et al. (1995), data from their study drew counter examples to three prevailing myths about cooperative learning:

Myth 1: There is a belief that in cooperative learning situations the student who messes about, or the student who won't work, or is not interested, will rely on the capable students in the group to do all the work (p. 456);

Myth 2: There is a belief that bright students will be held back in cooperative learning situations (p. 456);

Myth 3: There is a belief that the teacher has control of all knowledge (p. 457).

The Perry et al. (1995) program was clearly successful in increasing the students' concepts with respect to mathematics, and in "encouraging communication and collective ownership of solutions developed through classroom interaction" (p. 457). It is one thing, however, for adult student teachers to benefit from a cooperative group learning program provided for them during their tertiary studies, and another thing for them to be able to transfer this experience, suitably modified, to a school situation where the culture is quite different. One of the challenges is to design professional development programs to enable practising teachers to acquire the programming and theoretical knowledge to enable this transformation to take place (see Rice, 1993).

Concluding Comment

Figure 1 has provided a framework for discussion, and this has guided our thinking and our selection of articles for review. Perhaps the most important feature of Figure 1 is that it recognises that culture is all embracing, and that even mathematics is embedded within culture. So much of the research which we have reviewed testifies not only to the centrality of culture in mathematics education, but also to the fact that language is an integral and inseparable part of culture. Studies concerned with the learning of mathematics by people of all ages—from early childhood through to adult—have generated insights into the range and nature of social and cultural influences on learning, and there is an increasing recognition that unless this is done we cannot really understand how and why learning occurs.

Figure 1 also emphasised that curriculum theory, psycholinguistics, sociolinguistics, problem solving, and ordinary everyday language need to be considered before we can understand how mathematics is communicated to and between people. If we are to come to grips with the curriculum and teaching implications of what goes on in mathematics classrooms then, in our studies of classroom discourse, we need to take full account of the literatures from each of the separate domains identified in Figure 1.

It seems that mathematics education researchers have moved a long way from seeing learning as purely a question of how to transmit information from the textbook or from the teacher to the student in the most efficient way. Our survey of progress with respect to expanding the modes of communication in classrooms, and developing more appropriate assessment and evaluation procedures, suggest that mathematics education researchers need to take greater account of the fact that they do not work in a vacuum. Society still believes in the notion that mathematics is transmitted from teacher to student, and that a good test is one which finds out how much correct knowledge a student has acquired. From this perspective, although research to date could be described as in progress, much remains to be done.

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PART 3

TEACHERS AND LEARNERS

Research into the Professional Development of Mathematics Teachers

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Introduction

This review has been written in the context of a growing realisation in Australia as well as other parts of the world of the importance of professional development (in-service) for teachers in order to enhance mathematics education. Within Australia, reports at both national and state levels have for some time identified the ongoing development of teacher knowledge as being a priority area of need. For instance, the *Discipline Review of Teacher Education in Mathematics and Science* (Department of Employment, Education and Training, 1989) expressed the view that "there is a pressing need to revise and upgrade both pre-service and post-initial provision to meet the current and future needs of Australian society" (Vol. 1, p.xviii). Likewise, in the state of Queensland, the *Review of the Queensland School Curriculum: Shaping the Future* (Queensland Government, 1994), identified the need to upgrade the knowledge base of current teachers and recommended the "creation of collaborative partnerships between teacher education faculties and schools and systems" in order to give effect to such arrangements. Substantial federal funding in Australia has also been made available to assist in the achievement of these teacher development goals (National Professional Development Program, 1994-1996). With reference to another recent curriculum reform in the state of Queensland, Peckman, Robinson, and Underwood (1992) have reported that there is recognition within the teaching profession itself regarding the high value of professional development in the context of the implementation of new mathematics syllabuses. Related to these developments has been a debate about the nature and extent of collaboration among tertiary institutions and schools concerning their contribution to the education of teachers at both pre-service and in-service levels.

Mathematics teacher professional bodies have also vigorously emphasised the growing importance of teacher development. The Australian Association of

Mathematics Teachers, for instance, in its *National Statement on Girls and Mathematics* (AAMT, 1990) noted that there is a significant role for professional development in the area of gender equity within the life of the mathematics classroom. More generally, the *Professional Standards for the Teaching of Mathematics* (National Council for the Teachers of Mathematics, USA, 1991) has argued that "teachers' growth requires commitment to professional development aimed at improving their teaching on the basis of increased experience, new knowledge, and awareness of education reforms" (p. 125). Similar examples from other parts of the world also illustrate the extent of the general recognition of the need for such a commitment.

In pursuing these general objectives, the AAMT has been active in producing materials for the provision of in-service education for practising teachers (e.g., *Mathsworks*, AAMT, 1994). Other schemes for professional development of teachers have also been developed. For instance, Hollingworth and Robinson (1992) outlined a program to assist teachers to conduct their own professional development covering topics in mathematics teaching and assessment. This program also covered techniques useful for communicating with parents about mathematics in the school. Other similar projects are outlined in this chapter.

Sitting against these reports and recommendations, however, is the growing awareness that many programs of teacher development do not, in the opinion of teachers themselves, prove highly effective in enhancing the quality of professional practice (see, for instance, Swinson, 1993; Kanes & Nisbet, 1994; and Bishop & Larkin, 1994 below). This fact has afforded a common basis for much of the research reviewed in this chapter. It has also prompted providers of professional development to give careful consideration to the selection of an appropriate model for effective program delivery.

Overview of Research on Teacher Development

In reviewing research on teacher development it has been possible to group studies according to their principal focus of interest, as follows:

1. *Technical requirements for teaching.* In this grouping, the focus of studies has been on satisfying the technical needs of mathematics teachers in order to operationalise particular curriculum objectives.

2. *Professional needs of teachers.* In this grouping, questions addressed have been pertinent to investigations relating to the nature of current curriculum practice, how this relates to relevant criteria for teaching/learning effectiveness, and what can be concluded about the needs of teachers for teacher development in relation to current practice.

3. *Teaching context as a site for shaping curriculum change.* In this grouping, studies assume that teacher development is most effective when shaped by the joint construction of understanding among those engaged in the teacher development process with reference to actual teaching contexts.

4. *Models for teacher development.* In this grouping, studies reflect on the nature, rationale, and alternative theoretical models for the professional development of mathematics teachers.

The plan for this chapter will be to investigate the principal concerns of the research literature reviewed under headings suggested by these areas of particular focus. It is not suggested, of course, that any particular study will have confined its attention exclusively to the focus area ascribed to it.

Technical Requirements for Teaching

Characteristic of this focus for the promotion of teacher development is the view of the teacher as technician. Studies of this kind tend to be learning-product focussed in the sense that key questions relate to the development of an adequate means whereby specific teaching goals may be realised. For instance, Nulty (1993) outlined a scheme for professional development in the Northern Territory to assist teachers to (a) implement the National Statement on Mathematics for Australian Schools (Australian Education Council, 1990), national profile for mathematics (Australian Education Council, 1994), and the Northern Territory curriculum, and (b) develop a self-sustaining professional development among teachers. A series of workshops were organised to train tutors who would conduct professional development activities in their schools. The workshops covered many aspects of teaching, learning, and assessment in mathematics. Whilst teachers are frequently consulted about the issues and topics of concern to them (Peckman, Robinson, & Underwood, 1992), once these have been determined, little further reference to teachers is made. Little practical value is accorded to prior teaching experience in these investigations, although some attempt to involve teachers in more than purely transmissive ways are made. Adopting a similar perspective, O'Connor, Downton, Griffiths, and Hammond (1993) have outlined a professional development kit which provides teachers with presentation guidelines for the implementation of a problem-solving approach to mathematics. Whilst the design of this scheme allows for the course to be conducted in different ways, and includes components facilitating trialing, teacher reflection, and continued support, teachers are positioned as essentially passive agents over the duration of the process. Likewise, Hawton and Watts (1993) reported on a study which covered similar ground.

Professional Needs of Teachers

Research on the professional needs of teachers focussed on an exploration of teacher affect and attitude to teaching, teacher knowledge bases, teacher understanding of effective teaching, and educational change. These particular foci are discussed in turn. References to research relating to pre-service teacher education are made where these have significant implications for the professional development of teachers.

Mitchelmore (1992) investigated an approach to the development of confidence in primary (elementary) pre-service teachers. It was believed that lack of confidence was linked to students' perceptions of the nature of mathematics, and that it was necessary to help students understand mathematics as a human activity arising from real world experiences. As part of their pre-service preparations, students studied a mathematics education subject with several key elements, including cooperative problem solving workshops, application activities, tutorial discussions, and compilation of a literature journal. The subject was successful in increasing student confidence and relieving anxiety. Students perceived mathematics in a more creative and open way, and understood mathematics better from concrete materials, and working with applications.

Also considering questions related to teacher affect, Bleicher, Cooper, Nisbet, and Warren (1995) concluded that professional development of mathematics teachers in alternative methods of assessment is necessary to assist teachers overcome low confidence levels and break down resistance barriers with respect to some techniques of assessment, recording, and reporting. Their survey of 1242 mathematics teachers in primary and secondary schools across Queensland was part of the early stage of monitoring the progress of a professional development program associated with implementing Student Performance Standards (SPS) (Australian Education Council, 1994) in Queensland schools. They found that the implementation of SPS was relying on training mathematics teachers in assessment techniques that the majority do not yet use, nor have the confidence to use.

Turning to the investigation of teacher attitudes, various instruments for measuring these have been developed. Nisbet (1992) developed an instrument to measure pre-service primary teachers' attitudes to teaching mathematics. It consisted of a questionnaire which could be used to monitor attitudes of student teachers during their course of study. The instrument identified four subscales: (a) anxiety in relation to teaching mathematics, (b) confidence in and enjoyment of teaching mathematics, (c) desire for recognition as a competent mathematics teacher, and (d) pressure to conform to peer standards. These subscales were indicative of significant issues that should be addressed in pre-service primary teachers education. Results from administration of the survey revealed that student teachers' levels of confidence and enjoyment increased over three years of study, and anxiety levels decreased. Relich (1992) studied the mathematics and problem solving self-concepts of teachers and the possible modelling effects that depressed teacher self-concepts may have on students. The results suggested that such teachers may not provide adequate role models for their students, and that teacher education institutions need to consider implementing strategies which address the amelioration of low self-concepts. Relich and Way (1993) developed an attitudes instrument for use with pre-service primary teachers and proposed six subscales: ATM (attitude as a teacher of mathematics, incorporating confidence, anxiety and effectance motivation), MSC (self concept as a mathematician), MTMD (mathematics teaching as a male domain), UTM (usefulness of teaching mathematics), ETM (excellence as a teacher of mathematics), and OTM (other's perception of one as a teacher of mathematics). The authors stated that they intend to test reliability and validity of the new instrument as part of an on-going development program, so that teacher educators have a means to evaluate the success of their programs in this area.

Another topic of interest for researchers has been the understandings teachers have of mathematics and teaching mathematics. Kanes and Nisbet (1994), reported on a survey investigating the knowledge bases of primary and secondary school mathematics teachers. Their findings revealed that the mathematics content knowledge of secondary school mathematics teachers may not be sufficient, and that a majority of teachers are concerned about their level of knowledge relating to contemporary teaching methodologies. Differences were observed between teachers in the primary and secondary sectors (primary teachers having greater formal exposure to studies in mathematics pedagogy) and among teachers in the secondary sector (where levels of professional development and attitude to teaching mathematics were specific to schools rather than individuals or the system at large).

In addressing teacher understanding of effective teaching processes, Dengate (1993) piloted an instrument to measure teachers' perceptions of mathematics and pedagogy. It yielded four subscales: (a) the product-purpose of mathematics teaching, (b) the process-purpose of mathematics teaching, (c) the nature of mathematics, and (d) the value ascribed to mathematics. His study found that teachers of first year secondary students exhibited greater instructional conservatism in response patterns and less orientation towards problem solving than other teachers.

Educational change also received the attention of researchers. Peter, Clarke, Carlin, and Harmes (1992) gave an outline and evaluation of the Active and Reflective Teaching in Secondary Mathematics (ARTISM) Program of professional development (five 3 hour sessions on mathematics and teaching mathematics). Factors influencing successful implementation of new models of teaching were identified. It was concluded that elements central to the success of the program included: conscious recognition of classroom consequences of contemporary theory; extended nature of the program; collaboration among developers, consultants and schools; support and accountability provided by the school visits; participation of a substantial number of teachers from each school; practical nature of the program's work requirements; and emphasis given to teacher reflection on classroom practice.

Teaching Context as a Site for Shaping Curriculum Change

Studies adopting the notion of the teaching context as a site for shaping the curriculum, approached their analyses by emphasising certain complementary perspectives. Firstly, the production of teacher knowledge was seen as a collaborative activity involving teachers, researchers, and other facilitators. Secondly, the development of shared meaning for curriculum changes was seen as crucial to the effectiveness and appropriateness of professional development activities. Thirdly, teacher thinking and routine use of knowledge was identified as being critically shaped by the actual teaching context of teachers in everyday classroom sites.

Concerning the development of teacher knowledge, Barnett and Tyson (1993) described the use of "case methods" to enhance the quality of mathematics teacher knowledge and the capacity of teachers for autonomous action. This method consisted of discussions among teachers concerning a number of actual teaching situations. Led by a facilitator, the teachers were asked to clarify and elaborate their ideas, justify their positions, and critically examine alternative perspectives. In their findings, the authors reported that the processes of negotiation, debate, and joint investigation, led to a deepening of teachers' shared understanding of mathematical content knowledge, as well as the problems and tasks of teaching particular content material (pedagogic content knowledge, to use Shulman's (1986, 1987) terminology). Activity aimed at building shared meanings lay at the heart of this professional development strategy. Knowledge relating to mathematical concepts and processes as well as how to teach these was locally produced and maintained. No strong division existed between the theory and practice of knowledge relating to teaching mathematics. Moreover, it was found that teachers in this situation more often viewed mathematics as a changing body of knowledge dependent on social context, rather than as a set of externally imposed constructs.

Redden and Pegg (1993) reported case studies relating to the professional development of teachers which emphasised the importance of the joint identification of needs and production of shared understandings of the problems facing the mathematics teaching staff. One important conclusion in this study was that time spent carefully identifying the initiating problem or issue is crucial in developing a greater understanding of the matters to be addressed. Premature movement to the implementation phase of teacher development is to be avoided. The study found that Polya's four problem solving phases provided a useful framework to analyse the success or failure of in-service experiences and professional development programs.

In a study which focussed on the relationship between thinking and context, Crawford (1992) described a teaching experiment directed to the task of developing student-teacher knowledge relating to particular contexts within the school classroom setting. Whilst her paper reported on research on initial teacher education, it also provided valuable background for the issues of teacher development generally. The framework was derived from Leont'ev's activity theory of learning in which cognitive development is seen as the result of conscious intellectual activity in a social context. Thinking that occurred during the activity depended on the perceived needs and goals of an individual or group and the resulting ways in which they approach the tasks at hand.

Mousley (1992) conducted a study based on the assumption that there exists a dialectal relationship between the knowledge needed by teachers in development of their teaching skills and concepts, the context in which they actually teach, and theories of mathematics teaching. Mousley found that students manipulate not only classroom learning materials but also a range of other elements characteristic of their learning environment, for instance, teachers' directions, written text, group discourses and perceptions of the roles of both teachers and learners in classrooms. A case for loosely-defined tasks occupying a significant, though not central, place in the normal range of teaching practices was made. Consistent with these findings, Rice (1992) argued that if constructivist principles are to underpin mathematical learning environments for students, then they should also provide the basis for professional development programs for teachers. This may best be obtained, Rice suggested, through the development of a "professional development ethos" in which teaching is regarded as a dynamic, ever changing phenomenon. Teacher introspection, reflection, and self-direction were identified as behaviours most likely to be associated with such developments.

Models for Teacher Development

Teacher development has been considered from a theoretical point of view by a number of authors in recent years. For instance, Rice (1993) offered an analysis of models of staff development in terms of top-down versus teacher-directed approaches. Clarke and Peter (1993) presented a comprehensive model capturing the interdependence of four elements characteristic of professional development processes. These were identified as: teacher knowledge and beliefs, classroom experimentation, valued outcome by which these are interrelated, and the external agents and instruments of change. The model generated a framework in which transmissionist and meaning-centred models can be conceptualised. Alternative views concerning learning theory, the nature of knowledge in classroom processes, and the values and beliefs supporting these were encompassed within a single framework.

Clarke and Hollingworth (1994) also investigated models for professional development and characterised alternative conceptions of teacher change currently found in the literature. In considering the construction of professional development programs, the authors emphasised the value of maintaining an awareness of the various perspectives relating to the concept of educational change. These can be grouped broadly into transmissionist and meaning-centred categories (Kanes & Nisbet, 1995). Change as training, and change as systemic restructuring are examples of transmissionist type models; change as personal development, and change as local reform give rise to meaning-centred models. Kanes and Nisbet offered a general organisation for describing and analysing alternative approaches to teacher development.

Conclusion

As noted in Kanes and Nisbet (1995), two opposing approaches which frequently occur in studies relating to professional development have been the transmission of knowledge approach and the meaning-centred approach. On one hand, teachers are positioned as relatively passive recipients of purportedly expert knowledge. On the other hand, meaning-centred approaches emphasise the critical role of collaboration among teachers and researchers in developing effective concepts and skills for teaching. In the transmissionist mode it has been found, for instance, that whilst teachers are frequently consulted about the issues and topics of concern to them, once these have been determined, little further reference to teachers is made. In meaning-centred approaches, meaningful activity lies at the heart of professional development strategies. Knowledge relating to mathematical concepts and processes as well as how to teach these is locally produced and maintained and no strong division exists between the theory and practice of knowledge relating to the teaching of mathematics. Moreover, teachers in this situation are more likely to view mathematics as "fallible, changing, and like any other body of knowledge, the product of human invention" (Ernest, 1991).

Kanes and Nisbet's distinction between transmissionist and meaning-centred approaches to studies of the professional development of teachers can also be viewed from a theoretically more sophisticated standpoint than first proposed by these authors. In concluding this review, it does not seem out of place to provide an outline for just such an alternative schema, especially as this will also allow a more reasoned identification of gaps within the current range of research topics relating to this area of interest than might otherwise be possible. Grundy's (1987) adaptation of Habermas' theory of human knowledge-constitutive interests will be taken as a starting point for this approach. A reason for this is that Habermasian theory tends to provide a way of richly investigating interrelationships among formations of knowledge and domains of value of relevance in curriculum situations. In Grundy's adaptation, three possible kinds of curriculum interest are distinguished: technical, practical, and emancipatory. The first concerns the instrumental means of action required for a predesignated end, and is predominantly product oriented. In contrast, the second is concerned with the intersubjective process of interpretation, personal judgement, and reflective delineation, rather than with the objective products of learning. The third is concerned with notions of power and the way ideas can dominate the perceptions and actions of other less powerful members of society.

These ideas afford a framework against which the literature reviewed above may be assessed. Firstly, it is clear that the view of the teacher as technician (see the relevant section above) is very much in accord with the notion of the technical interests of curriculum formation. Such interests are concerned primarily with the delivery of a well-defined product in the most cost effective manner. It is clear that transmissionist approaches to teaching and teacher development are most consistent with this view of curriculum. Secondly, foci on the professional needs of teachers and the development of the context of teaching as a site for curriculum change, as illustrated in the relevant sections above, best relate to Habermas' concept of practical knowledge-constitutive interest. In examples of teacher development conforming to this category a high premium was placed on teachers taking responsibility for teacher development. Interpretation, personal judgement, and reflection leading to action featured prominently in studies sharing these designated foci. Finally, it is notable that no studies relating to teacher development fitted Habermas' emancipatory knowledge-constitutive category. Research into teacher development from this view point would seek to identify socially oppressive institutionalised practices, and suggest actions to be performed by teachers, administrators, researchers, and other stakeholders in order to overcome such practices. Two areas identified in the literature as being poorly represented within teacher development programs are gender issues (Bleicher, et al. 1995) and ethnicity (Bishop & Larkin, 1994). We recommend that research be conducted in these areas.

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Understanding the Adult Learner of Mathematics

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Introduction

Who are adult learners? Why do adult students return to study mathematics after it has been so unwelcoming and discouraging for many? What are their aspirations? In what courses do they enrol? What are their articulated needs? What are their needs as perceived by experienced practitioners? What is their sense of personal agency? What is known of their cognitive and affective needs? What are their socio-cultural backgrounds, and how can these be taken into account? Who are their teachers, and what are their needs? What are the philosophies underpinning teaching practice? This chapter explores these and other questions, through reviewing research in the field of adults studying mathematics, much of which is participatory in the sense that it represents the development, testing and publication of ideas by practitioners. It is not easy to define the parameters in a burgeoning sector such as this, characterised by complexity and heterogeneity, and there will be some themes which permeate other chapters.

Adult learners in mathematics are diverse in terms of their mathematics (and other) educational background, their life experiences, and their aspirations for the future. Levels of study range from basic numeracy, through vocational courses, extending as far as tertiary courses in statistics or calculus. Many have been marginalised along social, gender, and/or cultural lines. Not all adults studying mathematics have had the opportunity to learn beyond primary school years, if at all, and not all have used English as their first language of instruction.

This review will consider research undertaken or presented in Australasia since 1992, although earlier work which provides an historical context will be included. Because the area of research into adults learning mathematics is fairly recent, there are few dissertations world-wide (see, for example Wagner, 1995, where no code exists to signify adult learners). Accordingly, this chapter will report on papers presented and/or published in the Australasian region, some of which represent work-in-progress towards a higher degree. There are wide ranging issues which run right through the review, but it has been organised for convenience into five themes: (a) context, including definition and theoretical perspectives on adult learners; (b) basic education, including adult numeracy; (c) mature-aged students attending university, including bridging mathematics programs; (d) the vocational education and training (VET) sector, including workplace education; and (e) further research.

Context

Definition of Adult Learner

The definition of an adult learner is complex, and revolves around the student, not the level of mathematics being studied. Harris (1995) defined adult learners as students who have left full-time schooling and who are recognised as having adult status by virtue of having assumed, at some time, adult social roles such as independent living, marriage, parenthood, and/or full-time employment, so that adult education thus becomes "the set of activities or experiences engaged in by adults which lead to changes in thinking, values, or behaviour" (Cranton, 1992, p. 3).

Most adult students are returning to study after a break of at least one year, possibly decades, since their last formal study of mathematics. Although adult students are the focus of this chapter, the reality is that, apart from specific classes in basic numeracy or workplace education, most mature-age students are enrolled in classes along with young people continuing from school. Adult students bring perspectives qualitatively different from those of children attending school, and even from continuing students enrolled in TAFE and university courses.

Theoretical Background

The main roots of the field of adult education lie in the psychology of learning, but are also influenced by writings of critical theorists in literacy education. More recently theoretical foundations from sociology have been incorporated. Research studies in this chapter draw from both of these foundations.

History. Cranton (1992) outlined a brief history of the field of adult education. Dewey did not write specifically about *adult* education, but he regarded learning as a lifelong process. The practice of adult education has been traditionally associated with a humanist approach, and typical recommendations are made for the educator to: provide an atmosphere conducive to learning, including a warm, supportive environment, identify the learner's needs and interests, emphasise interaction among participants, de-emphasise or eliminate grading, and to try to accommodate individual learning styles. Some see the role of the adult educator as facilitating the development of self-awareness in the learner.

While humanists attempt to describe how learning takes place, developmental psychologists attempt to describe human growth to maturity. Some developmental psychologists emphasise the phases of an individual's life; others emphasise the stages of growth, from immaturity to maturity. Critical theorists, by criticising society and institutions, attempt to provoke change to overcome forms of oppression as they see it. Paolo Freire is probably the best known proponent, and his work has influenced many practitioners in Australasia.

The term *andragogy* was defined by Knowles as the "art and science of helping adults learn," and differentiated from pedagogy where the teacher assumed full responsibility for decisions about learning, thereby placing the learner in a state of dependency. Cranton (1992, p. 16) exhorted: "It is essential for all professionals to reflect critically on their practice, to be aware of their own values and assumptions, to make responsible choices based on their expertise and values, and through critical thought, become aware of and develop their philosophy of practice." Much of the research reviewed in this chapter has emanated from such critical reflection.

Theoretical perspectives. Ellerton and Clements, (1989, p. vii) stated that "the main lesson learned by most school leavers after years of being forced to study mathematics is that they can't do it." Many adults find themselves enrolled in mathematics classes, believing that they are incapable of learning the subject, and that the problem is their own lack of ability. Others claim that their teachers failed to "teach them properly." Rarely do they question the school mathematics curriculum, although the most commonly used descriptor is "boring" (FitzSimons, 1994a). This section addresses the need for, and development of, research into adult learners of mathematics from both socio-cultural and psychological perspectives.

Zevenbergen (1994a, pp. 188-189) argued that "the role of mathematics can be seen to be that of a social filter which legitimates social differences;" also that success and failure in learning mathematics is a political process. Their exposure to school mathematics has constructed some students as ineffective learners of mathematics (Zevenbergen, 1995), and this is related to the failure of teachers to address

> the wider social, cultural, historical and political context within which the processes of schooling and learning are situated. The non-random failure of working-class students, women, or students from non-Anglo backgrounds is interpreted within this framework and seen to be related to some deficit or difference within the individual rather than as a structural or discursive construction. [The lack of success is framed within psychological traits such as] ability/intelligence, self-esteem, levels of cognitive development, motivation, attribution theory where the blame is with the individual rather than social or structural causes. (Zevenbergen, 1994b, pp. 721, 722)

Researchers in the field of adult education are beginning to adopt a socio-cultural perspective in which to frame their work. See, for example, Buckingham (1995), Griffin (1994), and Stanton (1993, 1994a, 1994b), whose work will be discussed in more detail below.

According to Gordon (1994a, 1995), activity theory takes into account cultural factors and individual goals, and learning is viewed as a mediated activity within a socio-historical context. The meaning of the task to the student provides the framework within which the individual acts, and there is empirical support for the importance of goals as critical mediators of university students' performances. Activity theory differs from constructivism in that it emphasises the interdependence of socio-cultural and cognitive perspectives. Gordon (1993b) noted that this is what makes it "particularly relevant to the analysis of adult learning as adults are cognitively fully developed in the biological sense but generate new knowledge through participation in activities which are socially and culturally rooted" (p. 35). Educators need to understand who the learners are, and to acknowledge their experiences, beliefs and values; they also need to clarify their own goals, especially in relation to education versus training (Gordon, 1994a).

Cumming (1993) highlighted the need for a theoretical perspective on the development of expertise in mathematics for the adult learner. She asserted that such a theory is needed to incorporate cognitive theories of processing and higher order thinking, in order to encapsulate the existing expertise of the learner. Cumming claimed that Australian adult education and curriculum documents generally ignore the findings of research with respect to cognitive processing and the effects of instruction on performance. She recommended that the focus needs to shift from a curriculum perspective to a cognitive perspective and the processes of thinking mathematically. In this way the goals of the student will not be constrained by the intended curriculum.

Surman and Galligan (1993) contended that the development of metacognition is as important as cognitive development in students returning to study. They considered that it is unlikely that adults returning to study have had similar experiences to continuing students, and undertook a research project which incorporated explicit metacognitive strategies into student assignments. Galligan, Surman, and Harris (1994) reported on initial investigations into measurement of metacognition of adults studying a preparatory mathematics program in the distance mode. Southwell (1994) asserted that writing in mathematics is beneficial to adults as it assists them to become independent learners, and it also assists in the integration of ideas. Learners develop the power to reflect on their own experiences, and this is particularly helpful for those who may not have had the opportunity to do so in their earlier school days.

Returning to Study

FitzSimons (1994b) analysed the issues associated with returning to study from three perspectives: the psycho-social, the situational, and the institutional domains. She also outlined a cyclic pattern, beginning with the student's previous history of mathematics education (see also FitzSimons, 1995, July), the desire to return, entry to study, continuing with the class, and leaving the class. A literature review addressed the notions of a transitional activity, cognitive interests, role conflicts, implied deficiencies of adult learners, academic satisfaction, problems with science courses, sharing classes with younger students, social context, questioning by the teacher, and mathematics anxiety. The review also addressed constructivist theories of learning, which were tested in the practice of a numeracy class for women. Fell (1995) conducted a case study of women returning to study mathematics in a single-sex class, investigating previous histories, choice of class, preferred activities, and she outlined the positive affective and cognitive changes which had taken place.

Barriers to participation are numerous, and adult students need positive, ongoing support from family, close friends, and members of the community. One such barrier is geographical or social isolation, although distance education can provide a means for students to overcome such barriers. It is flexible, and timed to suit the pace of the learner, and can fit in with other commitments. The learner can receive individual attention, reducing the anxiety some people feel when part of a large group (Harris, 1994). However, Srivastava (1992) highlighted some of the difficulties which may be faced by adult learners, particularly in the distance education mode.

For further discussion of the factors affecting adults returning to study, and the impact of the teacher's belief in the value of adopting a constructivist perspective, see FitzSimons (1993a; 1993, March).

Basic Education

Adult Numeracy

Context. The situation in Australia is complex, with learners studying in a diverse range of situations. Seddon (1994) described perceptions held by outsiders of the Adult Literacy and Basic Education (ALBE) sector as its having a commitment to a view of education which aims at enhancing peoples' capacity to act, in the practical pursuit of a just and participatory society. Until the late 1980s it remained a marginal, but autonomous, sector in the field of education, engaged in the purpose of providing second chance education, re-entry for women, and language for non-English speaking background (NESB) migrants. Its marginal position resulted in vulnerable working conditions for staff, mainly women. In recent times ALBE has become part of the educational mainstream, with consequent complex changes in context. Now, in contrast to humanitarian approaches which provided empowerment to the individual learner, an era of economic rationalism, where education is big business, has emerged (Cumming, 1993). FitzSimons (1995a) presented an overview of adult numeracy in Australia, in both vocational and further education sectors, and highlighted critical issues for teachers, including the impending adoption of competency-based training and assessment.

Definition of numeracy. Cumming (in press) commented that although most writers discuss numeracy in mathematical terminology within a context of use and expectation of outcome, it is not the mathematics which is synonymous with school text-based performance. The 1959 Crowther report coined the term *numeracy* as a numerical equivalent of literacy, and which incorporated aspects of mathematics in work and everyday life; the 1982 Cockroft report specified a broader range of learning outcomes. According to Cumming (in press), it is difficult to separate generic competencies (such as the Mayer Key Competencies, 1992) adopted as part of the Australian government's national training reform agenda) from their applications, because descriptions of numeracy in the workplace link the two inextricably. For further information of the Key Competencies see Mayer (1992). (The Australian Association of Mathematics Teachers is undertaking a Department of Education, Employment and Training funded research project entitled *Rich Interpretation of the Using Mathematical Ideas and Techniques Key Competency*.)

Cumming (in press) also noted that, using Bishop's (1988) six universals as categories of mathematical activity, "it is apparent that most adults, even if they have experienced considerable failure within school-based mathematics, are quite likely to be mathematically experienced and numerate people." Bishop's universals provide a broader framework for instrumental goals and procedures than is often presently implemented in adult numeracy, according to Cumming. (See, for example, Stanton, 1993, 1994a, 1994b)

Numeracy assessment. Cumming (1994, p. 4) asserted that "assessment is now the most significant issue confronting the basic education sector." Increased funding and improved staff qualifications mean that adequate documentation of student progress must occur, and yet teachers in this sector, like teachers in other sectors, are inadequately prepared to address contemporary assessment issues. The student base is non-typical, population performance characteristics are ill-defined and known to be highly contextualised and there is a lack of coherent assessment practice. Traditional assessment practices are thus inappropriate and meaningless.

Cumming (in press) noted that:

> instructors in adult numeracy settings often do not appear to have explicit goals for individual students, rationales for assessment or understanding of principles of assessment. . . . Assessment is seen as threatening to the individual who is usually perceived by the teacher as having had negative experiences in school, particularly in the area of assessment. . . . Unfortunately, much assessment which does take place in adult numeracy reinforces school text approaches, using work sheets, focusing on written formal mathematics, and generally reinforcing negative experiences which students may have previously had.

Referring to contextualised assessment, Cumming (1995) noted:

> Notions of authenticity have to an extent led learning and assessment astray. In our desire to embrace competence, expertise, enable adult performance and simulation of the complexity of real life, we have lost sight of many of our short-term and medium-term instructional goals. ... Assessment has to be rationally tied to the purposes of instruction. ... The most important factors relating to assessment are the recognition of complex reasoning, the complexity of content for both learning and performance and the need to match assessment with learning and purpose.

McRae (1995) described a contextually based method of assessing students entering Adult Basic Education programs, on a one-to-one basis. Her method reflects a radical constructivist view and allows for active participation by the student, acknowledging their prior knowledge and experience, while at the same time providing information to the student about the class.

Grier (1994) investigated the dissociation between adult learners' informal and formal numerical ways of knowing and acting, distinguishing between mathematics used and learned outside and inside of educational institutions. Grier (1995a) then analysed a range of out-of-school mathematical practices of adult students, based on these distinctions. Her research demonstrated that it is extremely difficult to make definitive statements on differences between the two. Grier (1995b) asserted that the ability to recognise the mathematising in students' prior/out-of-school practices depends on the teacher being able to recognise the features and nature of informal mathematics. Assessment should be based on informed beliefs about the nature of mathematical knowledges, according to Grier.

Teaching in context. Helme (1994, 1995) investigated the impact of task context, in particular context familiarity, on the responses of adult women students. She noted the striking individual difference in both performance and perceptions in relation to the presence or absence of context. Helme concluded that teachers need to select contextual tasks which are appropriate to the experiences, goals, interests, and aspirations of their students, with particular sensitivity towards students' cultural and language differences.

Clarke and Helme (1993) highlighted the distinction between the situation and the context of a mathematics task. In the classroom a common situation may be presented to all students, but each will construe it individually, to make their own personal construction. They noted that more academically successful adults tended to give the least contextualised responses, possibly as a result of their mastery of the rules of school mathematics.

Critical Numeracy

Some lecturers of undergraduates in professions such as primary school teaching and electrical engineering regard the teaching of skills of critical numeracy as an empowering and integral part of the course (see Barnes, 1995; Yasukawa & Johnston, 1994; Yasukawa, 1995; Yasukawa, Johnston, & Yates, 1995). They assert that numeracy is a

> critical awareness [which] enables us build bridges between mathematics and the real world, with all its diversity. Being numerate involves not only having this critical awareness, but also involves the responsibility of reflecting that critical awareness in one's social practice. Thus, being numerate means being able to situate, interpret, critique, use, and perhaps even create mathematics in context, taking into account all the mathematical as well as social and human complexities which come with that process. (Yasukawa, Johnston & Yates, 1995, p. 816)

This definition appeared to be implicit in the approach adopted by Sefton, Waterhouse,and Deakin (1994) in working with automotive production workers (see below), and may well be applicable to all numeracy students.

Some regular university students may also find the need to enrol in numeracy classes in order to be equipped with the necessary skills and attributes to succeed in their courses, and beyond, as independent learners (Chapman, Kemp, & Kissane, 1990; Kemp, 1995). Many universities in Australia, New Zealand, and the South Pacific provide "support services for students to improve their mathematical thinking, knowledge and attitude towards mathematics" (Kemp, 1995, p. 380). Since 1990 there have been annual Bridging Mathematics conferences addressing the issues associated with provision of such support.

Mature-age Students at University

Bridging Mathematics Programs

Tolmie (1992) described bridging mathematics courses as those which enable pre-tertiary students to upgrade their mathematical skills to the level required for entrance to tertiary courses. They are also used as a background for other studies or to enhance self-knowledge or career prospects. Many papers have been presented on topics relating to this area, and a representative selection has been made to save space; some papers are referred to in other sections of this chapter.

Galbraith (1990), in summarising a symposium on bridging mathematics, noted several themes. Firstly, there were at least two conflicting ideologies.

> One is that which views education as a public good and intrinsically valuable in its own right; the other is that which views education as a valuable commodity, that applies accountability criteria, and that sees education as closely linked to the needs of the labour force and hence to skill development. (p. 141)

He warned that the latter left programs vulnerable to changes in funding, and recommended that the philosophy behind bridging courses be carefully examined and articulated from the outset.

Mathematical content levels in bridging courses are very broad and range from replication of primary school content upward, but with differing emphases on depth of approach, from acquisition of skills to underlying principles. Individual learning needs vary, in terms of reading ability, methods of processing information, and ability to cope with the language of mathematics. In some courses, motivated mature-age students are mixed with younger, less motivated students.

Sharwood, Emery, and Schumann (1992) listed several factors considered necessary for a successful bridging program. It should: provide a solid grounding for the field of study, teach certain technical skills, develop creative thinking skills, overcome weaknesses in spatial perception, improve linguistic competence, teach effective study methods, work in partnership with departments serviced, provide students with a credential, encourage student interaction with lecturers and each other in collaborative learning, incorporate aptitude testing and career guidance, have a selection system to identify students at risk, have constant evaluation of its effectiveness, employ dedicated staff, and have institutional support. They also listed several non-academic factors crucial to success.

Surman (1993) examined how bridging mathematics educators empowered their students, and how they themselves became empowered. She presented a model for adult learning which had three intersecting domains: content knowledge in the cognitive area of mathematics; self knowledge, where self-esteem is raised through increased self-confidence and self-awareness of the learning processes; and social knowledge, that is, knowledge of the institutional culture of higher education.

Barnes (1994) recommended that mathematics educators adopt a constructivist approach, and to try to understand students' conceptions. She noted that the assumption that the learner is always trying to make sense of information may not be valid for adult learners who have formed the opinion that mathematics does not make sense and is not meant to. It is imperative to challenge this belief in order to break the cycle of students employing strategies of rote learning and repetitive practice. Barnes also criticised the practice of using multiple choice diagnostic tests, as they fail to indicate what the student *can* do, or to reveal misconceptions (that is, what students *think* they know but don't).

Griffin (1994) argued that mathematics cannot, and should not, be separated from a student's background and sub-culture. Most students enter university with an inadequate understanding of the values and beliefs of the university sub-culture. The greater the number and difference of diverging beliefs and values, the greater is the impact on the student's chances of academic success. When they understand that there may be differences, students can preserve their self-esteem and develop coping strategies.

Milne (1992) described a pilot study investigating aspects of autonomous learning behaviours, attitudes towards mathematics, and problem solving performance of students enrolled in a year-long bridging mathematics course. Autonomous learning behaviours include independent learning, perseverance, taking risks, and willingness to choose challenging tasks, and are considered necessary in higher levels of mathematics. They are thought to develop over time, and be related to the student's internal belief system as well as to external or societal factors.

Gates, Robinson, and Blest (1995) used qualitative and quantitative methods to explore the mathematical knowledge that ten students, most of whom were mature-aged, enrolled in an introductory calculus program, brought with them. They argued that these methods yielded richer and more valid data than traditional, normalised tertiary entrance scores, and that this data could be used to contribute to improved teaching, assessment, and evaluation strategies.

Evaluation. For bridging mathematics programs to survive they must attempt to ensure continuation of funding. In an era of economic rationalism it is essential that they satisfy not only their student base, but also those who make financial decisions. Godden and Pegg (1993) argued that there is little evidence of formal evaluation of bridging mathematics programs whose emphasis is on flexibility and a student-centred approach, usually in the form of individualised or small group tuition, utilising a variety of teaching modes and resources. However, this emphasis presents obstacles to substantial evaluation, and to overcome them may jeopardise the very qualities which exist in such programs. Traditional evaluative techniques are inappropriate in this regard, and Godden and Pegg recommended that a new set of procedures, specifically tailored to this area, be developed. They noted that it is essential for each program to clarify its aims and objectives, so that success in meeting them can be measured. Students' perceptions of their needs may vary as they progress through the courses for which they are enrolled; their stated desires may be for immediate, instrumental understanding and hence at variance with the teacher's intention for longer term relational understanding. However, measurement of student retention and graduation require longitudinal studies, and these are rarely undertaken. Problems are compounded by student attrition and lack of reliable funding. This latter can create a cycle of minimal evaluative effort being made, resulting in the relative obscurity of the program's effectiveness, and a further decline in funding, due to the perceived lack of viability.

Gordon (1993a) evaluated the impact the Mathematics Learning Centre at Sydney University had on students taking second year psychology and found that it was effective in supporting their studies. Harnett, Graham, Coupland, Wood and Smith (1995) found that student perceptions of the bridging courses offered at the Mathematics Study Centre, University of Technology, Sydney were also positive, and they are using the feedback to become more responsive to student needs.

For a comprehensive review of the rationale, student participants, staff, and conduct of centres offering assistance to mathematically underprepared tertiary students, see Godden (1992).

Indigenous Groups

This section addresses issues associated with indigenous peoples such as Aboriginal and Torres Strait Islanders, Maoris and members of Pacific Island communities. Stanton (1994a, 1994b), in setting the theoretical context of working in mathematics with Aboriginal students, noted that there are two cultural worlds which have disjoint meaning realities. Stanton drew on the work of Michael Christie to make comparison: For example, Western traditions tend to rely on an atomistic view of the world, where identity is marked by boundaries, and value is placed on things which can be counted. In this tradition there is also a belief by many that there is an objective reality waiting to be discovered and described. However, according to Christie (quoted in Stanton, 1994a), "Aboriginal thinking prioritises neither atomism nor objectivism." Negotiation of an Aboriginal education commonground, specifically in mathematics, is taking place at Batchelor College in the Northern Territory.

Stanton (1993, 1994a, 1994b) reported on a course that was based on a philosophy of "both ways" education in which both Aboriginal cultural knowledge and western academic traditions are recognised and celebrated. The teacher education course is designed to prepare people identified as suitable and acceptable by Aboriginal communities to be teaching professionals to work in their own communities. Students are mainly female, from tradition-oriented communities, are diverse in age, and have mostly had a very limited formal education, sometimes well below the admission requirements of mainstream tertiary institutions. They enter the course with conceptions of mathematics as a technique-oriented mastery of skills which have no relevance to their lives, and which are "owned" by the dominant culture. Stanton described a curriculum which was designed to make mathematics accessible to teachers and students alike, and to allow the community to co-opt mathematics and its symbolic technology and machines for their own purposes. Unlike the traditional hierarchical content structure of Western pedagogy, the program investigated mathematical schema using Bishop's (1988) six universals as a framework.

Dawes and Dawes (1993) reported on a major research project to develop bridging courses for Aboriginal and Torres Strait Islanders to prepare them for entry into mainstream tertiary degree programs. Stewart, Dawes, and Dawes (1994) described one such approach. They argued that the key to its success was the incorporation of Aboriginal community values, based on solidarity and sharing, into the curriculum so that the knowledge was both relevant and meaningful to the students. Budby (1993) also argued that Murri (Aboriginal) students would be best served by a culturally supportive environment. Steen (1993) described her institution's attempt to increase the numbers of Aboriginals and Islanders graduating in the field of science and mathematics, by providing intellectual enrichment and exposure to successful professional role models, and to overcome perceived difficulties at the stage in school when most Aborigines drop out. Partlin and Nugent (1992) developed a selection procedure to assess Aboriginal applicants' readiness to undertake degree courses in the health sciences and to effectively assess their individual support needs. Partlin and Nugent found "classic" diagnostic tests to be culturally inappropiate, and they wished to allow applicants to assess their own readiness.

Corbett and Vassilli (1992) warned of possible misunderstandings of the needs of Urban Aborigines. They argued that it is offensive to question a person's claim to Aboriginality, and counterproductive in the educational setting. Problems such as those faced by Urban Aboriginal students are likely to be closely aligned to those faced by students from lower socio-economic communities everywhere.

Clark (1995) reported on a trial program which aimed at improving the participation and performance of Pacific Islander students in a first year statistics course, through special tutorial assistance which addressed cultural issues, and gave the students a sense of community.

The following research was intended to give encouragement to Pacific Islander school students to continue with their mathematical studies, and is included because it bears some relation to Clark's research. It also values the cultural knowledge of adult women in areas they would not consider to be mathematical, but which can be drawn on for application in mathematics classrooms. Finau and Stillman (1995) illustrated an approach to teaching Tongan secondary students, where mathematics lessons were based on the *tapa* designs of traditional Tongan craftwork, and McMurchy-Pilkington (1995) highlighted the mathematical excellence demonstrated by Maori women in a Marae kitchen. In this way, students are able to see their culture and gender as being valued in the classroom, and the women are able to view their cultural knowledge from a different perspective.

Language and Multi-Cultural Issues

Adult students in mathematics, whether continuing from school or returning to study, may not have English as their first language. Although some research indicates that being bilingual may be an advantage under certain circumstances, this is by no means always the case. In recent years there has been a huge increase in the number of full-time fee-paying students enrolling in the tertiary sector, many from Asian countries with linguistic systems distant from English. In this section, issues relating to studying mathematics in English as a second (or other) language are addressed from the perspectives of the participation of bilingual students in tertiary mathematics.

According to Galligan (1993), the precise nature of the mathematics register can cause problems not only with words, but also with semantic, syntactic, and pragmatic features. She investigated the problems confronted by students and the thought processes used in overcoming them, when communicating through the content area of mathematics. Galligan (1995) made a comparison of Chinese and English mathematics word problems, and outlined the difficulties that native English speaking students face in comparison to Taiwanese speakers of Mandarin, both in context-free and in contextual word problems.

Hubbard (1994) described an action research project in which instruction in mathematical language and strategies for adapting to the Australian university environment were integrated into mathematics classes for overseas students. However, Biggs (1994) noted that although Confucian-heritage culture students appear to have a commitment to rote-based learning, they in fact have a deeper, more academic approach than most Australians. Cameron and Norton (1995) provided further support for the success of international students, noting that there appeared to be significant motivational factors in operation.

Bull (1995) described an initiative to diagnose, using multiple choice tests, the level of mathematical preparedness for their proposed course of study in students sponsored by the Australian International Development Assistance Bureau (AIDAB). Students from the Pacific Islands and Papua New Guinea in particular are at risk in technological courses.

Baker (1994) and Wilson (1994) addressed the issues confronting teachers of ESL students based on their practical experience. Wilson outlined the needs and expectations of the students and Baker outlined a series of guidelines and strategies which could be of relevance to teachers.

Calculators and Computers

For many adults returning to study mathematics subjects, the use of technology can be an additional hurdle. For some, the issue of access can be problematic. Even if adults are familiar with home or office computers, the use of calculators, particularly those with graphing facilities, can appear threatening at first. Attitudes towards the use of technology are integrally linked with attitudes towards mathematics and how it is learned.

Calculators. Boers and Jones (1992) found that tertiary students who were permitted the use of graphics calculators in an examination did not use them effectively, and had difficulties integrating algebraically and graphically derived knowledge. They hypothesised that the graphics calculator confronted students with an independent source of information, which had to be reconciled with algebraic understanding. In addition, Barling (1993) noted that while a few students embraced the potential of graphics calculators with enthusiasm, most students compartmentalised their knowledge and failed to make optimal use of the technology available to them.

Britton (1994) reported on an on-going project investigating the use in tutorials of graphics calculators by first year tertiary students in a service course, approximately two thirds of whom were female. Although one of the intended outcomes of the project was an increase in understanding, Britton believed that most students viewed the calculators from a more instrumentalist perspective, as enabling them to perform better on examination-type tasks. She also asserted that the calculators increased confidence and decreased anxiety for many students.

Bradley, Kemp, and Kissane (1994) also reported on the introduction of graphics calculators into undergraduate service courses in mathematics. The two major reasons for the introduction of technology into the courses were to enhance student learning through the provision of active learning experiences in order to develop more effective concepts and procedures; and to educate students in the appropriate use of one form of technology currently available and which is relevant to a variety of fields where mathematics is used. Kissane, Bradley, and Kemp (1994) discussed the issues associated with equity in curriculum and assessment of courses incorporating the use of graphics calculators. This is of particular importance in the Further Education sector where problems of lack of funding for equipment can be severe. (See, for example, Aulich, 1991, p. 49). Kissane, Kemp, and Bradley (1995) reported further on student reactions to the use of graphics calculators. Written responses based on a Likert Scale indicated a very positive attitude toward their use, and also towards all aspects of the teaching of the course.

Schuck (1995a) investigated the attitudes toward calculator usage of a group of a group of first year preservice primary teachers. She found that, contrary to public recommendations (for example, AAMT/CDC, 1987; Australian Education Council, 1991), most student teachers had reservation about the use of calculators in primary school, and that these opinions appeared to be based on limited experience of calculators used solely as a computational tool.

Computers. Vistro-Yu (1995) examined the impact of technology on Philippines teachers who completed a post-graduate Certificate Program. The use of computers made some teachers realise the lack of effectiveness of their previous teaching methods, and they were open to the idea of using computers to motivate their students, even if their own experiences with the computers or with the classes were not very positive.

Adlem (1995) reported on the successful use of computer-aided review modules for first year university students. These were integrated with traditional lectures and classroom sessions.

Putt and Henderson (1993, July) and Putt, Henderson, and Stillman (1995) reported on the Remote Area Teacher Education Program to upgrade the qualifications of Aboriginal and Torres Strait Islander paraprofessionals to that of qualified teachers within their home communities. The teaching of mathematics and mathematics education is delivered at a distance using interactive multimedia (IMM), and other electronic technology, such as video, fax, teleconferencing, electronic mail, as well as conventional methods. Study centres house the technology, and provide facilities for the students to work in small groups, providing a learning environment which conforms to that of the culture of the students. Rehn (1992) also reported on the application of new technologies to distance education and bridging mathematical courses as part of Aboriginal programs. Lectures were transmitted live by satellite to students at learning network centres across Western Australia. Steen (1992) described the structure of an Advanced Learning Laboratory to meet the needs of Aboriginal students admitted under a special entry scheme to higher education. The laboratory is an open learning resource housed within, and supported by the library, and includes videodisc, CD-Roms, and a range of mathematical software.

The Affective Domain

It is important to understand a student's attitude toward mathematics prior to any course, as eventual understanding and success may be dependent on attitudinal factors, according to Taylor (1994). She compared two attitudinal scales to mathematics, the Fennema-Sherman 1976 scale and the Aiken 1979 scale. Construct validation procedures were then used by Taylor (in press) on Aiken's 1979 Attitude Scale with adults returning to study in a preparatory mathematics programme: Two factors of enjoyment and value were extracted with high reliabilities. Taylor (1995) described an investigation into these two aspects of attitude towards mathematics of 439 adults returning to study in the distance mode. She concluded that enjoyment and value of mathematics may not be as important for adults as for younger students, but that other dimensions, such as self-concept of mathematics may be more crucial. Miller-Reilly (1994), working with a group of similar students in New Zealand, reported on a pilot of a learning style inventory, and attempted to measure the following factors: enjoyment of mathematics, motivation, importance of mathematics, confidence in mathematical ability, and perception of mathematics.

Gordon and Nicholas (1992) investigated the attitudes and beliefs of 60 mature-age university students towards mathematics and statistics. The students perceived mathematics to be composed of the branches of arithmetic, algebra, and statistics; and to have the qualities of being (very) difficult, logical, and boring. Their attitudes to mathematics indicated primarily a lack of confidence. FitzSimons (1994a), and Wood and Smith (1993) also conducted surveys of student attitudes and beliefs.

Crawford, Gordon, Nicholas, and Prosser (1994) used a phenomenographical method of research to investigate the range of conceptions of mathematics, and general predispositions to learning mathematics, held by approximately 300 students entering university. They found that 77% of first year mathematics students held a fragmented conception of mathematics, and 82% adopted a surface rather than a deep approach toward learning mathematics. They also found that those with a cohesive conception and/or able to adopt a deep approach tended to achieve at a higher level after one year of university study. They argued for a more systematic examination of the relationship between teaching and learning, and the context in which they occur. (See, also, Gordon, 1992).

Carroll (1995) presented a review of research conducted on women and girls learning mathematics. It attempted to consider aspects of mathematics education which impinge upon the interaction between affective factors and performance in mathematics learning. A number of mature-age students, many of whom are women, are learning mathematics as part of teacher education courses.

Teacher education students learning mathematics

A considerable number of mature-aged students are enrolled in primary teacher education courses, and for many the prospect of having to study mathematics again is daunting. Nevertheless their beliefs and attitudes towards mathematics are likely to impinge on their pedagogical practice. A second reason for the importance of this section is that the research findings could inform the professional development of numeracy tutors; collaborative grants utilising the skills of both practitioners and researchers have enabled the development of exemplary professional development materials in the Adult Basic Education sector.

Carroll (1994) presented a case study of a female mature age pre-service teacher who experienced extreme anxiety, and who, although highly motivated, was unable to achieve at anywhere near a satisfactory level on a test of primary mathematics. The data were grouped into four categories: affective factors, cognitive factors, social factors, and teachers' questions. Carroll's study highlighted the importance of the quality of relationships between teachers and students, the role of peers in the learning process, the effect of teachers' questioning techniques, and the debilitating effect of negative affective factors on some students.

Schuck (1993, July; 1993, November; 1994, December; 1994, November; 1995, March) detailed the history and evolution of research based on a "grounded theory" paradigm into the attitudes and beliefs of pre-service primary teachers, some of whom were mature-aged. This grounded theory paradigm recognised the complexity and variability of the phenomenon being researched, according to Schuck, and the methodology allowed both teacher educators and prospective primary school teachers to develop insights into beliefs and attitudes held by the latter. One of the major issues to arise from the study was the dilemma faced by the lecturers of these students, and which was in fact the same dilemma facing the student teachers in the classroom. That is, there is a tension between comforting and challenging the learner. In other words, does the inculcation of a more positive attitude towards mathematics necessitate the avoidance of the complexity of the discipline? This is precisely the same question which confronts teachers of other adult students returning to study. A balance must be found between encouragement and challenge: Furthermore, students should also be made aware of the limitations of mathematics (FitzSimons, 1992).

Johnston (1992) conducted a case study of trainee teachers engaged in collaborative research to investigate their assumptions about mathematics, and its teaching and learning. She hypothesised that, as a result, there was a shift from an instrumental ideology towards an interactional ideology, which views knowledge as a human construction. Perry (1993), and Perry, Geoghegan, Howe, and Owens (1995) reported on a successful approach to enhance the mathematical backgrounds of teacher education students. They devised an alternative to the traditional model of teaching, built around an experiential learning cycle, and which used the principles of cooperative learning and a problem-centred approach. Mayers (1994) also reported on a similar study.

Although most teacher education has focussed in the past on didactics and pedagogical practice associated with the needs of children, attention is increasingly turning towards workplace education and training. Meanwhile, in Technical and Further Education (TAFE) institutes, the needs of students and practitioners in the realm of mathematics and trade calculations continue to be all but ignored by researchers.

The Vocational Education and Training Sector

Workplace Education

During the last decade Australian government policy has been to link pay levels in industry with educational qualifications, to encourage long-term unemployed people to undertake training in basic skills and/or retraining in appropriate vocational areas. There is an assumption that a more highly skilled workforce will contribute directly to economic recovery in Australia. The government's skill formation policies have arisen in response to its perception that Australia has lost its relative international trading position, and that this economic situation was caused by lack of appropriate skills in the work force (Kell, 1993; Stevenson, 1993).

Workplace settings. O'Connor (1994, p. 3) considered the workplace setting as "a site of constant learning, where workers through their interactions, relationships with others and job performance integrate affective, cognitive, reflective, and psychomotor learning on a daily basis." Within the workplace are a myriad of learning processes and activities, with the same workers occupying different roles as teachers and learners at different times.

Buckingham (1994a, 1994b, 1995) researched the culture of workplace education, particularly mathematics. In the technology of the workplace, the mathematics is only partly recognised, embedded in the procedures of everyday tasks. Decision-making arising from the implementation of work teams, in a flatter management structure, requires an understanding of the uses of measurement and statistical controls. According to Buckingham (1994a), research into the effects of workplace education is limited: Descriptions of learning/training tend to have a strong association with economic justification. Buckingham (1994b) examined a number of tensions between the participants and their contexts. These are associated with issues of access, between the situated learning needs of the workers and the provision of generic training packages, designed to train the broad range of industrial workers. Buckingham (1995, p. 141) commented that mathematics educators, "because of their distance from workplace culture, cannot easily learn how mathematics has been used in these settings." She asserted that the interest shown by workers in the uptake of numeracy classes will depend on their sense of personal agency; that is, "a sense of one's right to act on one's own behalf, or as a group, and a recognition that others have a right to do the same" (p. 135). The extent to which workers' contribution to, and involvement in, the education process will be recognised and rewarded by the workplace needs to be considered by educators.

Sefton et al. (1994) noted that mathematical concepts permeate all aspects of daily work in the foundry where part of their study took place. Workers frequently used mathematical applications related to their own work without much difficulty, but were not confident in other areas. A pragmatic, language-based integrated curricular approach, utilising only activities and content derived from the workplace was adopted, enabling participants to demonstrate skills and knowledge, and understanding of content.

In order to determine the mathematics content desired by learners in the workplace, and to find out why it was considered to be important, Beesey (1995) conducted a pilot study with a group of mature age first year Early Childhood Education students, who had volunteered to participate because they had not experienced success in mathematics at school. Although this pilot study may prove useful in community settings, decisions about content may not rest with participants enrolled in accredited certificate courses, and who are sponsored by their employers.

TAFE Institutes

TAFE Institutes (formerly Colleges) now form part of the Vocational Education and Training (VET) sector in Australia along with other industry and private providers in an open training market. Students may be continuing from school or drawn from industry, in which case it may be several years since they have studied mathematics, mostly with limited success. Major issues to be addressed are the need for immediate, contextual relevance where possible, the impact of new technologies, and consideration of the affective as well as the cognitive domains. FitzSimons (1994a) surveyed 73 school leavers and adults returning to study as they commenced a vocational course in science, and found that 56% considered mathematics to be comprised of numbers, rules and formulae, with a further 30% adding that it was usefully applied to solve problems, although the latter was weighted three to one in favour of recent school leavers. Two mature-age students saw mathematics at a higher level, as a way of thinking, possibly as a result of their broader perspectives on life. There is a paucity of research into the educational background and current needs of adult learners of mathematics subjects in TAFE institutes.

The initial preparation and continuing professional development of teachers of mathematics subjects in the VET sector, whether they are recognised as mathematics specialists or are trade teachers, is a subject that has largely been ignored by the sector itself and by the mathematics education research community. FitzSimons (1995, May; 1995b; 1995c) began to redress this. FitzSimons (1993b; 1993c; 1994a; 1994c; 1995b; 1995e; 1995f) and FitzSimons and Sullivan (1993) addressed practical issues of teaching students enrolled in TAFE middle level courses.

One of the major problems with obtaining research funding in the VET sector is that it is usually dependent on the support of an industry partner, and mathematics by definition has no industry, unlike hospitality, business studies, electronics etc. Although mathematics subjects are taught throughout most TAFE courses, they are not regarded as of much importance, compared to the more vocational subjects. Issues of curriculum, assessment and delivery are not seen as problematic, and there is little or no debate about the impact of introduction of competency-based training with particular reference to mathematics. Further, there is a trend towards the increasing use of self-paced materials in line with the national policy of flexible delivery.

Further Research

Since 1992 the adult basic education sector has "been catapulted firmly into the educational mainstream" (Seddon, 1994, p. 10). Accordingly it has moved into a national framework system of curriculum and assessment, written in competency based format, although the descriptors of the National Reporting System cast the learner in "an active, initiating, critical role" (Fitzpatrick, 1995), rather than adopting minimalist statements of competence. This mode may not be appropriate to all students. Research is needed to assess the impact of these changes to curriculum and assessment, the effectiveness of the concomitant professional development, and the relative merit of having numeracy subjects taught by literacy teachers, as is frequently the case.

As mentioned above, the VET sector is similarly moving into competency based training and assessment. Research into workplace education has begun, and needs to continue. With the emphasis on cost-saving, it is likely that, contrary to the integrated curriculum approach advocated by Sefton et al. (1994), providers will use generic packages, written predominantly in transmission mode, and attempt to inject context simply through the substitution of the name of one of the company's products where applicable. The trend towards the implementation of self-paced computer-assisted learning or text materials, eagerly supported by some industry personnel, needs to be examined for its educational consequences. At the higher levels of the Australian Qualifications Framework there is almost no research into mathematics teaching and learning, nor the professional development needs of practitioners at the trade or the professional levels—either in TAFE colleges or in the growing number of registered private and industry providers. This matter needs urgent attention.

In the university sector the bridging mathematics programs and the mathematics learning centre units may find themselves less marginalised as a result of external factors, such as the accountability of their institution to bodies such as the Committee for Quality Assurance in Higher Education, and funding sources such as the Department of Employment, Education and Training, AIDAB, or the benefactors of full fee-paying students. A considerable number of papers have been written documenting research in practice, researching the affective and cognitive domains, and the outcomes of some evaluations. Calls have been made for systematic evaluation and there is also a need for research into initial preparation and continuing education of practitioners.

Conclusion

Research into adults learning mathematics is in its infancy. This chapter has attempted to provide an overview of the activities of researchers in the Australasian region since 1992. The concept of lifelong learning is currently fashionable, and the next four years should see a dramatic escalation of research as the importance of this burgeoning sector is recognised on a world-wide scale. The 8th International Confress on Mathematics Education has inaugurated a working group: Adults Returning to Mathematics Education in recognition of the increasing importance of this field.

Adult learners have major qualitative differences from children. In one way or another, adults have usually made a conscious decision to re-enter study, although they may not have anticipated finding themselves once more enrolled in a mathematics-related subject. They have a greater experience of life which they, consciously or unconsciously, can bring to bear on the learning situation, especially in workplace education. They also bring to the learning situation prior experience of learning mathematics, which may be positive, but which is more likely to cause negative interference with current learning in the cognitive and/or affective domains. Frequently adults need to confront technology such as calculators and/or computers with which they are unfamiliar, yet which are commonplace to school leavers. Often they are in classes with younger, less motivated students; at the same time they can be made to feel inferior by the false bravado of youth. Students whose first language is not English, and whose cultural background may be quite distant from that which is dominant in Australasia, face an even more difficult task to be successful, whether in their own terms or in publicly acclaimed assessment results.

The quantity of available research among the three major sectors defined in this chapter has varied greatly. It is perhaps not surprising that the greatest body is within the university sector, as this is where most researchers are based and there is an assumption that research is to be valued, even though programs which offer access to learners from disadvantaged backgrounds are often marginalised. The basic education sector has a substantial research base, partly because of funded projects, and partly because of its historical background of having to continually justify its existence. There is quite a deal of interest in this sector by (some) universities, and a burgeoning interest in workplace education, with the availability of funding, such as that offered by the Australian National Training Authority (ANTA). The TAFE area of the VET sector is least well served by research, and, although there are policy documents and

ANTA funding to support research, it is difficult for teachers who are employed for an average of 21 hours a week in face-to-face teaching, 36 weeks a year, with numerous other scheduled duties for the remainder, to even plan a research submission. In any case, as mentioned above, the issues of teaching and learning mathematics are not even problematised in TAFE. Research papers are written are in the off-duty hours of the teacher, as is most, if not all, studying for higher degrees. Financial support for attendance at major conferences is rare, whether or not a paper is presented. In short, there is no substantial encouragement for TAFE practitioners to undertake research in mathematics education.

The next four years are likely to see changes to each sector arising from the recent election of a conservative government in Australia, where policies of privatisation and financial accountability are likely to override educational considerations to an even greater extent than has been the case previously. Those students least able to defend themselves are likely to be most at risk. The research community will be required not only to extend the current knowledge base on adult learners, but it will also need to be prepared to defend its territory.

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PART 4

SPECIFIC CURRICULUM AREAS

Research in Early Arithmetical Learning and Teaching

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Introduction

There has been a significant amount of Australasian research in early arithmetical learning and teaching in the last four years. As is appropriate, much of this research relates closely to and is informed by research from other countries. Nevertheless, a good number of the projects described here can be regarded as ground-breaking in an international sense. As well, many have distinctive foci or features which arise because of the uniqueness of approaches to early mathematics in New Zealand and within the separate states of Australia. Although this research will be of special interest to Australasian audiences, its distinctiveness will hopefully serve to increase its potential interest for audiences elsewhere. Examples of issues addressed by this research include: (a) the wide range in children's mathematical knowledge and strategies; (b) mismatches between curricula and children's knowledge; (c) the importance for learning of language; (d) the learning environment and teacher expectations; and (e) success of some intervention programs.

Studies reviewed in this chapter are classified into 11 categories. However, the reader should note that some studies fit well into more than one category. The categories in order of placement in the chapter are: preschoolers and kindergarteners; addition, subtraction, and mental computation; multiplication and division; fractions; calculators; conceptual development, representation and understanding; classroom-based teaching and curriculum studies; teachers' beliefs; interview-based assessment; intervention studies; and mathematics recovery and teacher development.

Preschoolers and Kindergarteners

Leder's (1992) study focussed on the quality and appropriateness of a preschool's educational environment for the mathematics learning of two four year-old boys, and its comparison with their home learning environments. One of the boys was classified as "bright" and the other "average". Data collection was undertaken for all but the first of four terms of the school year and involved regular observation in preschool, videotaping of more formal mathematics teaching sessions, and use of standardised tests to assess vocabulary, concepts, and mathematical knowledge. Of particular interest was how the students interacted with their teacher during the more formal mathematics sessions. Data on home backgrounds involved questionnaires, parental interviews at home and at preschool, and parents' diaries of activities for one week. Leder described differences in the students' backgrounds, their achievements on the tests and the ways they participated in lessons. The bright child showed more insight and perceptiveness. As well, subtle differences in home learning environments were observed, for example, encouragement of independence in the case of the bright child. Leder observed that "not only did the teacher initiate more contacts with [the bright child] …., but the quality of those interactions also differed" (p. 394) and concluded that not only can "there be substantial differences in the learning environment of students in the same formal teaching setting, but … [also in] the role played by teachers in contributing to these differences" (p. 394).

Hall and Elliott (1993) reported the findings of two studies designed to investigate the mathematical learning of "at risk" preschoolers in computer-supported interactive environments. In the first study, 50 preschool children were each assigned to one of three groups: an experimental group in which children participated in a metacognitively-rich mathematical environment based on software such as Number Farm; a second experimental group in which the teaching was largely informal, but caring and supportive in nature; and a control group which did not participate in any computer-supported mathematics activities. Children in the two experimental groups were paired and received 15 twenty-minute teaching sessions on the computer each. Hall and Elliott used the *Test of Early Mathematical Ability (TEMA 2)* instrument to measure mathematical achievement before and after the completion of the intervention program. They found that "computer mediated learning environments are successful in helping "at risk" preschoolers learn pre-mathematical and arithmetic concepts and skills" (p. 346) and that a metacognitive teaching approach was more successful than a caring approach typical of most preschool environments.

In the second study, 24 at risk preschoolers were exposed to a metacognitive teaching approach both in the computer context and in the general activities they experienced. Children's mathematical achievements were assessed using the TEMA 2 instrument prior to the change in approach and were reassessed six months later. The results indicated that a more focussed and metacognitively-rich pre-mathematical and mathematical program can be beneficial to children's mathematical development.

Wright (1994a) used interview-based assessments to document the arithmetical knowledge of 41 students at the beginning, middle, and end of a school year. The students were selected from three classes, a kindergarten, a Year 1 from the same school, and a kindergarten from a school serving a community considered to be higher in socio-economic terms than that of the first mentioned school. The assessment interviews were videotaped for subsequent analysis involving the determination of levels of arithmetical knowledge on each of five models. One of the models is based on the theory of counting types (e.g. Steffe, 1988), a second is based on children's development and elaboration of forward number word sequences, and a third is similar to the second but relates to backward rather than forward number word sequences. A fourth relates to children's ability to ascribe number to regular and irregular spatial patterns for the numbers from one to ten, and the last relates to identification of 1-, 2-, and 3-digit numerals. Documented in the report are kindergarteners' advancements in terms of the models and the usefulness of the models for documenting such advancements was examined. As well, relationships between advancements on the counting stages model and the number word sequence models were analysed. Significant mismatches between children's arithmetical development and typical curricula were described; for example, the tendency of curricula to underestimate kindergarten children's levels of arithmetical knowledge, and changes to current practice recommended.

In a study designed to investigate what enabled young children to perform dealing tasks, typically involving one-to-one sharing, when denied access to perceptual cues, Hunting, Pepper, & Gibson (1993) individually interviewed 30 children aged between three and five years. Of interest was whether young children could adapt by using a scheme that did not rely on perceptual cues such as comparison of heights of shared items, comparison of lengths, one-to-one matching across shares, or successive comparison using subitising which involves recognition of number patterns without counting them. Also of interest was the means by which children succeeded with tasks where schemes seeming to depend on perceptual feedback could not be used. They investigated the strategies used by young children on a series of tasks involving Money Boxes that were designed to progressively restrict children's access to perceptual cues. Hunting et al. analysed the strategies used to complete the tasks by both successful and unsuccessful children and found that certain "internal regulations of actions made it possible to succeed" (p.287). One such mechanism allowed students to monitor lots of multiple allocations. The researchers concluded that the development of a stable pattern of operations having an iterative structure was a critical difference between successful and unsuccessful children, but that the reliance on sensory feedback in the process of solving these kinds of tasks is unclear.

Davis and Pepper (1992) examined the problem-solving strategies used by young children when solving a "redistribution problem" (p.403), involving dealing — described as "a primitive action scheme … composed of repeatable units of action that are performed over until the task is completed" (pp. 397, 402), and which is important as a basis for the development of the concept of one-to-one correspondence in children. They discussed aspects of dealing such as its origins, commonality, and whether children use dealing as a "spontaneous" (p. 399) method of sharing. Davis and Pepper observed that little was known about the place of dealing in children's development but suggested that the ability to share by dealing may precede the ability for more abstract counting. Thus children who were "good counters" (p.402) would probably be good at dealing but children who were good at dealing may not be proficient counters. Davis and Pepper's study involved 74 children from three pre-schools who were categorised as " 'poor'; 'developing'; or 'good' counters" (p. 402) on the basis of Steffe and Cobb's (1988) "counting type classifications" (p. 402). The children were administered a 15-minute clinical interview to ascertain their counting ability. Interview tasks included counting unscreened, partially screened, and fully screened collections in a familiar setting. Davis and Pepper hypothesised that the children who were good at counting would perform significantly better than the children who were not. The children were then presented with a task requiring them to re-distribute 12 crackers to three dolls, when the crackers had previously been shared between two dolls. Davis and Pepper pointed out the problematic nature of this task, claiming that it required more than just the action of sharing out crackers in a repetitious, unreflective way, but that it made cognitive demands on the children. They detailed the strategies employed by the children, noting that there were five main strategies which were commonly used. The results of the study supported the researchers' expectations that children classified as good counters would more successfully complete the task than children who were classified as developing or poor counters. All the good counters solved the problem, whereas 61% of the developing counters and only 49% of the poor counters were able to do so. However, children who were not good counters and were able to solve the problem were able to do so at least as efficiently as the good counters did, suggesting that re-distribution tasks may be true "prenumeric problem solving tasks"(p. 413). Davis and Pepper also observed that a large number of solution strategies were used, indicating that the children's responses were largely "un-learned" (p. 413), although the most commonly adopted strategy suggested to them "that many children at this age are capable of apportioning discrete items in the ratio 2:1" (p. 413). In conclusion, Davis and Pepper conceded that they still knew little about the important aspects in the development of children's "dealing schemes" (p. 413) and emphasised the importance of a longitudinal study focussing on this.

Geoghegan (1993) and Geoghegan and Mitchelmore (in press) explored the possible relationship between experience with music and higher achievement in mathematics. Thirty-five preschoolers participated in a year-long structured music program involving an hour-long music lesson each week. Their early number concepts were compared with those of 39 counterparts with "limited musical backgrounds" (p.281). Concepts were assessed using the TEMA-2 test and included: "relative magnitude; counting skills, calculational skills, knowledge of conventions; and number facts " (p. 282). Participants performed significantly better than counterparts. Geoghegan then re-categorised the participants to take account of any other musical experiences these children may have had, especially in the home. Post-hoc analysis of the data indicated that there was no significant difference between the performance of participants with limited home musical experiences and counterparts. However, participants who had musical experiences at home performed significantly better than participants who did not. Geoghegan concluded that "a structured music program alone is less likely to contribute to higher achievement in early number concepts than musical experiences generated in the home environment or other pre-existing differences" (p.284).

Geoghegan and Mitchelmore (in press) attempted to determine areas of home musical experiences which may have promoted the higher mathematical achievement. Two areas were found to be significantly related to children's achievement. These were: children often listening to their own musical collections; and family members singing with or to the children. Geoghegan and Mitchelmore argued that two important aspects of learning could be inferred from these results: "listening" and "self-esteem" (p.7). They discussed listening in terms of "concentration and attentiveness" (p.7) which were prerequisites to children's construction of knowledge in a group learning situation and suggested that self-esteem could stem from parental interaction.

Dale's (1995) study of children's early number knowledge included the development of: (a) an interview schedule to assess the number knowledge of children on entrance to formal schooling; and (b) a curriculum statement, based on the United Kingdom National Curriculum Statement. Six girls and six boys of mixed ability from one school were individually interviewed, using 22 "pre-set questions" (p.217), each question consisting of several tasks. The tasks related to: counting; reading and writing numbers; recognition of pattern, money, and shapes; estimation, addition, and subtraction; and division as sharing. Findings of the study were that, whilst performance varied on some tasks, these children entered school with a substantial amount of number knowledge. For instance all children were able to count beyond 10, understand the principle of cardinality, solve addition and subtraction problems to 10 using concrete materials, and estimate objects to 10. Dale used her curriculum statement to compare these children's performance with that expected of children by the end of their first year of schooling. She found that most children already had knowledge of the number concepts in the curriculum statement. In conclusion, Dale emphasised the need for teacher awareness of children's prior knowledge so that skills already mastered are not re-taught; she suggested that even her preliminary findings lend support to curriculum reform.

Addition, Subtraction, and Mental Computation

McIntosh, Bana, and Farrell (1995) reported research which collected data on mental computation from students in Years 3, 5, 7 and 9. The 163 Year 3 participants were administered a survey of preferred method of calculation and mental computation tests. The researchers found that preference for mental computation increased with an increase in year level, that performance on additive tasks was better than that on equivalent subtraction tasks, that children who were competent at mental computation more readily selected this method than those who were not, and that children's confidence varied widely. McIntosh et al. conceded that their study "had several limitations" (p.36) but suggested that curriculum development and teaching need to: take account of the wide range in children's ability; help children to gain a better understanding of the number system; and provide opportunities to enable children to develop mental computation strategies.

Cooper, Heirdsfield and Irons (1993) reported the rationale and preliminary findings for a two year longitudinal study of children's mental computational strategies for 2- and 3-digit addition and subtraction. The study later expanded to include multiplication and division computations and was extended over five years from Years 2 through 6. Results from 130 Year 2 and 3 students showed that many children lacked basic understanding of concepts and procedures even after years of instruction, and that children used idiosyncratic and creative strategies to manipulate numbers. Some children used a variety of informal strategies succesfully even before instruction, but others became constrained by pen and paper instruction. Another aspect of this five year longitudinal study was reported by Cooper et al. (1995). It was found that children from Year 2 or 3 used varied and increasingly powerful strategies for solving 2- and 3-digit addition and subtraction word problems. Of interest was that children used non standard mental strategies (for example a strategy involving left to right calculation), but these were replaced by standard right to left strategies once instruction in algorithmic procedures occurred.

Boulton-Lewis and Tait (1993) investigated children's representations and strategies for addition. They hypothesised that some of the strategies and representations that teachers accept as good practice are not as successful as thought previously. They interviewed 55 children in Years 1 to 3 and their teachers. Children were presented with 27 test items (for example, 166+254) and asked to represent the operation and find the answer using any concrete aid they found appropriate including fingers, pencil and paper, and Multibase Arithmetic Blocks. Teachers were interviewed to determine the strategies they use to teach addition. Results were analysed from an information processing perspective and indicated that children prefer to use verbal and mental strategies rather than formal algorithms. The researchers concluded that children in the study were using their own strategies and thus reducing the processing load by using no materials or materials only.

Multiplication and Division

Mulligan (1992a) conducted a constructivist-based, two-year longitudinal study of multiplication and division strategies based on the study of Carpenter and Moser (1984). Children's solution strategies to a variety of multiplication and division word problems were analysed at four interview stages. The study involved 70 girls during Years 2 and 3, from when they had received no formal instruction in multiplication and division to when they were being taught basic multiplication facts. Ten problem structures, five for multiplication and five for division, were classified on the basis of differences in semantic structure. The relationships between problem condition (i.e., small or large number combinations and use of physical objects or pictures), performance, and strategy use were examined. Seventy-five percent of the children were able to solve the problems using a wide variety of strategies even though they had not received formal instruction in multiplication or division for most of the two-year period. Performance level generally increased for each interview stage, but few differences were found between multiplication and division problems. Solution strategies were classified for both multiplication and division problems at three levels: (a) direct modelling with counting; (b) no direct modelling with counting, additive, or subtractive strategies; and (c) use of known or derived addition or multiplication facts. A wide range of counting strategies was classified as counting-all, skip counting and double counting. Analysis of intuitive models revealed preference for a model involving equal grouping and repeated addition for multiplication, and a "building-up" model for division based on equal grouping.

Mulligan (1992b) analysed children's solution strategies for partition division problems included in the longitudinal study (Mulligan, 1992a) and found that children very rarely used a sharing one-by-one (that is, dealing) strategy at any stage. Instead, a variety of counting and grouping strategies such as estimation and grouping, one-to-many correspondence, and trial-and error grouping procedures was used. Knowledge of addition facts and skip (multiple) counting assisted children in forming equivalent groups. Lack of a sharing one-by-one strategy was attributed to the problem of identifying the total dividend to be shared, whereby children did not need to count out the total to be shared. Their understanding of "fair share" was demonstrated by the formation of equivalent groups and trial-and-error procedures to ensure that each group had the same amount. Some children who used immature strategies based on counting-all and modelling also used sharing one-by-one, and by the final interview this strategy had changed to a more efficient one based on estimation of equivalent groups. It appears then, from the analysis of individual profiles, that sharing one-by-one was used at early stages and that children progressed to more efficient division strategies with the development of skip (multiple) counting and addition skills. This research challenges traditional partitive and quotitive models for division and suggests that the development of problem-solving strategies for multiplication and division contexts can be encouraged from the pre-school years.

Mulligan (1993) used individual interviews in a follow-up study of 45 Grade 6 children who, in second and third grade, participated in a two-year longitudinal study (Mulligan, 1992a). This involved analysis of children's solutions to multiplication and division one-step word problems. Problem structures identical to those in the earlier study were used including problems involving decimals. A pattern of performance similar to that of the longitudinal study revealed a 60-85% success rate except for sub-division and decimal problems. Informal methods used by children in the early stages of the study dominated their understanding of multiplicative structures in Grade 6, and there was evidence of a lack of conceptual understanding of multiplication and division.

Mulligan and Mitchelmore (in press) analysed children's intuitive models of multiplication and division, how these relate to the semantic structure of problems and their development over time. An intuitive model is defined as "an internal mental structure corresponding to a class of calculation strategies". Instead of looking for implicit models from children's past which might influence current behaviour, the researchers modelled children's behaviour in terms of the strategies they use. Data were drawn from a longitudinal study of solutions of multiplicative word problems, of 60 second and third grade female students. From the correct responses, 12 calculation strategies were identified and grouped into categories from which the intuitive models of multiplication and division were inferred. It was found that the students used three main intuitive models: direct counting, repeated addition, and multiplicative operation. A fourth model, repeated subtraction, occurred only in division problems. All intuitive models were used with all semantic structures, their frequency varying as a complex interaction of age, size of numbers, language, and semantic structure. The results were interpreted as showing that children acquire an expanding repertoire of intuitive models, and that the model they employ to solve any particular problem reflects the mathematical structure that they impose on it. The study represented a detailed analysis of children's strategies over a critical period in which multiplication and division concepts develop. Implications for teaching and learning suggest far-reaching reform to the early mathematics curricula.

Fractions

Davis, Hunting and Pearn (1993) (see also Hunting, Davis, & Pearn, 1994) conducted a teaching experiment which focussed on fractions as operators and involved 10 third grade students. Teaching sessions were conducted twice weekly for periods of three weeks with each session lasting for about 20-25 minutes. A major feature of the teaching sessions was the use of a computer-based learning tool called CopyCat. This tool allows users to explore relationships governing inputs and outputs of discrete items and was chosen to activate the students' knowledge of whole numbers. Three cognitive schemes that two children used as they solved fraction comparison problems were discussed. A scheme is the "mental coordination of actions" (Hunting et al., 1994, p.2). The researchers argued that the types of tasks presented "during the course of the experiment assisted these children to refine and consolidate their knowledge of whole numbers and their interrelationships" (p.19). They concluded that rather than using a broad-based approach to developing rational number understanding, an alternative approach may be to develop a particular conception, and then expand that conception once children can relate mathematical procedures and their symbols to their informal knowledge.

In reviewing recent research into initial fraction concepts, Pitkethly and Hunting (1996) recognised two groups of researchers. The researchers in one group identify initial fraction concepts as emerging from the application of intuitive mechanisms, especially in the context of partitioning, and also leading to unit identification and unit iteration. Researchers in the second group identify ideas of ratio and proportion relations present in young children's early conceptions of fractions. In drawing together the research on initial fraction concepts, Pitkethly and Hunting highlighted the links and differences among different research projects. In so doing, they made a case for the 'emergence of a co-ordinated, global research program in this field" (p.49) and worked toward generating a theory of rational number learning.

Calculators

Groves (1992, 1993) and Groves and Cheeseman (1992) reported studies which formed part of a larger study of the long-term effects of calculator use on the teaching and learning of mathematics. In 1991, 1992, and 1993 third- and fourth-grade cohorts were tested for their use of calculators and were administered a written test. The 1991 third-grade cohort did not participate in the calculator project and served as the control group for the study. Groves (1992) examined the extent to which 55 Grade 3 and 4 children from six schools, who had not been involved in the calculator project were able to give reasonable answers to "five versions of a real world problem based on quotition division" (p.249), where each form of the problem was slightly more difficult than that preceding it. The children were given "simulated bottles of medicine" (p.249) showing the total contents and daily dosage and were required to calculate the number of days the medicine would last. They were provided with concrete materials, calculators, and pencil and paper. Children's strategies, calculating materials used and efficacy with calculators were observed during a 25 minute interview. The results showed that most children preferred to use mental computation, particularly repeated addition and repeated subtraction, and many were not aware when they gave unreasonable answers. Division was only used when the numbers called for the use of a calculator and many children had difficulty "attaching meaning" (p.255) when using calculators. Groves concluded that children can invent their own strategies for solving quotition problems, if "the numbers are not too difficult to handle" (p.255). She observed that many children fail to see the relationship between school mathematics and real world problems and suggested that giving children calculators to use is insufficient. Calculator use needs to be accompanied by changes in children's mathematical sophistication and overall number sense.

Groves' (1993) study involved 48 children from five schools and addressed the hypothesis that children who took part in the calculator project would be more capable of choosing suitable methods and be more competent in calculator use, when solving problems based on realistic situations. She compared the performance of 25 third graders who had participated in the calculator project for 18 months with that of 23 of the 1991 third graders (1992). Students were administered a 25 minute interview requiring them to solve a division task similar to the one in the 1992 study and a task based on "repeated-addition multiplication" (p.9), involving the calculation of the number of paper clips needed for *x* children to make a chain containing 17 paper clips, where x varied from 10 to 27, to 295 and finally, to 1 million. Results from the study supported Groves' hypothesis in some aspects. Participants in the calculator project performed better in the multiplication and more difficult division tasks than non-participants. There was no significant difference between the two groups' choice of "calculating device" (p.15), both groups using calculators with the same frequency. However, Groves pointed out that the experimental group was "better able to attach meaning and interpret their ideas" (p.15).

Another aspect of the calculator project is a focus on teacher expectations of children's performance and subsequent changes in teaching. Groves and Cheeseman (1992) found that seven teachers' expectations of children's performance over two years, with respect to negative numbers, counting and big numbers, increased slightly. These expectations were based on students' previous performance but were still "conservative" (p.327) when compared with actual performance. Five of the seven teachers reported a change in teaching approach to accommodate their increased understanding of children's number knowledge, made evident by children's calculator use.

Doig (1993a, 1993b) explored children's "beliefs" about mathematics in the context of responding to a set of "leading questions" (p.60) in written form or by means of drawings. Of the 200 Year 3 children who were required to answer questions incorporated into a short story format, based on "an alien's visit" (p.60), 100 had unlimited access to calculators and 100 did not. One study (Doig, 1993a) compared the responses of the two groups of children and focussed on children's ideas about number usage, the size of numbers, the "symmetric nature of the number system" (p.57) and the possible existence of other mathematical systems. Doig noted claims that calculator use may promote children's knowledge of decimals and negative numbers, before these topics are formally taught. He analysed the children's responses and set up a continuum which was descriptive of their number beliefs. Findings were that significantly more children who were calculator users gave negative numbers to describe the smallest number possible than those who were not calculator users. Similarly, four times as many calculator users believed in "a symmetry in the number system" (p. 64). None of the children used fractions for their answers. Doig concluded that the study supported the development of a "systematic set of beliefs" in children which was different for those who used calculators.

Doig (1993b) questioned children to elicit their ideas on calculator usage, on calculator correctness, on monitoring calculator answers, on learning from calculators, and on what makes calculators work. Doig again used a continuum for classification and description of children's responses. The results reflected the existence of a group of developing beliefs in children about calculators. Most children saw calculator usage in terms of calculation, felt that calculators were not always correct, and were unaware of what makes calculators work. Many children exhibited strategies for monitoring calculator answers and half of the children thought that learning from calculators was via seeing the correct answer. Doig discussed the implications of the results for calculator programs in junior classrooms, particularly emphasising the need for aligning teaching with prior knowledge. In view of the large numbers of uninterpretable responses in both studies, Doig conceded the need for further research including the refinement of questions and use of follow-up interviews.

Conceptual Development, Representation, and Understanding

Irwin (1992) outlined a framework for understanding how young children form mathematical concepts from the view that children are involved in an active sense-making process. Irwin discussed children's intuitive knowledge of number as protoquantitative knowledge based on the framework of Resnick (1992). This knowledge is based on children's schemas of comparison, increase and decrease and part-whole relationships, and usually develops prior to schooling. The notion of schemas is interpreted in light of the importance of developing mathematical relationships in the construction of number concepts. The child's actions on objects and understanding of transformations on objects are emphasised here. In her investigation of the part-whole schema, Irwin found that children needed to integrate their knowledge of counting, numbers, and quantities to develop quantitative and numerical knowledge. It was of concern that the teachers of the children interviewed in Irwin's study were not making use of this understanding. Irwin (1996) interviewed 107 children aged from four to seven years to ascertain their understanding of compensation in part-whole relationships. They were questioned about uncounted quantities, counted quantities, and numerical equations. Four-year-olds could usually predict the effect of changes to one or more parts of an uncounted whole, but were less competent in predicting changes to counted quantity. The ability to predict changes to counted quantity and give reasons for the change, increased with age. Six-year-olds were able to give reasons for these relationships that went beyond direct observation. Only seven-year-olds were able to use covariance and compensation in a purely numerical context, that of derived equations.

Mulligan and Killion (in press) explored mechanisms that intervene in conceptual development, for example, prior conceptions, implicit models, and spontaneous reasoning. Interpretations of the term *mechanism* were discussed. For example, a mechanism is a broader construct than a strategy. It was argued that researchers do not have direct access to mechanisms but can infer them by observing and interpreting children's behaviour. Questions concerning the interpretations of children's responses to situations were posed, for example, determining whether a child lacks conceptual understanding or fails to appropriately attend to the mathematical task. Also discussed were difficulties associated with mathematical tasks which focus too narrowly on the adult's representation, and implications for future research. Thomas and Mulligan (1994b, 1995) reported a follow-up study of gifted and talented students' representations of the counting sequence. An overview of research on imagery and its role in developing conceptual understanding in mathematics was provided and examples of dynamic imagery were supplied. Possible connections between children's development of structure of the number system and their representations were explored.

Thomas (1992) investigated links between children's understandings of the structure of numeration, and their representations of the counting sequence 1-100. From children's explanations and drawings of the numbers from 1-100, their internal imagistic representations were inferred. The analysis of verbal protocols, and children's productions of pictorial and notational recordings of the numbers 1-100 revealed three dimensions of external representation: (a) pictorial, ikonic, or notational characteristics; (b) evidence of creative structural development of the number system; and, (c) evidence for the static or dynamic nature of the internal representation. Children for whom a structured internal representation of the number sequence was inferred showed further evidence of a high level understanding of numeration.

Thomas, Mulligan, and Goldin (1994) provided an interesting analysis of the representational characteristics of children's counting. The researchers asserted that external representations can be used to gain insight into some aspects of children's internal representational systems. This provides some evidence of children's internal structure of the number system. However they pointed out that this is but a partial description of children's internal representational capabilities. Of particular importance was the evidence that children's representations of numbers are highly imagistic, and that these configurations embody structural development of the number system and vary widely across children often in unconventional ways.

In a study conducted by Boulton-Lewis and Halford (1992) the representations and procedures for place value of 28 Year 1, 2, and 3 children and their teachers were determined and analysed. They used Halford's structure mapping theory to explain the processing load implicit in the concrete materials used by the teachers. The children were interviewed prior to and after receiving instruction in place value concepts to determine their representations, procedures and knowledge of content. The results indicated that teachers can unknowingly increase the processing load for children trying to learn mathematical concepts if they use concrete representations that are unfamiliar to the children. Secondly, they found that teachers can cause "temporary confusion" (p. 21) in children if teachers do not match their representations and strategies for teaching place value as closely as possible.

Boulton-Lewis (1993) presented findings from an investigation intended to "test children's knowledge of place value and of the counting sequence in order to determine the relation between them" (p. 99). She individually interviewed 55 children in Years 1 to 3 in Terms 1 and 3. Classroom teachers were asked to identify their objectives for counting and place value and to describe their strategies for teaching it and the representations they would use. The results showed that at both testing phases, there was a close association between knowledge of place value and the level of the child's counting. They also found that most children counted well beyond that specified by the curriculum and that expected by their classroom teachers.

Classroom-based Teaching and Curriculum Studies

Bobis (1993) investigated the effects of teaching activities which were designed to enhance the development of "visualisation strategies" (p. 117) in kindergarten children. The activities focussed on identification of groups of numbers and part-part-whole relationships of number combinations, especially combinations to ten, rather than on count-by-one. Young children's ability to subitise, that is, to "instantaneously recognise number patterns without counting them" (p. 117), was discussed and Bobis asserted that intuitive skills such as subitising should be firmly entrenched before children are asked to deal with computation at a symbolic level. She suggested that, with practice, children may build up a repertoire of instantly recognisable dot patterns, which aid the development of concepts of number relations. Her year-long study involved 67 children from two mixed ability kindergarten classes and two classroom teachers with at least five years' teaching experience. Instructional materials consisted of "dot cards" (p.118), where the numbers from one to ten were depicted as an arrangement of dots, and "ten frames". Whole class and small group instruction were used and verbal interaction was encouraged. Teaching activities included subitising, counting, recognition of auditory patterns, and construction and recognition of number combinations. Children were also required to draw "the picture in their mind of their special number" (p.120). Ten-minute class discussions and short individual interviews about the children's drawings were audiotaped. Bobis included two excerpts of transcripts, each involving children's verbal responses when a ten frame was flashed. Bobis conceded that "counting is certainly important" but emphasised the importance of developing visualisation skills in young children. "With practice, children can become capable of mentally combining and separating patterns, thus able to instantly recognise the whole and its related parts" (p.121). She stressed the importance of such strategies for recall of addition and subtraction facts.

Higgins' research (Higgins 1993, 1994) focussed on children's and teachers' behaviours and task effectiveness in situations where games were used for concept learning and for the development of children's autonomy as learners. Both studies were based on observations of eight New Zealand classrooms involving 5- and 6-year old children, during Terms 2 and 3 of the 1993 school year. A total of 113 one-hourly mathematical lessons were observed, video-taped and audio-taped. Higgins (1993) examined the effectiveness of games in developing mathematical processes. Of interest was whether games would enhance children's motivation to learn, enabling them to learn mathematical concepts and develop skills for effective group operation. Higgins used transcripts to illustrate the interactions which occurred between children in small groups as they played mathematical games, with particular emphasis on children's perceptions of the activities as *winning games*, on group discussion of the basis of winning, and on the different ways children coped with a competitive situation. She found that children had differing viewpoints on the desirability of the game being a winning game and some children had difficulty accepting the competitive situation. Some children were able to sort out conflicting viewpoints through discussion whereas others were not. Cheating occurred on several occasions, was acknowledged as such, and rationalised by the children. Higgins questioned the value of games as they are currently used in junior (i.e. K-2) classrooms. She noted that games were aimed at skills reinforcement and practice and mostly "won on the basis of luck rather than skill" (p.7). Higgins claimed that games would be more effective if they were adapted to promote the use of such mathematical processes as checking, problem solving, and explaining. Her concluding remarks suggest that games need to be restructured, not only to encompass mathematical concepts, but to enable children to more easily make decisions, such as what constitutes winning, and more easily "get access to the mathematics" (p.8).

Higgins (1994) emphasised the importance of identifying "key factors" (p.339) which enhance the effectiveness of small group learning, and discussed the importance of children's interactive behaviour, the role of the teacher and the nature of the task. Effective small-group instruction was illustrated in a game in which children were required to match larger cards showing workers with smaller cards depicting tools. Transcripts are used to illustrate effective teacher behaviour and effective group interaction, and elements of tasks are discussed. In the case of the teacher's introduction to a task, Higgins claimed this was effective because the teacher modelled reflective discussion, highlighted possible decision-making thought processes, and only briefly referred to rules of the game. In the case of children's interactions while playing a game, Higgins noted the supportive attitude of the children and the amount of discussion they engaged in. The task was considered effective because it generated discussion, encouraged children to think about the mathematical ideas embodied in the task, and provided an opportunity for all children to participate by giving rise to multiple sub-tasks. In conclusion, Higgins re-emphasised the importance of task design, appropriate teacher actions and effective children's interactions in promoting cooperative group work.

In investigating the degree to which children's mathematical understanding was enhanced by the talk they engaged in during independent group work, Thomas (1993) considered the following four issues: (a) the nature of children's talk; (b) the extent to which this talk facilitated mathematical understanding; (c) the classroom conditions needed to promote the desired kind of talk; and (d) the methods teachers might incorporate into their programs to foster the development of mathematical discussion. Talk was categorised as task-related or non-task-related (p.3). Task-related talk was categorised as social or cognitive (p.3), and cognitive talk was further categorised as action or abstract (p.3). Thomas illustrated the kind of talk children predominantly engaged in during baseline, progress, and final phases of the study (p.2). The target group consisted of two boys and five girls, aged between six and seven years, who were videotaped while they participated in group work, over 16 sessions. At the end of the baseline phase Thomas found that most talk was task-related but only 27% of this talk was related to the mathematics of the task. At the beginning of the progress phase the classroom teacher initiated two changes,which were designed to challenge children and to promote better cooperative work. In the final phase there was an increase in the proportion of action talk and a decrease in social talk, and this was attributable to the changes. Thomas subdivided social talk and found that children were now more concerned with task completion and helping others than with the mechanics of the task. She then subdivided action talk and found that children in the final phase were reflecting more on their own or other's actions than they had during the other two phases (p.8). Thomas noted that children very rarely engaged in abstract talk (p.10), which she argued "involves the clarification of one's ideas" (p.3). She concluded that activities can be re-structured to allow higher levels of pupil involvement in independent group work, but maintained that these children may be "unable to engage in substantive abstract talk with their peers" (p.10).

Tait and Boulton-Lewis (1993) described a project involving classroom teachers and academics in collaborative action research. On the basis of previous research findings (Boulton-Lewis & Halford, 1992) they predicted that, if teachers were aware of the cognitive load imposed on children's processing capacities when unfamiliar concrete representations were used, then they would be in a better position to improve their teaching. Two teachers interviewed 27 children from Years 1 to 3 to assess their understanding of place value prior to and after an instructional phase. While the results of the study concurred with previous research findings, the real value was in the way theory was linked to practice for the participating teachers. Tait et al. reported that the self-reflective nature of the enquiry was enhanced by the collaborative relationship between the classroom teachers and the academics, with the ultimate goal being to enhance student learning.

Wright (1993d) reported a classroom-based study involving a problem-centred approach to mathematics in the first year of school. In this study a researcher, a research assistant and the class teacher collaborated to plan and evaluate three lessons per week, for 17 weeks of the school year. In the first nine weeks the instructional approach was predominately whole class teaching. In the tenth week a lesson format was introduced and used for the remaining seven weeks. This involved an initial period of whole class teaching, an intermediate period during which children worked in pairs to solve arithmetical problems, and a concluding period involving teacher-led discussions of solutions. All 34 children in the class were assessed at the beginning, middle, and end of the school year, via individualised, interview-based assessments. This enabled documentation of levels of arithmetical knowledge, arithmetical strategies and responses to arithmetical tasks, and the profiling of each student's progress over the course of the project. This progress is tabulated in the report and comparisons drawn with progress of students who participated in an earlier study. Virtually all students made notable progress and relatively large gains were made by students who were initially the most advanced, that is, gains which were significantly beyond those typical of advanced children in the first year of school. This can reasonably be attributed to the instructional approach being more challenging and extending than what is typical in the first year of school.

Burnett (1993) studied the use of instructional mathematical games with a first-grade class of mixed ability. Burnett argued that children's progression from informal mathematical knowledge to more abstract mathematical ideas is facilitated when they make important connections between their everyday language and the more formal language of mathematics. Further, instructional games enable children to use and understand mathematical language and learn patterns of social interaction, within a context that makes sense to them. The activities used were hands-on, problem-solving games played in small groups. Children's responses were recorded via videotape, journal entries, and formative assessment. Transcripts were presented to illustrate various levels of social and mathematical interaction. Burnett argued that the activities and learning environment "contributed to the development of the children's numerical concepts" (p.142) and that discussion generated by the games and the social context (p.142) is an important facilitator of learning.

Visser and Bennie (1993) reported on a longitudinal study of the effectiveness of the Beginning School Mathematics (BSM) program, in terms of pupil learning and attitude. Five children were randomly chosen from each of 40 New Zealand schools. All 40 teachers used BSM in their classrooms, nine exclusively and 25 as a major resource. Most of the teachers used BSM as specified in the documentation. Information about the 200 students was collected from teachers, parents, and school principals, and students' mathematical skills were assessed via interview at school entry and five to ten weeks later. An extended version of the interview was administered 18 months later to 155 children who remained in the study. Results indicated that "children enter school with a vast range of abilities" (p. 4) and learn a great deal in their first 18 months of school. Questions designed to elicit students' attitudinal responses showed that more children held positive attitudes towards school and mathematics after 18 months of schooling than did initially. A questionnaire was used to obtain parents' and caregivers' views about mathematics, and more particularly BSM, in the junior school. These were circulated approximately three months prior to the children's final interviews. Indications of the study were that about two thirds of the parents were happy with their child's mathematical program but approximately one third of the parents felt that they were insufficiently informed to comment. Only about one third of the parents felt they knew enough about BSM to comment but most of these "liked it" (p. 14). Visser and Bennie concluded by suggesting that schools should provide parents with more information about the teaching and learning of mathematics at school.

In a review of mathematics in the lower primary years, Wright (1994c) drew on current research to explicate twelve assertions relating to curricula, teaching, learners, and learning environments in lower primary mathematics. The review addressed the following topics: unchanging and under-challenging curricula; the need for greater emphasis on developing children's verbal number strategies and number sense, and activities specifically suited to prenumerical children; curriculum constraints on teachers; the role of problem solving and differing interpretations of problem solving; the need to better understand how children learn mathematics; differences in children's knowledge; "anti-interventionism", discovery learning, constructivism, children's autonomy, and developmental learning; the need for compensatory programs; and learning in collaborative settings. Wright concluded that learning and teaching lower primary mathematics continues to be an important area of focus and challenge for teachers and researchers. (See also Wright, 1992b for a discussion of several of these issues.)

Teachers' Beliefs

Faire (1992) explored the relationship between six to seven year-old children's understanding and knowledge of place value and the classroom mathematics programs in which they participated. The children, who were in their third year of school, were drawn from three New Zealand schools. Clinical interviews to ascertain their understanding of the place value nature of multi-digit numbers were used at the beginning of the year and six months later. Faire also used observation, informal conversations with teachers and children, and open-ended interviews to examine teacher methods used to promote the learning of place-value concepts. From the initial interview, Faire categorised children's place-value understanding in terms of three levels of increased sophistication. At all levels children were able to count a collection of 16 counters, draw them, and write the numeral 16. However, it was only at Level 3 that children were able to represent the tens digit as a multiple of ten. Children at Level 2 interpreted the tens digit "at its face value" (p. 3), while children at Level 1 assigned no meaning to the individual digits. Observing that children at two of the schools progressed more than those at the third, Faire compared the teachers' classroom strategies and teacher beliefs. Teachers at the schools of greater progression espoused a constructivist view of children's learning and organised their classrooms and teaching strategies accordingly. Faire concluded that teacher beliefs of how children learn mathematics may facilitate their development of place-value concepts and suggested that teachers may need to re-examine their beliefs and re-organise their classroom strategies in order to allow children to generate their own mathematics.

Wright (1992c) used open-ended, individual interviews in an investigation of teachers' theories about how children's concepts of number develop and advance. Transcripts of the interviews with 33 teachers in the kindergarten year, were analysed and as a result, 12 categories of responses were determined. A category labelled *experience with numbers* had the most frequent number of responses — 54%. Categories next in order of frequency were *role of language* (27%), *prior learning* (27%), *counting* (24%) and *materials* (21%). In comparing teachers' theories of young children's arithmetical development with those of researchers, Wright suggested that teachers tend to regard problem solving as an end rather than a means in early arithmetical learning. The notion that young children learn arithmetic through problem solving did not feature in any of the teachers' responses. The report concluded with an overview of researchers' concerns about early childhood mathematics curricula, and outlined two current investigations into young children's arithmetical learning aimed in part at addressing these concerns. Finally, the use of models of children's arithmetical development by researchers and their potential for use by teachers was addressed.

The beliefs that primary school teachers have about children's prior mathematical knowledge and its relationship with their classroom practice was the focus of research by Brooks, Wright, and Hunting (1995). They interviewed kindergarten teachers using open-ended questions concerning the teaching of number in the first year of school. The findings indicated that teachers "believe there are four basic influences on children's prior mathematical knowledge; positive parental interaction, a wide variety of real life experiences, language, and attendance at pre-school" (p.21). They also believe that these influences can vary widely and may have either a positive or negative impact on the children's mathematical knowledge. The study found that teachers did not seem to link their beliefs about what children knew as they entered school with what they actually taught in the first year and concluded that it is vital to consider the programs these children will undertake in school when assessing their prior mathematical knowledge.

Interview-based Assessment

A report by Hunting, Doig, and Gibson (1993) focussed on the development and validation of clinical tasks for use in interview-based assessment aimed at determining mathematical strengths and weaknesses of students identified as having difficulty in mathematics while Hunting and Doig (1992) discussed issues of validity relating to interview-based assessment. Mulligan and Thomas (1995) outlined limitations of assessment practices in early arithmetical learning and discussed assessment via interview and instructional settings in which problem-based learning was featured. They presented theoretical models for mathematical learning and problem solving based on characteristics of children's representations, and outlined implications for professional development, mathematics curricula and assessment, promoting the growth of mathematical understanding. In a pilot study using interview-based assessment and drawing on the model of arithmetical stages (e.g., Steffe et al., 1988; Wright, 1993c), Hughes (1993) reported links between fourth-graders' abilities to solve single and two-digit addition and subtraction problems and their understanding of place value. Notable differences in the levels of children's arithmetical knowledge are detailed via transcripts of the assessment interviews and the usefulness of the assessment for identifying students at-risk is examined.

Wright (1993c) outlined an approach to assessment in the lower primary years in which students are individually interviewed and interviews are videotaped for subsequent analysis. This approach has been used extensively by researchers and teachers, and has been found to be useful and informative. The approach is task-based and involves determining levels of knowledge on several topics such as arithmetical strategies and levels of facility with forward and backward number word sequence and numeral identification. A profile is formed summarising the child's arithmetical knowledge in terms of the levels and describing the child's strategies and responses on a wide range of arithmetical tasks. Also included in Wright's report are two case studies of student progress over the course of a school year which illustrate use of the assessment approach. Wright also discussed of the purposes of assessment in the early years, the contrast between normative based assessment and assessment which places importance on developmental learning, and an investigation into teachers' views of assessment at this level. Of interest in the latter were: (a) that for some teachers it is important that reports to parents contain only positive statements about a child's abilities; and (b) evidence of a profound difference between what teachers know about the relative progress of students and what they typically report to parents.

Intervention Studies

Young-Loveridge (1993a; 1993b) explored the effectiveness of two kinds of intervention programs to improve the number skills of five-year-olds. Her intervention study involved 246 children all but one of whom were aged between 5.0 and 5.7 years. On the basis of individual, task-based interviews, children were classified as target, that is, having low to average numeracy levels and therefore eligible for intervention; and non-target, that is, having high numeracy levels and therefore not eligible. Target children were similarly assessed on three more occasions: Posttest 1 — after a two-month intervention period; Posttest 2 — six months after completion of intervention; and posttest 3 — 15 months after completion of intervention. Posttest 3 involved 131 target and 33 non-target children — the latter being those non-target children just above the cutoff point for inclusion in the target group. There were six project schools in all and two types of intervention. Intervention A — school-based intervention, occurred in two schools; involved 13 pairs of target children; was of seven weeks duration; and involved daily, 30-minute instructional sessions featuring number rhymes, number stories and games involving early arithmetic (e.g. board games using die, card games etc.). Intervention B — home-based intervention, occurred in two schools different from those used for Intervention A; involved 24 target children; featured weekly home visits for the first two weeks, parental attendance at weekly workshops for five weeks and a final home visit; and involved parents in playing games with and reading stories to their children. Target children in the two schools without intervention programs constituted an across-schools contrast group. Non-participating target children in the four schools in which intervention occurred also constituted contrast groups. In the year following that in which Interventions A and B occurred, a program labelled *combined intervention* was conducted. Findings included: (a) the effects of school-based intervention were initially large and remained statistically significant for at least 15 months; (b) the effects of home-based intervention were modest overall with effectiveness increasing with parent commitment; and (c) significant differences typically relating to more than one of; gender, socio-economic status, and ethnicity. For example, Maori girls in the contrast group were substantially ahead of boys at every assessment point. The combined intervention program, found to be highly effective, featured a meaningful context, that is, books and games; an early beginning, that is, new-entrant level; an orientation to prevention rather than remediation; emphasis on co-operative learning and social interaction; adults' structure and support of children's learning; matching level of challenge to the learner's needs; and use of teachers trained in research.

The effectiveness of a combined intervention program was the focus of a study involving 139 seven-year-olds and three schools (Peters, 1993). Intervention A was used with the three classes from one school, involved maths games, and did not involve parental help. The combined intervention involved maths games and parental help and was used with two classes from a second school. Identical maths games were used in the two intervention schools while two classes from the third school served as a contrast group. Data collection included pre- and post-interviews to assess mathematical knowledge, post-interviews of children focussing on attitudes to mathematics and solution strategies, and interviews of teachers. The results indicated that there was very little difference in the gain scores across the three groups of children. Peters conceded that the interventions used in this study were unsuccessful but pointed out the value of games for assessing children's mathematical understanding and providing "a suitable context for the development and reinforcement of mathematical skills" (p.22). She reported that 78% of the children showed positive attitudes towards mathematics and concluded that intervention with seven-year-olds may not be "too late" (p.22). Peters highlighted several factors of importance when planning intervention: the duration of the intervention, the availability of sufficient materials to suit all children; assistance from motivated parents, teachers who believe in the intervention and are able to motivate parents, parents working with pairs of children of similar ability, and training and support for parental helpers. Finally, Peters suggested that using "parents sessions" (p. 24) as an intervention method is worth continuing, but research into methods to help teachers work with parents is required.

Young-Loveridge, Carr, and Peters (1995) investigated how mathematics concepts and skills of four-year-olds are enhanced at the preschool level. Their study involved four preschools and 154 children aged from 3.9 to 4.9 years for a three-month period of intervention in the form of teacher development focussing on construction of new resources. Pre- and post-testing involved individual task-based interviews, teacher interviews and observation of four children with high numeracy attainment and four with low numeracy attainment at each pre-school. Observation of children's activities involved narrative recording in conjunction with approximately 70 hours of audiotaped language in 15-second intervals, recorded via remote microphones worn by individual children. Each interval was coded according to categories of area of preschool, mathematical purpose, mathematical skill, manipulation of quantity and social group. Major findings of the study included: (a) a high degree of variability in preschoolers' concepts and skills; (b) relatively little mathematical activity in the preschools; (c) virtually no difference between high and low attainers, in engagement at preschool, in mathematical activities, skills and purposes; and (d) that high attainers spent more time with adults than did low attainers. Included in the report are: (a) individual case studies of control and intervention children, and of preschools and their teachers; (b) tables of data relating to children's performance; and (c) a detailed discussion of implications of the study for preschool programs.

Pearn (1994) reported on a pilot Mathematics Intervention Program which had as its main aim the development of a selection schedule and effective program for children at-risk. A screening interview using tasks relating to the five counting stages (e.g., Steffe et al., 1988; Wright, 1993c) was administered to 27 first-graders and 23 second-graders. For instance, children were required to count visible and partially or totally screened collections and solve written algorithms. Children's forward and backward oral counting skills by ones, twos, fives, and tens and their comprehension of terms such as *after, before, between* (p.465) were also assessed. Pearn observed: (a) a large variation in children's number knowledge; (b) language problems, such as confusion between *before* and *after*; (c) a variety of strategies; (d) and difficulties common to many of the children. Six second-graders were selected to participate in an intervention program working in groups of three for three half-hour morning sessions per week for 20 weeks. A group of three first-graders was similarly taught for eight weeks. Instructional activities were designed to enhance the children's counting skills and included the use of concrete materials and mathematics games. Verbal interaction was encouraged. All participants showed substantial gains in terms of verbal counting and arithmetical skills. The Mathematics Intervention Program continued in similar vein in the following school year (Pearn & Hunting, 1995). This report underlined the importance of intervention programs for students at-risk, the benefits of clinically trained teachers working in early intervention, and the need for additional research.

Mathematics Recovery and Teacher Development

Wright (1992a, 1993b, 1994a) described contextual and theoretical aspects of the Mathematics Recovery Project, a project which focusses on advancing the arithmetical knowledge of low-attaining first-graders and related specialist teacher development. The project involves collaboration with regional government and Catholic school systems. This collaboration takes several forms: (a) substantial resource contribution in the form of teacher time throughout the three years of the project; (b) collaboration in planning at a macro-level with senior administrators and consultants in the school systems; (c) collaboration at micro-levels with school administrators and class teachers; (d) on-going collaboration with specialist Mathematics Recovery teachers who contribute substantially to the development of practical and theoretical aspects of the resulting Mathematics Recovery Program. Mathematics Recovery is an applied research and development project because it aims to solve a practical problem, i.e. that too many children experience chronic failure in school mathematics. The notion of recovery education differs from early intervention because the latter refers to preschool children for whom identification of potential learning problems has occurred in the first three years of life. Recovery education is distinguished from remedial education because the former involves identification after one year at school of children apparently unable to benefit from classroom teaching; intensive, individualised teaching aimed to advance the child to an average level; and intensive and long-term teacher development. Recovery education is most well known in the area of children's learning to read. Development of Mathematics Recovery has adopted some of the key organisational aspects of the Reading Recovery program (e.g. Clay, 1993; Lyons, Pinnell, & Deford, 1993). The theoretical and methodological aspects of Mathematics Recovery are in the results and methods developed by Steffe and colleagues (e.g. 1988) known as constructivist teaching experiments. Mathematics Recovery rejects the assumption that a good number of students necessarily will find mathematics very difficult, are destined to failure and after 10 or 12 years of schooling in mathematics have little understanding of many of the key ideas in elementary mathematics. No doubt this is a convenient assumption for educational professionals but it does not always rest well with students and their parents. In many cases the parents of these students perceive that schools are failing with too many children or their school has failed with their child, and that their only recourse is to vote for legislators who promise to improve schools through mandated testing and emphasis on basic skills and habituated knowledge.

Details of organisational aspects of the mathematics project are provided by Wright (1992a, 1993a). The project can be regarded as an extension of a collaborative program between a university faculty of education and regional school systems which, prior to the project, focussed on professional development and preservice teacher education. Key structural aspects of the project are exemplified in the following overview of the third year of the project, that is, the 1994 school year. An infants level teacher from each of ten regional schools was allocated half-time to the project for approximately three quarters of the school year. Teachers attended a workshop at the university each morning for an initial training period of six weeks' duration. This was followed by implementation of a Mathematics Recovery Program in their school which involved teaching four children individually for 30 minutes each day, for four days per week, and attending an on-going professional development program on the fifth day. Teachers were initially taught an interview-based assessment procedure. Student selection and pre- and post-assessment were undertaken co-jointly by project teachers and researchers in consultation with class teachers. Progress of participants was documented and compared with that of non-participating counterparts. The on-going professional development meetings constituted a context for intensive collaboration between researchers and specialist teachers focussing on student assessment and teaching. For example, an integral part of the project was routinely videotaping by teachers of all assessment and teaching sessions. On-going meetings typically involved short reports by teachers on student progress and contextual issues; longer reports focussing on students' strategies, knowledge and progress; discussion and critique of teachers' assessment and teaching; and discussion of theoretical issues and instructional principles and techniques.

Wright and colleagues (Wright, 1994d; Wright, Cowper, Stafford, Stanger, & Stewart, 1994; Wright, Cowper, Stanger, & Stewart, 1995; Wright, Stanger, Cowper, & Dyson, 1993) reported student progress during each of the first three years (1992, 1993 & 1994) of the Mathematics Recovery Project. In 1992, on the basis of performance during an interview-based assessment (e.g. Wright, 1993c; 1994b), four first-graders in each of six schools were judged to be amongst the lowest-attainers of their school cohort and were selected as Mathematics Recovery participants. Each participant was individually taught 30 minutes per day, for four days per week, for an eight-week teaching cycle, by an infants teacher from their school who was undertaking a professional development course to prepare specialist Mathematics Recovery teachers. In each school, the interview-based assessment procedure was used for post- as well as pre-assessments of participants and non-participating counterparts. It included simple additive and subtractive tasks involving screened and unscreened collections, saying forward and backward number word sequences, stating the word before or after a given number word, and identifying one-, two- and three-digit numerals. A videotaped record of each assessment interview was analysed. This led to documentation of student progress in terms of four models, each of a particular aspect of early arithmetical development (e.g. 1993c, 1994). Wright et al. (1993) reported progress of participants and their prenumerical counterparts in the 1992 cohort, in terms of one of the models — stages of arithmetical strategies. In brief, a student is prenumerical if they do not spontaneously use a counting-on strategy when solving an additive or missing addend task. In each of the six schools, participants' progress notably exceeded that of counterparts. Four case studies are included in this report. The case studies illustrate in detail the progress of individual students, and include descriptions of their strategies and responses to assessment tasks, as well as overviews of teaching sessions. Wright (1994) reports similar progress across all four models — the other three relate to numeral identification, and facility with forward and backward number word sequences. Progress of participants and counterparts from the 1993 and 1994 cohorts are reported in similar vein (Wright et al., 1994, 1995). The numbers of teachers and participants for 1993 and 1994 were respectively 10 and 32, and 12 and 89. For each of the three years overall progress of participants was judged to be excellent. Approximately 75% reach at least an average level for their class and one third of these reach levels significantly above average. In addition, significant peripheral benefits resulted from schools' participation in the project. Principles and techniques learned during the professional development program were found to be easily and powerfully applied to classroom teaching by Mathematics Recovery teachers and their colleagues, and were as equally applicable to average and high-attainers as to low-attainers. Finally, the response to the project by students, teachers, parents, and school and regional administrators was, almost without exception extremely positive.

Conclusion

Recent research in Australasia, into early arithmetical learning and teaching has been wide-ranging. Prominent areas of significant and distinctive research include: longitudinal, classroom-based studies; studies documenting children's knowledge, strategies, and progression over time. Studies of intervention programs and recovery programs for at-risk students; and the role of calculators. These areas are illustrative of the important need for on-going research in early arithmetical learning and teaching.

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Space, Geometry, Measurement, and Visualisation

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Introduction

In this chapter, the term *space* is used to refer to mathematics in the following areas: (a) understandings of two-dimensional (2D) and three-dimensional (3D) shapes; (b) relationships in space and shape such as parallelism; (c) spatial thinking, reasoning, and visualising; (d) solving, conjecturing, and proving in geometry; and (d) topics in non-Euclidean geometries such as topology. The term *geometry* is frequently used to cover some or all of these areas especially in connection with high-school mathematics. In order to convey the full coverage of this chapter to potential readers both words are used in the title. These areas and the related areas of measurement and trigonometry have been the foci of Australasian research over the past four years, building on the research reviewed in 1992 (Davey & Pegg, 1992).

The chapter initially covers the broad issues of problem solving which have emerged from a number of studies involving students solving spatial problems. These studies have developed from learning models such as the generative learning model of Osborne and Wittrock (1983) (referred to in the last review), the Coatsruam Framework of Lawson (1992), and the internal representation model of Goldin (1987). In addition, the new research studies have generated their own models and explanations.

Visual imagery and drawing have been of special interest as they were in the 1992 review. Some studies have considered external representations such as drawings either as a problem-solving tool or as part of communication. Studies have covered the effect on cognitive load of diagrams and verbal explanations, the correlation between problem solving and being able to interpret figures, and intuitive approaches in interpreting graphs. Other studies have focussed on internal representations, in particular imagery. The chapter reviews not only studies on visualisation in space mathematics but also the fascinating use of visual or spatial imagery by young students for number concepts and word problems. Some of the reviewed studies have elaborated the ways in which students use schemata or mental models and others have elaborated the importance of selective attention and responsiveness during problem solving. A number of studies have considered the impact of context and social interaction on language, problem solving, and conceptual development.

The second section of this chapter covers studies on the development of space and measurement concepts; research in these areas has continued since the last review in 1992. This section covers (a) the concepts of parallels, perpendiculars, angles, and symmetry, (b) 2D and 3D thinking, (c) levels of geometric understanding (studies based on the van Hiele theory have continued), and (d) the concepts of length, area, and volume.

Five other issues have been raised in spatial research during 1992-1995and are covered in the final section. These are gender issues, the application of research to curricula and teaching, the impact of technology, the impact of culture on mathematics learning, and the nature of assessment and data gathering.

Problem Solving and Learning

This section reviews a number of studies which have investigated either problem solving or its role in the development of specific concepts in space mathematics. Visualisation and the role of diagrams are two aspects covered. The use of diagrams in teaching has also been the focus of some investigations.

Problem Solving

The effective use of schemata or mental models for solving geometry problems was investigated by Chinnappan and Lawson (1994a, 1994b). They showed that more successful students activated available knowledge more often and used these items of knowledge as cues in order to produce information. Certain management activities were also evoked by more successful students when a certain path did not lead to the goal state. A teaching experiment followed in which a class of 27 Year 10 students were taught for two weeks. Half were given a workbook which encouraged the use of generative activities (such as the use of diagrams to access other related information) in problem solving; the other half of the class were matched on previous performance and formed the control. Despite the small sample of students, the use of procedures for accessing and using information resulted in significantly better posttest scores for both the high and low achieving groups. A similar experiment was carried out with another class; in this class, the experimental group was given a workbook stressing management strategies (such as the importance of careful planning of a solution path).

By considering the correlations between scores on a range of tasks, Chinnappan and Lawson (1994a, 1995; Lawson & Chinnappan, 1994) were able to show that problem-solving performance was particularly related to the organisational quality of students' geometric knowledge as measured by the speed and the variety of recall of relevant mathematics and appropriate examples. An analysis of think-aloud data from pre- and post-test interviews with ten students, half of whom had been given instruction in using general problem-solving processes indicated that generating information and improvement in solution outcomes were the only statistically significant aspects for which the treatment group was better than the other group. For other aspects such as management strategies there was no difference. However, given the small sample size, these results would need to be treated with caution.

In order to assess what types of schema students might access, Chinnappan and English (1995) showed that, for a plane geometry problem, a trigonometry schema was the most frequently considered by 36 Year 10 students.

Nearly 200 students from eight primary school classrooms participated in an experimental study by Owens (1992a, 1992c, 1993a, 1993b) to compare spatial thinking scores of students undertaking a series of 2D spatial problem-solving activities with those of students undertaking a series of number problems over a six-week period. For the students involved in spatial problem solving, the mean score on the 2D items of the delayed posttest *Thinking about 2D Shapes* (Owens, 1992b) improved significantly more than the mean score for the number problem solving group. There was no statistically significant difference in the means of the two groups for the six items on transforming 2D shapes into 3D shapes.

A qualitative study (Owens, 1992d, 1993a; Owens & Clements, in press a, in press b) of students' learning generated a model of problem solving illustrated by a cycle in which the context of learning, especially the classroom interactions and concrete materials (Owens, 1994a, 1994d), influenced the students' cognitive processing and their responsiveness which in turn influenced the classroom context. Responsiveness is a personal involvement emerging from a full range of cognitive and affective processes. From the analysis of video-tape records and students' comments, Owens inferred students' responsiveness resulted from selective attention, imagery, affective processes, checking and other heuristics. Owens' (1993b) study showed that an important aspect in recognising and using concepts and images was that of selective attention which was influenced by perceptual strengths, prior knowledge, comments or actions of others, arrangement of concrete materials, visual imagery, self-monitoring, expectations, and intentions such as trying out ideas. There was no evidence of certain processes always preceding others and the levels of cognitive development were not in a fixed cycle with imagery preceding conceptual modes. Owens' study extended research based on the generative learning model (see Happs & Mansfield, 1990; Osborne & Wittrock, 1983) and representational theory (see Goldin, 1987).

A study by J. Lewis (1995) of 23 students aged seven and eight showed that spatial thinking scores on the 2D items of the test *Thinking about 2D Shapes* (Owens, 1992b, 1993b) were correlated with the use of strategies in solving two complex 2D spatial problems - a tangram task and a mapping task. Scores such as time for completion and use of cues for a 3D task - building a model from a picture - did not correlate significantly with the 2D test items. Lewis further considered strategies used during the tasks. She found, for example, that focussed use of the picture in the building task and the strategy *Sliding in points* in the tangram task correlated with success on the tasks. Based on her study, Lewis recommended that teachers encourage students to check pictures and other clues in order to solve problems efficiently. She also noted that there was no particular strategy use that seemed to correlate with completion rate or earlier completion of the tangram task. Lewis linked the results to the Coatsruam Processing Framework (Lawson, 1992) that shows management and attention, within a context, linked to each of the processes of orientation, analysis, transformation, storage, retrieval, and utilization. The generality of this framework is reminiscent of that developed by Owens (1993a) which shows context influencing cognitive processing and hence a student's responsiveness. Owens noted that monitoring strategies (management in Lawson's terms) and selective attention as important aspects of cognitive processing.

Various theoretical approaches taken to describe problem solving, such as those used by Polya, were compared by Owens (1994a) and considered in terms of a representational model of conceptual processing, visual processing, affective processing, and heuristics used in spatial problem solving.

Visualisation

Owens (1993a, 1994b, 1995a; Owens & Clements, in press a, in press b) established that visual imagery is an important part of problem solving and conceptualising. The imagery varied; some was dynamic involving internal changes but at other times, imagery was better described as concrete (pictorial), pattern (involving a spatial pattern which was often verbalised), action (involving bodily movement), procedural (involving a number of actions), or imagery related to a past episode. (Similar descriptions are found in Presmeg, 1986). Visual imagery was inferred from representations portrayed by verbal description, manipulation, and drawing. Studies by Thomas (Thomas, Mulligan, & Goldin, 1994; Thomas & Mulligan, 1994a, 1994b) also showed that visual imagery in the development of early number concepts was dynamic or static. Thomas and Mulligan (1994b, p. 5) found that among their sample of 77 high ability Grade 5 and 6 students "children with a high level of understanding of the numeration system showed evidence of both structure and dynamic imagery in their representations" (see chapter on early number concepts).

External representations of drawing, picture, and verbal responses were used to classify responses and internal processes of students in Taplin and Robertson's (1995) study. In this study the three different ways were also used to present the problem which involved spatial patterning. Students solved the problem by counting on, doubling, separating the first term, using a pattern linked to previous problem experiences, and finding a pattern (see section on SOLO taxonomy).

Irwin (1995) reported on the responses of 36 students asked to explain a picture associated with what comes between the numbers 0 and 1, 0.1, and 0.01. Students' visualisations varied. While most successful explanations were linked to small parts of the number line between 0, there were some other interesting visual images. An adequate link was made to the age of a baby; but another one was a spatial representation of 0, 1ths, 10ths, 100ths, 1000ths, 1. Most students could associate a picture with their understandings. Irwin specifically noted the influence of schooling on the number line imagery and of language of Pacific Island cultures on making parts. In particular, in trying to represent a tenth, some students tried to take halves and halves again. The word *afa* may be used as half or part in their languages; a similar point was also raised by Owens (in press b) in reference to Papua New Guinean lingua franca.

Mansfield and Happs (1992c) illustrated the importance of mental imagery in estimation from their study of sixth and eighth grade students. Many students had constructed their own mental image of a unit sometimes based on informal units in the classroom or on a metre. For a room's 9 metre length, their estimates varied from 6 to 18 metres. For their estimates of angles, some imagined a protractor while others used a right angle or half turns as a benchmark; still other students used an angle of a polygon.

Images also prove to be important signals for work in algebra. Arnold (1992, 1995) found that certain types of algebraic equations or expressions elicited certain actions, either graphical or symbolic manipulation, depending on the algebraic image. An expression like $4 - 3x$ aroused strong negative feelings because no action was immediately perceived as possible. Arnold noted that these expressions were the usual representation in algebraic software packages and that they would not signal appropriate algebraic action. He also noted that the function machine image was flexible and able to support cross-representational thinking better than a graphical image but no one form should be used because of its likely limitations on algebraic thinking.

In an article on the effects of computer-based instruction,Jones and Lipson (1993) suggested that it is important to use computers to illustrate experimental sampling distributions but they add that the computer images need to use the dynamics of the computer screen to show changes in the distribution with each sample. Mason (1992) provided a theoretical exposition of the importance of imagery and of the role of actions associated with computer screens, especially when students were using the computer mouse and the program *Cabri géomètrie.* In particular he noted the effect of dynamic imagery on construing mathematical ideas. By contrast, Adlem (1992) made no mention of imagery in describing his computer-aided review of statistics although he noted many important principles for program design such as uncrowded screens, comments pertinent to the question for those who do not give the correct response, diversity in answers, some (but not too much) motivation, and careful but independent modules of learning.

Another study of the effects of using computer graphics was carried out by Andrius and Boocok (1995). After reviewing some of the literature on visual imagery and tests of spatial ability, they developed their own computer-based tutorials and test of interpreting diagrams of 3D objects. By comparing the results of pre- and post-tests of 20 trained and 9 "control" Technology Education students, they showed that the computer programs improved both visualisation scores and response time scores. The program itself was self-paced but consisted only of static representations of isometric and orthogonal views of 3D objects.

Bobis (1993) described the effects of activities using cards and "ten frames" with dots and counters on kindergarten children's visualisation of number connections. Bobis concluded that encouraging visualisation aided students' intuitive images of patterns in part-whole relations for numbers (see the chapter on early arithmetic development).

A study by Ohtsuka (1994) showed that spatial mental models differed according to the text genre used. It was found that descriptive genre for a passage which used directions and landmarks yielded 2D spaces but passages using procedural and narrative genres apparently produced one-dimensional mental models. These inferences were based on data showing longer times and more errors in answering order-inference questions. The differences disappeared on the second reading of the texts.

Drawing diagrams. Several studies have considered the role of drawing in either spatial or other problems. Drawing diagrams was seen as one way of generating more information in Chinnappan's studies (reviewed earlier in this chapter) but Diezmann (1995) suggests that diagrams may not be well used by students. She considered the role of drawing in spatial problems and other problems and evaluated the effectiveness of the strategy *Draw a diagram* in problem solving. She noted that end product measures such as performance scores, frequency and spontaneity of diagram use, and solution time were not reliable indicators of students' effective use of a diagram. Her case study illustrated the richness of how a student was thinking but ineffectually using a diagram. Diezmann (1994) also evaluated the use of diagrams by young children at different ages. She did not find babies acting towards diagrams as if they were objects and so she concluded that these very young children already had an understanding of diagrams. In addition, she found that the responses of students in grades 2 and 5 could be classified hierarchically as (a) needing an induction into 2D representation of 3D objects, (b) confusing some properties of the referent and diagram, (c) representing objects by only some features of 2D representations of 3D objects, (d) overcoming perceptual conflicts but not being expert in drawing diagrams, or (e) understanding alternative representations.

Veloo and Lopez-Real (1994; Lopez-Real & Veloo, 1993) found that 38% of primary school students who were prompted to draw a diagram in explaining how to do the problem went on to satisfactorily solve the word problem. Sometimes the students had miscued the words, sometimes one concept they needed to use was clear but the other was not without the diagram. The diagram helped them to focus attention on this other concept, especially when it involved relationships between the numbers of the problem. This occurred whether the diagrams were concrete representations, semi-concrete, or symbolic representations. In their high school sample (1994) the use of diagrams varied with the questions but many students used some type of diagram and then went on to solve the problem. In other words, the diagram was not just an aid for the poorer problem solvers.

Outhred (1995) analysed the responses of children in Years 1 to 4 to word problems including equivalent groups, arrays, and Cartesian products (e.g., the number of combinations of three shirts and two jeans). The representations that children drew to solve or to explain solutions were categorised in terms of structural characteristics such as a linear structure (e.g., a child might draw a line of circles with no groups delineated), non-linear groups, or linear groups in which the elements were aligned in one or two dimensions. Logically, representations that are structured in one or two dimensions would seem to link to array and area models of multiplication. However, the form of the representation seemed to be affected by the wording rather than the type of the problem. Many more children drew non-linear groups for the equivalent group problem than for the array problem while more children drew linear group representations for the array problem than the equivalent group problem. The representations drawn for these two problems were strongly influenced by key words in the problem (e.g., "group" or "line"). Children's responses were also examined to assess if the structure of the drawn representation was related to success in solving the problems. There was a statistically significant relationship only for the Cartesian product problem.

Drawing diagrams and learning style. A study by Lowrie (1994) involving students in the last year of primary school considered the use of visual imagery in problem solving. By using Suwarsono's (1979) problems and interview schedule he ascribed a score for visuality, that is whether visual, verbal, or both types of thinking are used. Through interviews Lowrie probed to check whether students could use both visual and non-visual approaches, and he found that students would tend to use a non-visual approach if the problem seemed easy. When prompted to draw a diagram students corrected their answers. From the analysis of the students' responses he rejected the notion that students had preferred learning styles (visual or verbal) but suggested that students choose a visual or analytical method depending on the problem. (Such a finding echoes Veloo and Lopez-Real's findings mentioned above.) Less competent students often just used the numbers in the problem in some way rather than visualising the meaning of the problem. More competent students who chose an analytical approach could also use a visual approach when requested.

The possible influence of learning style was also investigated by E. Lewis (1994) in a small study on the use of computer screens and concrete materials. He proposed that students learned better if the teaching style matched the students' learning style but concluded that modality use (visual or kinaesthetic) was also influenced by type and level of difficulty of the problem.

These last two studies would bring doubt to the notion that learning styles strongly influence problem solving. Dawe and Anderson (1993; Dawe, 1993), who proposed that the use of imagery may be associated with holistic learning styles rather than to analytic styles which break up a problem into discrete entities, did not pursue their study.

By contrast, a small ethnographic study of adults (Daniel, 1995; Marshall, 1994) suggested that people could be placed on a circular rather than a linear representation of three specific types of mathematical behaviour: (a) the rationalisers who analysed problems, (b) the pictorial pragmatic group who showed a strong visual tendency, and (c) the spatial abilities or harmonic group who tended to work in their heads and give intuitive responses rather than using drawings or pencil-and-paper methods. When these same students undertook three non-routine spatial problems, their responses were classified placing the students in similar positions on the circle of types. Daniel claimed that such a result indicates that students bring their same learning style to a range of problems.

The difficulty that engineering students had in reading diagrams was an issue confronting Churches, Magin, and Barratt (1994). They found that scores on tests of mental rotation and mental cutting had low but significant predictive correlation coefficients for examination results in the subjects Drawing and Descriptive Geometry. The authors felt that self-paced computer-aided instruction in 3D to 2D representations would assist students with low spatial ability to improve their visualisation since they currently struggle with their studies.

Diagrams and cognitive load. Several studies have considered the effect of diagrams on learning. A study by Bobis (Bobis, 1992; Bobis, Sweller, & Cooper, 1993) investigated the effect on performance of Year 4 students on a paper-folding task presented in diagrammatic form when redundant material, both written and diagrammatic, was also given. One presentation provided diagrams with written instructions, another diagrams with additional diagrammatic information, and another written information on the diagrams. Bobis drew the conclusion that redundant material reduced acquisition and recall of the task. One useful finding from this study was that information given on the diagram was most effective in reducing cognitive load and split attention. Although there were no statistically significant differences between the groups using *t*-test analyses of time for completion, the proportions completing the task within a given time were significantly different.

Mousavi, Low, and Sweller (1995) were also working within the cognitive load paradigm in designing experiments in which acquisition problems, followed by test problems, were given to students under various conditions. Top Year 8 students were given congruent triangle geometry problems and Year 4 students were given perimeter of a rectangle problems. It was found that presenting visual diagrams and auditory text, either simultaneously or sequentially, for the worked examples was more effective than two visual presentations. The argument was that using both modalities increased working memory capacity. These results are important for either teacher or computer presentation of worked examples in geometry which are dependent on diagrams and either oral or written text.

Ayres (1993, 1995) also presented data to support cognitive load theory. He found that when given goal-free geometry problems (in which two unknowns could be found) students performed marginally better than with fixed-goal two-move geometry problems. He explained the results in terms of means-ends analysis, that is students would have to keep in working memory not only their fixed goal but also their current problem-solving step in order to solve the fixed goal problems. The number of subjects in the later comparison groups was small and he cautioned that the effect may not apply to problems with many unknowns.

Boulton-Lewis, Wilss, and Mutch (1994), following their study of young children's measurement concepts (see the review later in this chapter) have also recommended that teaching strategies should change to take account of existing mental schemata in order to reduce processing load. For example, young children seem to enter school aware that rulers are used to measure but not informal units so that gaining an understanding of the concept of measurement would be easier if teachers referred to the incidentally understood ruler than to informal units. In these last three studies explanations were given in terms of cognitive load: The load may have prevented efficient learning.

Concrete representations. Experiments used to support cognitive load theory have also shown that concrete materials may hinder learning (see, for example, English, 1994). A study that did not use a cognitive load paradigm was carried out by Tsakalos (1993). The use of investigation with concrete materials was part of her study of one class of Year 8 students undertaking a course designed to develop spatial thinking and problem-solving strategies. She tested the effect on several spatial abilities but found a statistically significant improvement only for interpreting figural information. There were no significant differences in scores for problem solving and use of visual imagery as measured by Suwarsono's (1979) test of word problems and immediate-recall checklist of visual to analytic processing. Tsakalos' qualitative analysis suggested that those who did well were able to impose necessary structure onto the concrete materials. These students generally had good spatial abilities and were mainly male students (the small group of able female students did not show an improvement) but it was found that extra time and direction enhanced effective use of the equipment and concept development. The group of male students with good spatial ability but low choice of visual methods on the pre-tests did increase in their use of visual methods and problem solving scores. It was found that students who used visual means to solve ratio problems were more successful than those who tried to apply a rule. Tsakalos noted students did not discuss much, used equipment to aid calculation rather than problem solving, and those with lower spatial ability scores used the equipment ineffectively. Tsakalos suggested that the lack of investigation may explain the lack of improvement in problem-solving skills.

By contrast, in Owens' (1994a, 1994b) study, the use of concrete materials within a problem-solving context was seen as important. The materials were used and discussed as part of the spatial tasks; problem solving engaged the students' thinking. Features of the materials frequently became the focus of attention and hence aided the problem solving. It is Owens' contention (Owens, 1993b; 1995b; Owens & Clements, in press a, in press b) that selective attention (van der Heijden, 1992) and responsiveness influenced by the materials and other aspects of the classroom, expectations, and intentions could assist students to locate concepts and images affecting their problem solving. This was argued as an alternative explanation to cognitive load theory in explaining the effects of materials on learning (Owens, 1995).

Language and Social Interaction

Several researchers have drawn attention to the lack of adequate language and classroom interaction in learning about spatial mathematics. Tsakalos (1993) noted the effect of lack of classroom interaction on learning. During Owens' (1993b) study, language associated with concepts was limited and students tended not to give verbal reasons based on physical representations suggesting a prior lack of discussion about parts of shapes and about comparisons of shapes. Nevertheless student-student interaction and student-teacher interactions, both verbal and non-verbal, were shown by Owens to encourage students' analysis of shapes and their monitoring during problem solving. Bell (1994) found vocabulary about 2D shapes among primary school students to be quite limited and Robertson (1992) showed that pre-service education students needed considerable development in understanding terminology.

Social context and learning. Previously it was mentioned that Boulton-Lewis et al. (1994) had noted the importance of prior experiences in reaching an intuitive understanding of measurement and that Irwin (1995) had explained the effects of schooling and language or culture on making parts. Zevenbergen (1992) noted that past experiences and social contexts are key aspects of learning and problem solving. Zevenbergen (1995) considered the effects of everyday experiences on spatial language and social equities. Like Owens and Clements (in press a, in press b) and Boulton-Lewis et al. (1994), Zevenbergen emphasised the effects of expectation and social discourse on learning.

Macmillan (1994) analysed the discourse of learning in a preschool play setting in which students were building with blocks. Macmillan commented on the use of language about position and size of blocks to establish social position and cooperation. Thorpe (1995) conjectured, as a result of her observations, that children's learning was strongly influenced by their language usage. Mainstream children used language mainly as a vehicle for communication among themselves whereas special-needs children used language mainly to summon adult help.

In Peter's (1992) study, the girls among the six four-year-old children showed a wide range of spatial skills. Those who scored better on the spatial tasks spent more time in play judged to be high in fostering spatial skills. Girls played with equipment likely to develop spatial skills even though it is traditionally regarded as belonging to the male domain.

The importance of informal experiences and contextual influences is supported by two other small studies in formal school settings in which students interpreted pictorial representations of data. Neal (1995) showed that four-year-old children will attempt to interpret pictorial representations of data but many of their responses can be classified as intuitive, that is linked to personal experiences, some students gave responses that could be classified as procedural knowledge or in transition between interpretive and procedural levels. Asp, Dowsey, and Hollingsworth (1994) also found that younger students interpreted graphs from their own world view but older students were able to use many conventions of graphing.

Affective Factors

From two qualitative studies, one with primary school students (Owens, 1993a) and one with teacher education students (Owens, Perry, Conroy, Geoghegan, & Howe, 1994), Owens concluded that affective processing was important in students' responsiveness during problem solving including spatial problems. Affect was linked to self-monitoring and reassurance but also to willingness to tolerate open-ended problems and persistence in problem solving.

Two other studies which looked at affect and geometry are described later in the chapter; Southwell and Khamis (1994) noted gender differences and Brodie (1993) noted poor teacher attitudes to the space curriculum.

Spatial Conceptual Development

A number of Australian researchers have investigated the development of specific spatial concepts: parallels, perpendiculars, angles in general, and symmetry. Early spatial development has also been investigated in some of the more general studies that have already been reviewed. Owens (in press a) summarised results on spatial items of three *Basic Skills Tests* used in NSW and noted the difficulties with items involving symmetry and interpreting diagrams.

Parallels

Mitchelmore (1992c) sought an explanation for an earlier finding that when young children copied parallel lines crossed by an oblique transversal, their copies were usually not parallel. He found that Grade 2 children in fact recognised their copies were not accurate, but were reluctant to alter their first attempts. When they were explicitly given the chance to improve their drawings, they eventually drew accurate parallels. Mansfield and Happs (1992a) were interested in older children. By asking Grade 8 children to judge whether given pairs of curves were parallel, they showed that—although most of them could give an acceptable definition of parallel lines—their conceptions of parallelism were still basically global. Teaching activities designed to make properties of parallels (and, in particular, the concept of the distance between two lines) more explicit were found to be effective, at least in the short term.

Perpendiculars

Mitchelmore (1992a) found that children in Grades 2 and 4 have far greater difficulty recognising oblique perpendiculars than parallels. He surmised that children normally see right angles only as intersections of horizontal and vertical lines, and therefore do not relate them to angles in general. Mitchelmore (1992b) suggested various teaching activities designed to help children see right angles as special angles.

Angles

Mitchelmore and White (Mitchelmore, 1994b; Mitchelmore & White, 1995a) have been carrying out an extensive investigation of the development of general angle concepts among young children, based on a reformulation of the theory of abstraction in mathematics learning. This theory regards a concept as a product of the recognition of deep similarities between superficially different objects, events, or ideas. In particular, the angle concept is thought to develop gradually as children recognise more and deeper similarities between physical angle experiences, going through three successive stages: classification into physical angle situations such as walking up a slope and using scissors; then into separate angle contexts such as sloping and crossing; and finally into a general angle concept which includes all contexts (Mitchelmore & White, 1995b).

An initial classification of physical angle contexts was presented in Mitchelmore (1993b). Understanding of six of these contexts and the relations between them was then explored among children in Grade 2 (Mitchelmore, 1993a, 1994a, 1993c, in press) and Grade 4 (Mitchelmore & White, 1995b). It was found that children had a good global understanding of all the contexts presented, and improved significantly in their recognition of angle-related similarities between Grade 2 and Grade 4. The major factor hindering recognition of similarity was whether both lines are physically present (e.g., crossings), one line is present and the other has to be imagined (e.g., slopes), or both lines have to be imagined (e.g., turning). Children most easily recognised similarities between situations where both lines are physically present; they found it difficult to relate turning to other angle contexts. Implications for teaching about angles were discussed in Mitchelmore (1993d) and White and Mitchelmore (1995).

The most recent study in this series (Mitchelmore, 1995a) has confirmed that primary-aged children relate situations involving turning about a fixed point to each other more easily than to turning around a corner, and suggested that the physical presence of the two lines in the latter situation encourages a static interpretation. It appears that children learn to relate turning to static angle situations by interpreting turning as a static angle rather than by interpreting static angles as turns.

Symmetry

Cooper (1992) asked whether students would make the same errors in constructing mirror images in three dimensions as have been observed in two-dimensional tasks. Students in Grades 7-11 were asked to draw the reflections of points and lines depicted on the faces of various cubes drawn in perspective. Four mirror planes were compared: horizontal, vertical, and parallel to a receding edge, vertical and diagonal, and diagonal and sloping up and away from the observer. There was no discernible age trend but a strong tendency towards horizontal or vertical sliding in the diagonal cases.

Two-Dimensional Representation of Space

In a small study, Leeson (1995c) interviewed young kindergarten students and asked them to (a) identify which net made from squares and triangles matched each of the presented 3D shapes, and (b) say how many blocks were in a stack when shown isometric drawings. Results showed that either initially or after intervention: (a) nearly all students chose the correct nets for the cube and pyramids but only two thirds chose correctly for the triangular prism; and (b) about two thirds could give the answer for the four- and five-cube stacks but only a third for the ten-cube stack.

Owens (1992b, 1993b) also found interesting results for items on the test *Thinking about 2D Shapes* which asked whether pentomino-shapes (five squares joined together) were 2D nets of open cubes and whether four triangles joined together formed triangular pyramids. Some Year 4 students chose nets which were in the familiar T-shape but no other possible nets, probably resulting from the teacher only using a T-shape to make cubes. Many students estimated results in that they were able to choose shapes correctly but could then not mark the base correctly. Students had more difficulty in mentally folding the triangles (except the large equilateral triangle). These comments were based on overall test results, observations, and interviews of 18 students who, immediately after completing the items, were asked how they were thinking.

General Spatial Development

Thorpe (1995) became a participant observer in three early childhood centres and investigated students' learning and use of spatial concepts in their interactions during play. Her sample consisted of special-needs children and mainstream students. The special-needs children concentrated on topological activities but older ones explored 2D and 3D shapes. The mainstream children were more often able to use their knowledge of shape to solve spatial problems.

A study on 2D shape concepts was carried out by Levins and Pegg (1993). The study suggested that descriptions of 2D shapes given by sixth- and seventh-grade students could be categorised into five groups. These groups were regarded as representative of developmental cycles although the study did not consider any developmental factors other than logical inference. It was suggested that there were two cycles for the one level of development. The study was focussed on the concrete symbolic level in which each cycle began with unistructural comments, then multistructural comments, followed by relational comments. The first cycle was said to focus on features of geometric shape which were seen as necessary while the second cycle considered other properties.

Levels of understanding. Research related to, or associated with, the van Hiele theory (Pegg, 1995a) continues; Pegg (1992b) summarised much of the current research world-wide in terms of features or properties associated with the van Hiele levels. This overview identified a considerable degree of empirical support for features of the theory as well as detailing several difficulties and research directions that appeared promising. Those aspects which have been explored and are commented upon below include: (a) a more robust description of the levels, including the influence of probes and prompts; (b) the relationship between the van Hiele levels and those in the SOLO Taxonomy; the notion of cycles of levels; and (c) an exploration of higher-order functioning.

Lawrie (1993) explored the responses of students to a written version of the Mayberry test which was developed and used in the early 1980s. By using an equivalent, but much larger sample than Mayberry (1981), she was able to explore underlying assumptions as well as the validity and reliability of the Mayberry items. Lawrie was able to identify important features that would detract from accurately describing students' thinking and which cast doubt on some of Mayberry's behavioural indicators of levels. This work was extended in Lawrie's (1994) study when the focus shifted to items that Mayberry had classified as requiring thinking at Level 4. Once again problems were identified which enabled a more precise interpretation of the meaning of the levels as well as the role certain prompts play within a question.

Prompts are not only evident within question formats but, together with probing questions, can also occur in teacher-student discourse. If these are taken into account when working with students they offer a more dynamic and realistic means of exploring student understanding. Pegg (1994), with the use of the NUDIST software package, developed a graphical procedure which summarised teacher-student interaction on an extended geometry problem. This approach offered a means of illustrating students' functional and optimal levels of attainment by taking into account input from the teacher or researcher.

Despite the apparent theoretical difference between the focus on thinking of the van Hiele levels and the focus on responses of the SOLO levels, comparison of the two approaches has implications for geometry research. Pegg (1992a, 1995b) has interpreted van Hiele's five levels within the response categories of the SOLO Taxonomy (Biggs & Collis, 1991). Pegg suggested that Level 1 (the visual level) be equated with the ikonic mode; Levels 2 and 3 with the concrete symbolic mode; and Levels 4 and 5 with the formal and post formal modes, respectively. This comparative analysis offers a broader view of geometrical growth by allowing (a) multi-modal functioning and (b) the use of cycles of levels within modes to explain performance. The former aspect is valuable in explaining and exploring how visual imagery supports later levels of growth. The latter aspect provides a more fine-grained framework to describe student performance.

An alternative approach to this link between cycles and modes has been given from a pilot study by Taplin and Robertson (1995). These researchers reviewed some of the findings given in the literature on spatial processing and visual imagery and then considered responses of 40 Grade 7 students to a patterning task presented in three different ways, concretely, pictorially, and verbally. They described the responses of students and then attempted to classify them in terms of the SOLO taxonomy unistructural-multistructural-relational cycle as the major level which transcended the independently described ikonic or concrete symbolic mode. "Responses at the unistructural and multistructural levels are consistent with Biggs and Collis' (1982, 1991) description of the ikonic mode, with some concrete symbolic support. . . . At the relational level there is some suggestion of transition to concrete symbolic mode" (p. 48). Interestingly, the responses classified as multistructural could be as diverse as (a) constructing the fifth step in a pattern and then using multiples; (b) noting the number of matches for the first step, recognising the pattern for the following steps, and adding on; and (c) looking for a pattern based on probable previous experience.

Pegg (1992a) and Levins and Pegg (1993) identified two cycles of SOLO's unistructural, multistructural, and relational levels in the concrete symbolic mode. Pegg and Woolley (1994) extended this work by examining sets of deductive exercises with over seventy senior secondary students in an individual situation. Their findings identified an early cycle of growth in the formal mode (i.e., van Hiele Level 4). The characteristics of this cycle are the development of the skill of applying some relationships (namely, congruence) to sets of deductive activities. It was not until the beginnings of the second cycle that necessary and sufficient conditions were able to be spontaneously used by students.

The research of White and Pegg (1994) considered geometry at the upper end of Level 4. They tested third-year university students who had completed a course on the development of non-Euclidean geometry. The focus of the research was on an analysis of written and verbal responses to questions concerning notions of restrictions to assumed knowledge. The results showed that an indicator of advanced formal thinking lies in the ability to develop a proof that accommodates cases which take into account different relationships (as opposed to properties) between necessary and sufficient conditions.

Measurement Conceptual Development

Research on measurement among secondary school students has been summarised by Mitchelmore (1995b), who also explored its implications for teaching. At the primary level Outhred (in press) has examined the results of length, area, and volume items for three years of Basic Skills Testing in NSW in the light of research findings. A report on primary school students' understanding of science and technology in Australia (Pattie, 1995) revealed that the effect of teaching was evident for the sample of 1161 children from 34 government and non-government schools. The study indicated that students performed at the higher levels when measuring temperature, length, and mass, reading a classification chart, and using space relationships—mathematical skills used in science and technology. However, such skills need to be further related to the scientific context. There was no overall difference in performance between males and females, or between children in urban, suburban and rural schools but there was a high correlation between socio-economic status and children's understanding of science concepts and their performance of science skills.

Length

Boulton-Lewis et al. (1994) asked students to measure the length of two lines made from joined matchsticks; each configuration had a recognisable pattern. Younger students were likely to choose the familiar ruler rather than the unfamiliar measurement units (sticks) in order to attempt to compare the lengths of the two lines. Boulton-Lewis et al. suggested that the idea of introducing measurement with arbitrary, informal units may not be appropriate for students if they are to grasp the concept of *measuring length* because they do not have a mental model based on familiarity and past experience of the arbitrary units.

Area

There have been several studies on various issues related to this concept. Doig, Cheeseman, and Lindsey (1995) investigated the effects of different material—paper squares, Dienes' blocks and wooden tiles—on children's success in measuring the area of a rectangle. They found that children were least successful when they used the paper squares and most successful when they used the wooden tiles. Children who used Dienes' blocks were most likely to confuse measurement of area with that of perimeter or length. However, Doig et al. thought that use of paper squares seemed to provide more insights into children's understandings of the nature of units than the other two media.

Clements and Ellerton (1995) interviewed a large sample of students on several test items used in basic skills tests in Australia. They showed that there were notable proportions of mismatches between correct/incorrect answers and non-understanding/understanding as probed during interviews. One of the items was to find the area of a trapezium consisting of a square and a triangle (half the size of the square) with only some lengths given. In another study Clements (1995) illustrated a lack of conceptual understanding for a student able to calculate the area of a triangle.

Owens (1993a, Owens & Outhred, 1996, submitted) examined the drawings of students in Years 2 and 4 who were asked if specific units (squares, rectangles, right-angled triangles, and equilateral triangles) could be used to tile figures and how many units would be needed. Three factors that seemed to influence children's responses were summarised by Owens and Outhred (1996):

1. *Size of tiles.* While children seemed to know that there was a pattern for filling the space, some seemed to retain the shape but not the size of the tiling unit. Children who drew tessellated tiles without regard for size usually felt that the space was being adequately filled.
2. *Recognition of tile features.* Students frequently used the sides and corners to begin filling in spaces with tiles. The type of corner seemed important to some students in deciding if a particular tile would be likely to fit. Students had difficulty recognising a trapezium as a composite of tessellated right-angled triangles, despite prior experience with concrete materials.
3. *Judgments about drawings.* Some students decided that overlaps or size or gaps did not matter if the result was "close" enough, that is, they based their judgments on their spatial sense and ignored slight discrepancies in their drawings. Children who thought the tiles should fit often filled gaps with additional tiles, disregarding shape or overlap. Others were influenced by inaccuracies in their drawings and said that the shapes could not be made by fitting tiles together.

While Owens was carrying out her study, Outhred (1992, 1993a, 1993b, 1993c) was independently exploring children's difficulties in representing arrays and how such difficulties are related to performance on area measurement tasks. Her research suggests that knowledge of array structure provides a link between measurement and multiplication concepts in the context of rectangular area measurement. She found that knowledge of array structure was essential for children to relate the lengths of adjacent sides of a rectangle to the number of squares that would cover it. These results indicate why activities with concrete materials may not be sufficient to help children understand the formula for the area of a rectangle. When measuring the area of a rectangle using concrete materials children do not require awareness of row and column structure because the structure is determined by the materials, rather than by the child's thinking.

The effects of specific types of instruction on children's use of lines to represent rows and columns was investigated with children in Years 1 and 2 (Outhred, 1993b). The findings suggested that teaching children that squares in an array were all the same size was not the most effective method to help children to perceive array structure. Teaching children that the squares are aligned or that there are the same number of squares in each row (column) seemed to be more effective methods for moving children from drawing squares individually to representing rows (columns) of squares using lines.

Volume

Toh (1993) compared the effectiveness of computer-simulated experiments with that of parallel instruction involving hands-on laboratory experiments for teaching volume-by-displacement concepts. The purpose of the simulation was to have students test their misconceptions rather than simply being told about erroneous misconceptions. The study consisted of 389 students from 6 Malaysian schools. The results indicated that the computer-assisted group were significantly better in terms of learning gains in the cognitive categories of knowledge and application.

Other Issues

Gender Issues

In the study described in the last paragraph, Toh (1993) found that female students performed better on knowledge and application questions on volume by displacement than males in both the hands-on and computer-simulation groups, especially in the latter group.

Owens (1993b) found that there were no initial gender differences on the test *Thinking about 2D Shapes* in her sample of 200 students in Years 2 and 4 and that gender did not significantly explain variance in scores after a series of spatial problem lessons except on the items requiring 2D to 3D transformations. Lewis' (1995) study of a kindergarten class also found no difference between male and female students in spatial ability on the *Thinking about 2D Shapes* test or in completion of tasks. However, boys were much more independent (asked less of the teacher) than the girls and they were able to use more effective, focussed strategies with regard to using pictures and clues during the 3D model-building task.

A review by Leeson (1995a, b) compared male and female performance on multiple-choice test items from a national competition (1990 to 1992) which was completed by about 30% of Years 5 and 6 NSW students and some students from other states. While boys' scores were significantly higher than girls' on most items, girls performed better than the boys on two reflection items and two figure-identification (embedded or hidden figures) items. Leeson considered this result may be linked to girls' work on design but the small number of items makes this finding tentative.

Another investigation of affective factors in mathematics by Southwell and Khamis (1994) involved the analysis of responses of 2000 secondary students to an attitude and beliefs questionnaire which contained several items on geometry. Geometry was generally not considered a part of mathematics. There were significant differences between male and female students on whether one can discover things about geometry without being taught, whether it is necessary to memorise the way to do geometrical constructions, and whether learning geometry is a means of better understanding mathematical thinking.

Curriculum and Teaching in Schools

A National Statement of Mathematics for Australian Schools (Australian Education Council, 1990) and *Mathematics in the New Zealand Curriculum* (Ministry of Education, New Zealand, 1992) include activities related to a broad range of spatial processing and conceptualising. In the Australian statement, there is an emphasis on visualising, representing, and locating in the process strand called Working Mathematically and in the Space content strand for which a professional development package was developed to cover ideas raised by research (Mansfield, 1994). Mansfield and her colleagues particularly noted the value of using concept maps (Mansfield, Happs, & Briscoe, 1992) and the importance of visual processing in learning about shapes and estimating measures (Mansfield, 1993a, 1993b, 1993c; Mansfield & Happs, 1992a, 1992b).

The NSW K-6 Syllabus (NSW Department of Education, 1989) dramatically changed the mathematics curriculum with its emphasis on activities, language, assessment, and mathematical thinking processes. However, unlike the later national document, visualising was not mentioned among the processes. The Space strand covers 2D Space, 3D Space, Position, and Graphs. Brodie (1994) surveyed a stratified random sample of primary schools in one region of Sydney and found from the responses of the 138 teachers that (a) teachers' attitudes towards the space strand were significantly less positive than towards the other two strands (Measurement and Number); (b) time spent on it was less (modes for Years 2 and 4 were 20% of the time, and for the other years 30%); (c) their theoretical knowledge of how students learn spatial mathematics was poor (e.g., less than 20% had heard of van Hiele); and

(d) their understanding of the breadth of geometry correlated with their own school mathematics.

Each of the three experimental studies by Lowrie, Tsakalos, and Owens (described earlier) involved a series of lessons designed to improve the use of spatial thinking or visualising. Owens noted the importance of problem solving in engaging students' spatial thinking and attention (Owens, 1992c; 1993a; 1994a; 1994b). Concepts such as *shape* developed through activities centred on making pentominoes, and making shapes with pattern blocks, matchsticks, and tangram pieces. Comparisons of overall shape, lengths of sides, areas, and angles were made spontaneously during problem solving. The activities encouraged the understanding of families of shapes, and equality of shapes in different orientations. The need for recognition of parts of shapes was evident but lack of recognition often limited use and development of concepts about 2D shapes.

Lowrie (1994) showed that the use of a teaching schedule encouraging visual representations of problems and discussion of imagery positively influenced students' use of imagery and scores on problem solving. Tsakalos (1993) showed that there was little improvement in using visual imagery but she reported that teachers needed to learn how to manage and use concrete materials so that they might be used effectively.

Using technology. Earlier in this chapter, reference was made to computer-based learning experiences at high school and university. Yelland has carried out a number of studies of children learning LOGO. Her early studies focus on gender differences, and are summarised in the chapter on gender and mathematics. Yelland (1995) summarised LOGO research and the role of length, angle, and shape concepts in LOGO activities. Yelland (1994) describes how several pairs of seven- and eight-year-old children carried out two tasks in a new, more interactive version of LOGO. She concludes that a technologically based learning environment could help to make mathematics more meaningful to children by encouraging them to make connections between spatial and numerical concepts.

Trigonometry. A comparative study (Kendal, 1992) of two approaches to teaching trigonometry, the unit circle approach and the ratio approach, concluded that the ratio method was significantly more effective. The finding was based on an analysis of test results and of responses to an attitude survey given to eight classes randomly allocated to each method. However, as the ratio method required advanced algebraic skills, the author suggested that the definitions of trigonometric functions based on the unit circle could be connected to the ratio definitions and then algebraic techniques should be used.

Culture

Culturally based mathematics is often intuitive and spatial as well as developed. Several studies have noted the spatial mathematics of indigenous societies in the region. Other studies have encouraged an expansion of indigenous mathematics into the school curriculum (e.g., Yirrkala Community School, Thorton, & Verran, 1995;

Stanton, 1994a, 1994b). The Garma Maths program at the Yirrkala Community School and local adult education courses take into account traditional culture on locating. Location and land are integral parts of the culture and provide a way of classifying and explaining. Other support documents for schools in the Northern Territory are also being developed. An extensive record of Aboriginal mathematics, especially space mathematics (maps and orientations and shapes), was summarised and contrasted with Western-European mathematics by Harris (1992).

Finau and Stillman (1995) studied the designs of Tongan women in making tapa and linked these designs to geometry, matrices, vectors, and sequences. Repeated patterns obtained by sliding, flipping, and rotating were a feature of the designs. The triangles were modified by use of parallel lines, and deletions of part of the area. The use of colour on some motifs and not others increased the variation of design and mathematical implications. Owens (1995a) noted the different uses of language words in describing everyday directions in Papua New Guinea as well as the diversity of designs involved in pandanus weaving and string-bag making. These designs were based on square motifs which may be subdivided as rectangles, triangles, or trapezia to create a diversity of design based on the same interlocking weave. The history and systems of Maori navigation provide examples of the actual use of mathematics in non-Western societies and the intuitive nature of the mathematical knowledge of navigators from these indigenous nations (Barton, 1992).

McLeay (1994) explored the variety of ironwork lace patterns in Australia and noted their links to reflections and other transformations.

Assessment and Data Gathering

An interesting issue raised in a number of reviewed studies (e.g., Clements & Ellerton, 1995; Doig, Cheeseman, & Lindsey, 1995; Owens, 1992b) has been the limitation of paper-and-pencil testing, especially multiple-choice test items, to investigate students' learning. As a result, researchers have used these items in interview settings to gain further insights into students learning. The importance of probing and prompting was also raised by Pegg (1994), and Lawrie (1994) gave new interpretations of data gathered from a well-known test used to measure development of geometric thinking.

Conclusion

During the past four years, Australasian research into the mathematics of space has been extensive, covering age groups from early childhood to tertiary students. New insights have been given into aspects of problem solving, in particular, how students organise their geometric knowledge and the effects of context on cognitive processing. Studies have drawn attention to the importance of visual imagery, providing an understanding of its diversity and pervasiveness in mathematical thinking. The relationship of visual imagery to problem solving has been addressed in spatial, numerical, and algebraic contexts, as well as how such imagery might be influenced by the structure of the problem text.

Communication and aids are fundamental to classroom learning environments. In this chapter the complexity of factors affecting students' generation and use of diagrams have been considered, for example, the task itself, the structures of drawn representations and students' knowledge of two-dimensional representations of three-dimensional objects. Specific psychological aspects such as spatial abilities, mental models, selective attention, and cognitive load have featured in these studies. Researchers have focussed on (a) the effects of cognitive load in processing diagrammatic information, (b) the role of concrete materials in developing spatial thinking, and (c) social interaction, language and affective factors, especially in relation to the responsiveness of students.

There has been considerable research into conceptual development in both space and measurement. Topics investigated in spatial conceptual development included ideas of parallels, perpendiculars, angles, and, symmetry. Ways of categorising such development have included the van Hiele levels and the SOLO taxonomy. Conceptual development in students' understandings of measurement have focussed on length, area, and volume. The research into the development of these concepts indicates why they are difficult for students to learn.

Finally, this review has referred to studies of the role of technology, gender, and culture in learning spatial concepts and how these factors may be reflected in curriculum and assessment policies.

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Research into the Learning and Teaching of Stochastics

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Combinatorics has been taught in some schools in Australia and New Zealand for more than 60 years, but the topics of probability and statistics only became common in school curricula during the 1960s (Douglas, 1970). By the 1990s they had become enshrined in national curricula statements (Australian Education Council, 1991, 1994; New Zealand. Ministry of Education, 1992).

Some research into the learning of these topics was done in Australia from as early as the 1950s, but general interest was limited until the 1990s. Hence, earlier MERGA reviews did not deal with them explicitly or systematically. The first review (Briggs, 1984) mentioned four works on probability: Swinson (1978) on page 9, G. Jones (1977) on page 23, Koop (1977) on page 40, and Kempster (1982) on page 85. Of these only Jones' work was summarised in detail and Koop's was neither a research report nor a systematic review of the literature. Subsequent reviews (Blane & Leder, 1988; Atweh & Watson, 1992) made no mention of such research. So this chapter covers a longer period of time than others in this volume in order to provide a comprehensive basis for future reporting. Work done is reported under six headings—Cognitive Studies, Assessment, Curriculum and Pedagogy, Affective and Ethnographic Issues, The Use of Technology, and Historical Evaluation. It has often not been possible to draw links between different work, so some sections are inevitably rather disjointed. In this developmental stage of research comprehensiveness was seen as more important than trying to identify a pattern which is simply not there.

Different expressions for the joint term *probability and statistics* have recently come into use, of which the most common is *chance and data*. Both terms are lengthy and neither specifically includes *combinatorics* which has traditionally been associated with Probability and is also discussed in this survey. There is a perceived need for a comprehensive generic term. We have chosen to follow recent European and North American practice (e.g., Benz, 1982; Szendrei, 1990; Shaughnessy, 1992, p. 466; Shaughnessy & Bergman, 1993, p. 178) and employ the generic term *stochastics*. While this term is not widely used in Australia and New Zealand at the moment it is certainly used from time to time (Watson & Collis, 1994; J. & K. Truran, 1995).

Many teachers seem to feel insecure about teaching stochastics, so research findings could be of special practical value, but neither local nor overseas research has been well disseminated. Some articles of the "this is how I do it" genre have been more widely distributed, but because their ideas have not been supported by formal research studies they are not discussed here. However, they represent important studies of a different kind, and, given the speed at which stochastics teaching is changing and the high quality of some of the material published, there is a need for a separate summary of such material and perhaps a widening of what is understood by research. Such a need is perhaps more marked with a topic like stochastics where there is not a well-established received wisdom about what constitutes good classroom practice.

Cognitive Studies

Early investigations into the understanding of stochastic ideas were few and disparate, and influenced mainly by the work of Piaget and Inhelder (1951) or of Tversky and Kahneman (1974). But by 1995 stochastics was much more commonly taught in schools and was attracting more research interest. By this time too, tertiary teachers of service statistics courses were becoming concerned with the quality of their students' understanding of statistical ideas and were starting to undertake research to understand how their students were thinking.

Very early Goodnow (1958) had provided an extensive summary of people's reactions to chance processes from a psychological perspective and argued that people were less concerned with understanding the structure of random generators than with being able to predict their outcomes correctly. Her work, which presaged many later discussions among educationists, was published within the psychological tradition, and had little impact in educational circles. G. Jones (1974, 1977) prepared a doctoral thesis in the USA which investigated the influence of grade, IQ and the nature of embodiments on young children's probabilistic understanding. He found that understanding developed from Year 1 to Year 3, that IQ was a useful predictor of low levels of performance, and that different probabilistic embodiments stimulated different levels of thinking. This last result was a particularly important early result in probabilistic research, but has tended to be neglected by both researchers and curriculum designers. J. Truran (1985) summarised results of clinical interviews examining children's understanding of symmetry among random generators and proposed a hierarchical classification of the responses which children gave when asked to comment on the likelihood of outcomes from tossing coin or dice.

In the 1990s a number of research strands developed. One followed the work of Tversky and Kahneman (1974) in investigating representativeness and availability heuristics. People using these heuristics to estimate probabilities are unduly influenced by either the results of a small sample or by the ease with which they can recall such situations happening in the past. Peard (1992) compared the use of these heuristics by pre-service secondary mathematics and science teachers and pre-service teachers with no tertiary mathematics training. The heuristics tended to be used only in more complex situations. All subjects had an adequate mathematical background to answer the questions and prospective secondary mathematics teachers employed the same incorrect heuristics as those with no specialised background.

Another strand followed from early work of Piaget and Inhelder (1951) and is reported here in logical, rather than chronological, order. Toohey (1995) examined the meaning of *randomness* and then designed a test to assess adolescents' understanding of the term. Their understanding involved the coordination of a number of different related aspects such as absence of pattern, unpredictability, and generating process used. He found that young adolescents appear to be better at detecting randomness than their older counterparts and offered possible explanations for this. He defined eight ways in which understanding of randomness in adolescents might be developed within the classroom. He further argued that many earlier, frequently cited investigations, such as that of Green (1989) had failed to consider all of these aspects, and so their results should be called into question. K. Truran (1994, 1995, in press) has investigated how some 300 primary school children view the behaviour of various random generators. Children did not always recognise that mathematically identical generators were identical, nor did they respond to similar generators like 6- and 12-sided dice in similar ways. Some attribute animistic properties to generators and some believe that they can control the generators' outcomes. Peard (1995b) investigated the probabilistic misconceptions of students in Years 9 and 10 using a card game which he designed for the purpose of introducing elementary concepts of probability. The experiment was administered over two 80 minute sessions during which long periods of playing the game were interspersed with teaching designed to make the underlying structure of the game better understood. Students preferred to take the part of the player over the part of the dealer even though analysis of the game showed that the odds are clearly in favour of the dealer. All players were able to develop a simple game strategy which improved their chances of winning. These findings have strongly supported those of G. Jones, and may be regarded as strong enough to have general application, but they have had little impact on classroom practice or other research. J. Truran (1992, 1994e) analysed the responses of children in primary and secondary schools to simple chance situations. He argued that experiments which ask children to choose which of two urns containing coloured balls would give the better chance of drawing out a particular colour are assessing their understanding of probability and not just of proportion, as has been argued by some. He also questioned a common view that children predict random events by using a negative recency heuristic (i.e., the probability that a certain event will be predicted decreases as the number of recent occurrences of the event increases). His data suggested that the nature and effectiveness of previous predictions were also important influences on children's predictions (J. Truran, 1992).

The investigations of Taylor and Biddulph (1994) led them to question whether many children have sufficient number sense to engage in activities in a probabilistic way. In subsequent investigations Taylor (1995) identified some of the same heuristics as those reported by others, and went on to question what might realistically be achieved in probability education up to about Year 5. She produced a strikingly unconventional classification of young children's intellectual strategies—do whatever comes to mind, just guess, do not pause to reflect, and regard tasks as separate activities. She found little improvement with age, because the children held tenaciously to their beliefs and their idiosyncratic, unrelated interpretations of what adults would see as related activities. If she is correct, then the assumptions of many workers in this area will need to be reassessed.

Several investigations have dealt with statistics. A report by Pereira-Mendoza, Watson, and Moritz (1995) into children's interpretations of simple graphs found that they tend to believe that a graph must have a pattern, that they have difficulty interpreting intermediate values, and that they tend to make unwarranted assumptions from the data. Callingham (1993, 1994) used a questionnaire to investigate understanding of the arithmetic mean among 136 pre- and in-service teachers and interpreted her results within the neo-Piagetian SOLO taxonomy (Biggs & Collis, 1982). The respondents used a variety of modes of functioning, according to the complexity of the questions, but the ikonic mode was the preferred initial approach. It was suggested that the more content knowledge a teacher has, the easier it is to work in the less demanding ikonic mode. A theoretical model was proposed to explain the interaction between cycles in both ikonic and concrete-symbolic mode; recommendations were made from this for teacher development and further research. Reading and Pegg (1995) have investigated the ability of children in Years 7-12 to use measures of central tendency and dispersion to summarise data. They have concluded that students found raw data easier to summarise than graphs, and that they preferred measures of central tendency to measures of dispersion. Girls were better at summarising graphs than boys. The responses to the questions were also able to be set within the theoretical hierarchical framework of the SOLO taxonomy. J. Truran (1995b) analysed incorrect responses of first-year tertiary statistics students when asked to interpret, in a meaningful context, the value of a correlation coefficient. He found that few had a good understanding of the statistical significance of a correlation coefficient and that most did not understand what was meant by the idea of explaining the variance of a variable. This report also listed a large number of idiosyncratic answers which could provide a basis for a more detailed examination of students' understanding of this concept.

The idea of confidence limits plays an important role in integrating the topics of probability and statistics. Dalgleish (1994) examined the estimates of confidence intervals made by fourth-year honours statistics students. He found that having concrete experience of the sampling distribution improved students' estimates. All students allowed for sample size in making their estimates, but tended to over-estimate the confidence intervals for small samples. J. Truran (1994a) argued that children in Years 4-10, after simple experiences with an asymmetrical two-outcome random generator, demonstrate a naïve understanding that results from trials will vary from group to group, but their understanding has a number of inconsistencies. He suggested how these findings might be used to design appropriate forms of classroom activities.

Finally, little attention has been given to combinatorics, despite the fact that early overseas workers like Piaget and Inhelder (1951) argued that its understanding was a necessary pre-requisite for understanding of probability. English (1988, 1991, 1993) has questioned their findings that children aged less than 11 are unable to acquire a systematic combinatorial heuristic, and suggested that their tasks did not encourage such development. She presented 7-12 year old children with two- and three-dimensional combinatorial tasks set as novel, non-routine problems where children did not have an efficient solution procedure. The children showed considerable skills in developing appropriate, though not always optimal, strategies when presented with problems close to their "level of potential development".

It can be seen that by 1995 a much wider research base into stochastic understanding has developed. A significant amount of this has been done using the SOLO theoretical model, but the projects remain relatively isolated from each other. Only now is the amount of raw data available becoming anywhere near sufficient for more general conclusions to be drawn and the work reported here needs to be viewed essentially as part of the world-wide data gathering process which is the most urgent need for stochastics research.

Assessment

A critical aspect of the data-gathering which is necessary at the moment is the design of appropriate assessment instruments. The non-deterministic nature of much of stochastics makes it particularly difficult to assess, and its strong linguistic component makes it an especially interesting topic for study by feminist workers.

A major funded longitudinal study on assessing statistical understanding of teachers and students from Years 3-12 is being done within the framework of the SOLO taxonomy (Watson, 1992, 1994). Such studies are relatively rare in educational research because they need a long time-frame and involve special organisational difficulties, but they are of great value for observing developmental trends. The questions asked in this project are linked to current curriculum documents; many have been previously developed by other researchers, and the intention is to develop items suitable for classroom use by teachers, similar to those written by Collis and Romberg (1992) (Watson & Collis, 1993). The researchers' concern is especially with the way in which people in stochastic situations use multimodal functioning which is the use of both the ikonic (intuitive) mode and the concrete symbolic mode to assist in solving a problem (Watson & Collis, 1994). In trials of the 20-item questionnaire little difference was found between the responses of Year 6 and Year 9 students (Watson, Collis, & Moritz, 1994a). Two questions on luck have been classified using the SOLO taxonomy, and examples provided of students' responses at each level, together with a discussion of the implications of these results for curriculum planners and for teachers (Watson, Collis, & Moritz, 1995).

An attempt to address the issue of linking research with classroom practice has led to the design of "authentic assessment" materials. These reflect educational objectives and encourage open-ended responses which may be ranked according to a hierarchy consistent with a model of cognitive development (Watson, Collis, & Moritz, 1993). This work has led to the development of materials based on the media and concrete embodiments (Watson, Collis, & Moritz, 1994b, 1994c). Every effort has been made to link this work with the National Statement (Australian Education Council, 1991).

Another assessment instrument which is being developed is that of "data cards" (Watson, Collis, Callingham, & Moritz, 1994). This pilot study of higher order thinking in statistics consists of an interview protocol based on a set of 16 cards containing information about imaginary children; the trial used students in Years 6 and 9 and adapted the instrument for group work with two classes of Year 6 students.

Some of the other work on assessment of stochastic understanding has examined differences between traditional assessment and less structured tasks. Forbes (1994) investigated the internal assessment mode for public assessment of Year 12 statistics courses. She believes that this mode advantaged non-Maori female students and written examination advantaged non-Maori male students. The gender differences are less marked for non-Maori students who also tend to do better than Maori students. She argues that the skills being assessed in the internal mode are closer to those required by practising statisticians. Clark (1994a, 1994b) provided supporting evidence for this position at both secondary and tertiary levels. She showed that female tertiary students had most success, and prefer questions which involve people or the environment. She also found some evidence of gender difference when examining the relationship between the order of questions in a paper and the choice of questions, when this was permitted. Smith, Wood, Gilles, & Perrett (1994) have suggested that difficulties with the encoding and decoding between English and mathematical expressions plays an important role in determining the difficulty of statistics questions for students. This may be independent of the linguistic difficulty of the examination questions themselves as measured by lexical density.

Two other researchers have used concept maps in preliminary studies to assess tertiary students' understanding of statistical ideas. Lipson (1994, in press) has been establishing a theoretical model for assessing statistics teaching which distinguishes procedural and conceptual knowledge and has found concept maps effective in measuring both forms of knowledge. Williams (1995) has used concept maps to assess understanding of hypothesis testing and has found them better for assessing conceptual than procedural knowledge. She has also developed principles of administration to maximise the value of the assessment procedure.

J. Truran (1993, 1994d) has proposed a provisional structure for a theoretical base to assess the ideas of probability being examined in a test. This structure incorporates variables which research has shown to be relevant, such as the nature of the random generator, the presence or absence of rewards, the operator of the random generator, and the form of questions asked. Application of the structure to some commonly used tests shows that they tend to assess probabilistic understanding in a restricted was by using only a limited number of random generators, questioning formats, and probabilistic situations.

There is, of course, a lot more which still needs to be done. But the work reported here is within the highest traditions of education research—it is disciplined, concerned with equity, sound learning, and relevance to classroom practice. The researchers are concerned to delve deeply into students' understanding. They care far less about assessing whether an answer is right or wrong than about understanding the reasons for the answers given. They are interested in breaking down the difficulties which are inherent in using language as a medium of communication in learning and assessing environments, and are also trying to make their work suitable for supporting good classroom practice, particularly in a relatively new topic.

Curriculum and Pedagogy

Not only has probability and statistics been a relatively recent addition to the primary and secondary mathematics curricula in Australia and New Zealand but statistics has increasingly become a service component of an increasing number of arts subjects in tertiary institutions. The need to present the subject in so many new ways has produced some imaginative and different pedagogic approaches. Some of these have been subject to formal assessment by researchers.

Swinson's (1977) substantial literature review led him to conclude that probability could be taught to primary children, and that adequate pedagogic techniques were available. Kempster's (1982) work on whether probability could be taught in primary schools included an early test of probabilistic understanding to assess the teaching programme. He used practical work in probability to prepare a course for upper primary children to be taught by their usual teachers and found that the course was moderately effective. J. Truran (1992) wrote and tested a distance course in Year 11 probability which incorporated some research findings as well as being based on a rigorous mathematical foundation. Neither Kempster's nor Truran's work have had wide influence. Recent Australian curriculum material for classroom teachers (Lovitt & Lowe, 1993a, 1993b; Finlay & Lowe, 1994) has been extensively trialled in the classroom, and to this extent warrants its inclusion in a research summary. However, it is not closely based on more formal research findings, partly because it reflects the authors' applied perspective.

Since about 1992 school curriculum development has been dominated by the national curriculum statements (Australian Education Council, 1991; New Zealand. Ministry of Education, 1992). These have received little critical attention at the detailed level but Watson, Collis and Moritz (1993) have observed that many important questions were not asked before the Australian statement was prepared and J. and K. Truran (1994; K. & J. Truran, 1994) have questioned the scope and sequence of this statement. The latter researchers argued that the profiles failed to distinguish different types of probability, they failed to pay sufficient attention to accurate probabilistic language, that the Work Samples provided examples of poor practice, and that the links between the Chance and Data strands and other strands were poorly articulated. The documents showed little awareness of either the special difficulties of teaching stochastics or of research findings about children's learning of probabilistic ideas. Callingham and Watson (1995) found that while teachers are confident about teaching graphs and less confident about other topics in stochastics, their confidence is sometimes misplaced. Teachers' understanding varied according to both school type and gender. The need for more work on the relationship between curriculum documents and teachers is clear.

Other investigations have dealt with individual topics within stochastics. In qualitative studies it was found that advanced Year 8 students can develop intuitive ideas of average and dispersion, but have some difficulty in expressing these ideas precisely. With care they are able to learn to use and interpret box-and-whisker plots effectively and their performance and understanding are enhanced when they collect their own data and, when working initially with small data sets, mark all of the data on the plot as well (Carr, 1994; Carr & Begg, 1994). Asp, Dowsey and Hollingsworth (1994), in a small preliminary study, suggested that prior knowledge, scale, pattern, and possibly also gender were significant variables affecting the ability of children in Years 4-6 to interpret bar graphs and pictographs. Encouraging a child to reflect, after giving an initial answer, usually elicited further responses.

The role of theoretical probability in tertiary statistics courses has been of special interest to tertiary teachers. Pfannkuch and Brown (1994) conducted a pilot qualitative study, which included a teaching component, to assess the understanding of variability and probability among first-year psychology students enrolled in a stochastics course. They found that the theoretical section on probability which is included near the beginning of many statistics courses was the first barrier to students' understanding and questioned whether it is necessary. Similarly, Lipson (1994) argued from her research that simulation is a more appropriate way for developing probability concepts than the learning of probability theory. Other points noted by Pfannkuch and Brown were the difficulty of making the transition from descriptive to inferential statistics and students' preference for deterministic rather than probabilistic thinking. Roberts, Martin, and Pierce (1994) reported on the incorporation of practical work (worksheets or practical sessions for collecting data) into a traditionally presented tertiary course in basic statistical inference. Students preferred the practical classes and were positive about the approach. However, examination results dropped significantly. A possible explanation for this may lie in the changed nature of the student intake during the experiment. Whatever the reason, this report highlights some of the difficulties of effectively assessing the results of changes in pedagogy.

Flanders and Fuller (1995) have reported on the increasing place which probability is taking in business-oriented tertiary courses, and on the ways in which business leaders are influencing the nature of what is taught and how it is taught. They emphasise that there is a need for a high level of relevance in such courses, and for close links between industry and universities, and claim that current teaching methods are not meeting the needs of students. However, the basis for these claims is limited, and the report should be seen as preliminary only.

In the 1960s and 1970s some North American research on theories of teaching used a specially constructed course on probability to test these theories on the grounds that the subject matter would be unfamiliar to students. Of course, such experiments also revealed something about children's understanding of probability.

Within the same tradition, Ingram (1995) has carried out a qualitative study on the effectiveness of cooperative learning in two all-female Year 10 statistics classes. She found the students' responses to be of a high level, and attitudes to the subject to be positive.

The research summarised in this section has a more pragmatic bias than that in the earlier section on Cognitive Studies, but there is of course some overlap. It shows that at the tertiary level, where teachers enjoy a relatively high degree of autonomy, there are some committed attempts to match good pedagogic practice with a soundly constructed curriculum. However, even at the tertiary level, there is still a lot more exploration to be done, and at the school level the links between pedagogic practice and curriculum are, to say the least, tenuous. The importance of establishing such links is highlighted by those researchers addressing affective and ethnographic issues.

Affective and Ethnographic Issues

There has been relatively little work done in this area, but what has been done shows that perceptions about stochastics which students have developed within their home culture can influence their ability to develop formal stochastic ideas.

Peard (1994, 1995a) has explored how social background contributes to intuitive knowledge in probabilistic reasoning by comparing a group of 20 Year 11 students from families where track gambling formed an important part of their culture with a group from families where gambling in any form was absent. When relating mathematics to a variety of gambling and non-gambling situations he found differences in the consistency and type of language used by the two groups and in the ways in which they processed probabilistic ideas. Gamblers obtained more correct solutions than non-gamblers, but there were no differences discernible when comparing school achievement or gender. Both groups tended to use representativeness and availability less than has been reported in other studies.

Gordon (1993, 1994, 1995a, 1995b) has been concerned with comparing the perceptions of willing and unwilling enrollees in tertiary statistics courses. She has found that their perceptions of mathematics and statistics strongly influence their approach to and learning of statistics. Students view themselves as relating to "people" or "numbers" and believe that these categories are disjoint. If this perception is widely held, then it is of great importance in considering the relationship of computers and other technological aids within the stochastics curriculum.

The Use of Technology

The introduction of the personal computer into classrooms has provided new ways of exploring statistical data. While many successful experiments concerning the use of computers have been reported, few have been subjected to critical research analysis.

P. Jones (1995) has postulated a theoretical model of an "intelligent partnership" between a student and computer-based technology, in particular, statistics packages. This model aims to optimise the statistical performance of the partnership while conceding that such a partnership may not be the most appropriate for allowing a student to develop concepts.

Naidu (1994) used a variety of qualitative methods to investigate the effectiveness of computer simulations in helping upper secondary students to understand the law of long-run probability and the central limit theorem. Both students and teachers agreed that computer simulations were more effective, more flexible, and more enjoyable than a teacher directed theoretical approach.

The impact of technology on the stochastics classroom is clearly an area where more research is needed. Such a paucity of research in such a significant, developing area highlights the limitations of the unstructured way in which educational research tends to take place. This chapter concludes with a summary of the history of stochastics research in Australia which shows how there has in the past often been a poor match between research practice and the needs of classroom teachers and curriculum developers.

Historical Evaluation

When stochastics entered Australian schools overseas trends were considered, but what was likely to work well in the classroom was generally seen by teachers as more important than theoretical justifications. Early stochastics teaching was within the "pure" tradition; in recent years the "applied" tradition has become more influential (J. Truran, 1994b, 1994f). Curriculum change was slow and uncertain (Douglas, 1970; Swinson, 1977, 1978). Research findings have had little effect on curriculum change and there has been little dissemination of research to teachers (J. Truran, in press). In Victoria after 1990 stochastics in Victorian Certificate of Education school mathematics courses decreased but this trend has since been reversed. (P. Jones & Lipson, 1994). Parallel changes have been found in New Zealand (Begg, 1988). J. Truran (1995) has argued that the Australian developments are better explained by an ecological model of curriculum development, than by the frequently proclaimed "colonial echo" one. An ecological model suggests that curriculum change should consider both mundane forces like the amount of time necessary to develop a curriculum as well as higher level forces such as educational reasons for including a topic in a curriculum.

In an examination of research and curriculum development in Australia and New Zealand J. and K. Truran (1995) observed that most research has been idiosyncratic and unconcerned with directly influencing curriculum structures. They attribute this to the fact that most researchers, with the exception of Watson and her colleagues, are working alone for further qualifications or for the improvement of their own teaching, rather than within a well-funded project supported by a major research grant.

Conclusion

Early research into stochastics teaching and learning in Australia and New Zealand was exploratory and concerned with whether aspects of the subject could be taught in primary and lower secondary schools. The almost universal introduction of probability and/or statistics into secondary schools in the 1970s had almost no influence on research at all. It was only when stochastics was becoming a significant part of national curricula statements that research interest started to increase. At the same time, the increasing number of service statistics courses taught in tertiary institutions led a number of tertiary teachers to develop a research interest in the topic. Such research tended to be closely linked with ways of solving specific classroom problems.

Researchers in these two countries have investigated a wide variety of aspects of teaching the subject, but in a rather uncoordinated way. It may fairly be claimed that these researchers are making a large and significant contribution to such research throughout the world and that some of the work is particularly imaginative. The building of links between researchers into the learning of adults and researchers concerned with the learning of children has the potential to become very fruitful.

There are, however, many important questions which are not at the moment being adequately addressed. They are unlikely to be addressed in a systematic way until curriculum developers and teachers are more aware of what it is that researchers have to offer them.

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Research in Teaching and Learning Algebra

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Before the 1990s, research studies into the teaching and learning of school algebra tended to focus on students' errors and their causes. Failure to learn was usually attributed to psychological characteristics of students (Booth, 1986; Collis, 1975b). Certain misunderstandings or errors, particularly those relating to the concepts of generalised number and variable, were seen as reflecting particular levels of cognitive development. Since 1990, many researchers have redirected their attention from the student to the teaching. Instead of asking why the student did not learn, they ask why the teaching was not effective and whether a different approach might be more successful. These changes in research directions are affecting current thinking about the content and purposes of an algebra curriculum, as well as being influenced by it.

A National Statement on Mathematics for Australian Schools (Curriculum Corporation, 1991) emphasises the role of school algebra as a language for describing generality in patterns and relationships. The recommended teaching approach centres on the study of functions and their graphical and written representation. Its intention is to expand the notion of algebra from a set of manipulative techniques for solving problems to a versatile tool that supports and extends mathematical thinking. This broader view of school algebra is reflected in the diversity of research topics covered in this chapter.

The chapter is divided into two main sections. The first section deals with studies of students' understanding of algebraic concepts and the causes of particular difficulties. The second section deals with aspects of the algebra curriculum and approaches to teaching, concluding with a review of research studies on teaching with new calculator and computer technology.

Students' Understanding

Specific aspects of students' understanding that have concerned Australasian researchers in the period reviewed (1991-1996) are (a) interpretation of algebraic letters, (b) concept of a variable, (c) recognising and describing relationships, (d) causes of the reversal error when writing equations to describe a relationship between two variables, and (e) cognitive factors affecting reasoning in the context of algebra.

Interpretation of Algebraic Letters

Students' interpretations of algebraic letters were investigated by MacGregor and Stacey (1993c, 1994a) and Quinlan (1992a, 1992b, 1992c). MacGregor and Stacey collected test and interview data from a large sample of students in Years 7 to 10, and found that some students' incorrect use of letters stemmed from a variety of sources in and out of school; for example, experiences with puzzle codes in which letters have specified values (e.g., A = 1, B = 2, etc.). As well, there were many students who made guesses about how letters are used or who invented their own notation to express algebraic ideas. This behaviour suggests that their initial learning experiences in algebra had not been effective. The findings supplement previous explanations of students' errors, particularly the hierarchies of understanding proposed by Küchemann (1981) which did not take account of interference from other knowledge or the effects of teaching.

Pegg (1992b) also points out that the interpretation and understanding of letters in algebra is not a simple task for many students. Since it seems likely that mathematical skills of the future will involve the interpretation and development of algebraic formulas rather than their manipulation (which will be done by machines), understanding the meaning of letters and symbols will be of crucial importance.

In Quinlan (1992a, 1992b) a synthesis of hierarchies of understanding for algebraic symbols as expressed earlier by three other researchers - namely, Collis (1975a), Harper (1979), and Küchemann (1981)—led to the identification of five hierarchical levels. These levels were:

Level 5.	Class of numbers: no need for use of trial numbers;
Level 4.	Class of numbers: readily checked by using trial numbers;
Level 3.	Use of trial numbers a necessary process;
Level 2.	Use of only one trial number seen as sufficient;
Level 1.	Letter as meaningless object, or stands for object, or value from place in alphabet, etc.

Studies of students' understanding of algebraic symbols prior to 1992 generally treated data in terms of individual test or interview items. In the Quinlan (1992a) study, data were collected from 517 students in Years 7 to 12 by written tests, interviews, and lesson observations. Procedures involving seven methods of factor analysis followed by the grouping of test item responses produced 38 statistically acceptable scales. These comprised 14 Correct Response Indicators (based on correct responses), 11 Progress Indicators (based on nearly-correct and some correct responses), and 13 Hindrance Indicators (based on incorrect responses). These scales helped to identify the major constructs embedded in the mass of data from the written tests, and incorporated interrelationships between measures formerly considered only in separate studies or treated in isolation within one study. The five levels for hierarchies of understanding were found to be applicable to the sample and fell into an identifiable learning sequence. The most difficult challenge for students was attainment of an understanding of algebraic symbols as representing numerical variables (Level 5). There was a tendency to regard symbols as standing for objects rather than numbers of objects. Those students who made the distinction in favour of numbers were generally more experienced with algebra and/or were in advanced classes. At Year 7, about one-third of students understood the meaning of algebraic symbols after three weeks of algebra instruction. In Years 8 to 12, about half appreciated the number aspect of the symbols. Interviews indicated that some students could have been intuitively thinking beyond numerical variables to consider the possibility that algebraic symbols could stand for variables in a wider sense than simply numbers. Students with better levels of understanding of the meaning of algebraic symbols were more likely to have higher degrees of success with algebraic tasks.

Concept of a Variable

In an action research study of Year 12 students, White (1995) concluded that the ability to think of letters as representing numbers is a key factor in solving linear programming problems. Tularam (1992) found that some students at Year 11 still had the naive belief that letters stand for objects and therefore cannot be operated on. Coady (Coady & Pegg, 1993) used items devised by Küchemann (1981) with a sample of students in first-year tertiary courses, and looked for evidence of the interpretation of letters as variables rather than as specific unknowns. Students' explanations for their responses appeared to be in two distinct categories, the distinguishing factor being the ability to take into account the range of possibilities and limitations associated with an expression containing variables. Coady concluded that the concept of variable requires complex reasoning skills that are not necessary for interpreting letters simply as generalised numbers. From a later study of students' ability to deal with second-order relationships in algebra, Coady and Pegg (1995) concluded that the majority of the 147 first year university students who took part were unable to operate with the notion of variable at the formal level. Quinlan (1992a) also found evidence that not all students would achieve this understanding by the end of Year 12. However he concluded that the majority of "advanced" students, as well as some others, would leave school with an understanding that algebraic symbols in the context of generalised arithmetic represent numerical variables.

Recognising and Describing Relationships

Many errors in algebra can be traced to limited knowledge of the properties of numbers and the operations of arithmetic. MacGregor and Stacey (1993a, 1995b) found that some students do not understand that a single rule or relationship can apply over the whole range of numbers. Consequently they mix counting-on procedures (for small numbers) and direct proportion calculations (for large numbers). It seems likely that they do not know that they are applying a different rule when they use proportion and believe they are taking a legitimate short cut to the answer. For example, the student will count on to complete the pattern (1,5), (2,6), (3,7), (4,?) ... for small numbers, but then apply a proportion rule to get (30,70) instead of using the correct rule "add 4" to give (30,34). Other aspects of arithmetic knowledge, essential for algebra in the middle and later years of secondary school, include familiarity with exponential notation (MacGregor & Stacey, 1994a) and operating with fractions (Bell, MacGregor, & Stacey, 1993).

Blood (1992) found that 12-year-olds' success in recognising relationships between variables in patterns (e.g. geometric patterns of matchsticks) conformed with a theory of van Hiele phases. As in Coady and Pegg's (1993) study, two groups of students were identified - those who were able to see how independent and dependent variables were related, and those who only looked at changes in one variable. Students' perceptions of patterns and their development of algebraic rules were further investigated by MacGregor and Stacey (1993a, 1995b). Written tests and interviews were used for students in their third year of algebra learning. The test items required finding a pattern in a function table, working out higher values to extend the table, and generating a rule connecting the two variables. As expected, one major cause of difficulty in generating rules was focussing on changes in one variable (e.g., "keep on adding 3") and not looking for a link between the two variables (e.g., "to work out *y*, you would multiply *x* by 3 and add 2"). However there were students who could calculate further values in the table by unconsciously using a rule but were not able to describe, either verbally or symbolically, the rule they had used. Evidence gathered so far, by Quinlan, MacGregor and Stacey, and Pegg and co-workers, suggests that the verbal description of perceived relationships in a pattern or data table is a crucial step towards formalising the structure of a rule and writing it as an equation.

Working with senior students, Arnold (1992a, 1992b) explored the ways in which they think about functions in terms of both images and definitions. The results of the study suggested that most students think of functions primarily as algebraic objects that can be graphed and that describe a rule or relationship. Students at certain schools showed no knowledge at all that a mathematical function was any more than "a relationship between things." Students who did not know either an algebraic or a geometric definition of a function had difficulty in distinguishing examples from non-examples. Two important conclusions from Arnold's study are (a) the need to teach students that not every relation is a function, and (b) the need for students to develop a broad base for thinking about functions, incorporating multiple representations. The study also led to concerns about the effect of graph-plotting software on students' thinking. Arnold suggests that students may come to depend on a graphical representation unless encouraged to move between algebraic, graphical, tabular, and spreadsheet forms.

Bell and Malone (1992) report a preliminary study of students' ability to comprehend, construct, and transform algebraic expressions and equations. The aim of the study was to investigate the steps by which students come to use the algebraic language with some degree of mastery. The data suggest that four different modes of thinking were used, and that the dominant mode used by experts for transforming formulas was to visualise physical movement of symbols. This finding raises the important question of whether the visual symbol-moving method should be encouraged, since it may be associated with loss of awareness of the underlying mathematical operations.

Reversal Error when Writing Equations to Describe a Relationship between Two Variables

It is well known that students make errors when writing even the most simple equations from verbal statements about two variables. In the international research literature (summarised in MacGregor & Stacey, 1993b) it has been assumed that a major cause of error is the attempt to translate the English statement into algebra by substituting mathematical symbols for key words sequentially from left to right. This procedure has been called "syntactic translation". MacGregor and Stacey (1993b) designed simple test items so that syntactic translation would yield a correct equation (e.g., "the number y is eight times the number z", which translates syntactically to the correct equation $y = 8$ ¥ z). Most students tested did not use the syntactic method, and wrote incorrect equations such as $8y = z$ in which the numeral 8 is associated with the larger variable y. MacGregor and Stacey suggest that such equations are produced when students attempt to represent on paper their cognitive models of compared unequal quantities, and that syntactic translation is not the predominant cause of reversal.

Data collected by Crowley, Thomas and Tall (1994) on the "reversal error" supports the findings of MacGregor and Stacey but raises questions about the different cognitive models constructed by process-oriented thinkers and "those who think in a flexible proceptual manner" (p. 246). Crowley et al. suggest that process-oriented thinkers prefer [illegible] left-hand-side of an equation to be a "process" and the right-hand-side to be an "answer". Analysis of the data found that, when verbal problems become more complex, students tend to revert to a process-orientation when formulating equations. Thomas (1994, in press) concludes that two major factors influencing students' errors in the construction of equations from word problems are (a) well-formed cognitive processes based on a preference for the process-answer model, and (b) when the written form of the word problem is complex, the use of syntactic translation leading to the word-order-matching error.

Quinlan (1992a) used Odds Ratio analyses (McPherson, 1990) to show that students were more likely to make the reversal error if they had an "object view" of symbols rather than a "number view." He used a revised version of the "professors-and-students" problem (Rosnick, 1981). Combining data from this item and another that was an adaptation of a MacGregor (1991) item, he found corroboration with MacGregor's conclusion that the tendency to reversal was not influenced by whether the context was abstract or concrete.

Exploring further causes of reversal error, Lopez-Real (1995) suggests that different cognitive models are produced from different syntactic arrangements of the same information. His data from 577 students in Hong Kong support the hypothesis that some syntactic structures produce a cognitive model of a comparison relationship whereas others produce a model of an equality relationship. He investigated the effect of giving students "lead-up" questions before they wrote equations about comparative situations. For example, two lead-up questions precede the instruction to write an equation in the following:

In a classroom there are 6 more girls than boys.
If there are 15 boys, how many girls are there? ......
If there are N boys, how many girls are there? ....
If there are N boys and M girls, write down an equation showing the relation between N and M. ....

The occurrence of reversal errors was greatly reduced by the inclusion of the lead-up questions. Lopez-Real suggests that students were successful because they had been forced to think about the meaning of the given statement and perhaps mentally reorganise its structure. Evidence for restructuring was seen in the variety of forms of the correct equations written. MacGregor and Stacey (1994a) used the same problem and also found lead-up questions effective. As Lopez-Real suggests, the function of these subsidiary questions may be to delay an immediate response and force students to think about the meaning of the information given.

Cognitive Factors Affecting Reasoning in the Context of Algebra

Several studies explored cognitive factors that may affect algebra learning. Prasad (1994) obtained evidence supporting Küchemann's (1981) hierarchy of cognitive levels. English and Warren (1994, in press) and Warren (1992) investigated the interaction between algebra understanding and the spatial, logical, analogical, and patterning processes involved in general reasoning. They concluded that understanding the variable concept does not seem to be a natural progression from generalising a pattern. Their findings (see also Warren, 1995) suggest that variables are better understood from tables than from spatial patterns. Generalising from spatial patterns was also investigated by Taplin and Robertson (1995) who linked Year 7 students' performance to the SOLO taxonomy proposed by Biggs and Collis (1982). Students' responses indicated a unistructural-multistructural-relational cycle in their attempts to recognise generalisations from their representations of patterns. A unistructural type of response was efficient for extending a pattern (e.g., a geometric step pattern) when simple calculations were involved, but when trying to calculate higher terms (such as the hundredth term in the pattern) it was not possible for students at this level to make generalisations. The findings of Taplin and Robertson support the suggestions of English and Warren (see above) that generalising from a spatial pattern requires cognitive skills that some students have not developed.

Tularam (1992) investigated students' abilities to apply their algebraic knowledge and reasoning strategies to the solving of non-routine word problems. Looking for connections between aspects of language skill and success in learning algebra, MacGregor and Stacey (1994b) found that metalinguistic awareness of symbols and syntax could be an important factor controlling and limiting students' understanding of algebraic notation. Students with low levels of metalinguistic awareness used automatic unconscious natural-language processing rules, which are not applicable to algebra, and they had not learned how changes in the order, position, or grouping of mathematical symbols affect meaning.

It is widely accepted that algebraic concepts may be acquired and interpreted in two ways: as procedures and operations or as structures and objects. An investigation of first-year university students' algebraic concepts was carried out by Coady (1995). The results indicated that, although these students had been exposed to both the procedural and the structural aspects of algebra, a deep ontological gap existed between the two interpretations with the possibility of a bridge between the two, in many cases, being extremely remote. Interviews with individual students verified that the procedural aspects of algebraic learning, requiring that numbers must be manipulated, predominated.

Teaching and Curriculum

Approaches to Algebra at Years 7 or 8

The restructuring of the algebra curricula in NSW and other states, appears logical and is appealing to many teachers. However, as Pegg (1992a) points out, research into algebra learning lacks a strong theoretical foundation and consequently, despite some important findings, our knowledge has not progressed very far.

There is much debate about the merits of alternative approaches to algebra and the value of concrete materials. MacGregor and Stacey (1992, 1995b) found that a pattern-based approach, currently recommended in curriculum documents for teachers, did not automatically lead to improved understanding of functional relationships and simple equations. In some classes it was successful and in others it was not. These researchers point out that there are critical steps in a pattern-based approach that are emphasised by some teachers and not by others, and that there is more than one cognitive route from perception of a pattern to writing an algebraic rule.

Quinlan (1994a) reported outcomes from a project which were in favour of a concrete approach rather than an arithmetic approach for developing an understanding of equivalent first-degree algebraic expressions. The same teacher taught two Year 7 classes, one with an arithmetic approach and the other with the aid of an objects-and-containers analog. No significant differences between classes were detected at the start, but, after four lessons, the students in the class using the concrete analog were significantly better than their counterparts on both attitude to algebra and content-specific achievement. The evidence pointed to the likelihood that the use made by this teacher of the objects-and-containers analog assisted the cognitive and affective development of the students. Quinlan (1995) discusses this analog in terms of Gentner's (1982) structure-mapping theory of analogs, presenting a strong theoretical basis supporting the use of such a concrete approach. Quinlan (1992e) reports interviews with 52 Year-7 students some six months after a three-week teaching session using the objects-and-containers model. Almost all these students remembered how to use the model correctly, but were not dependent on them. The model was completely effective as a self-correcting aid. Further studies by Quinlan (1994b, 1995) showed the effectiveness of concrete analogs for teaching powers, factorising, and expanding expressions.

Atweh, Boulton-Lewis, Cooper, and co-workers (1994, 1995) suggest that the cognitive load imposed when students use concrete materials in certain circumstances outweighs their potential benefit. The teacher of a class observed by the researchers did not make a systematic attempt to develop the required connections between the materials and the corresponding algebraic representations. When using the materials (coloured discs and cups) to represent equations and solve them, students were slow, hesitant, and often incorrect. They were more successful when they used remembered symbolic routines and mental strategies. Atweh et al. point out that teachers should not assume that students will make the connection between a concrete representation and its corresponding algebraic representation, particularly when the connections are arbitrary rules (e.g., green discs represent positive units, yellow discs represent negative units). The researchers recommend that further research be carried out to clarify the roles of concrete, pictorial, and mental representations in algebra learning. As Quinlan (1995) has remarked, there is little point in promoting the use of concrete materials as such - a lot depends on the teacher's input and the characteristics of the analog used. Analogs that help clarify the underlying arithmetic structures and express isomorphism between the concrete and the algebraic structures are the most likely to assist learning.

Specific Obstacles Addressed

So-called "backtracking" is a useful way for beginners to solve equations. However, as MacGregor and Stacey (1995c) found, for some students it has become a complicated graphic ritual that has lost its meaning and purpose. This was an unexpected finding when 66 students in Year 10 at a particular school were tested on their use of algebra to solve problems. Seventeeen of these students drew a two-row flowchart using boxes and arrows for a simple equation that could be solved mentally, and four of these diagrams led to incorrect solutions. Several other students who tried to use a written backtracking method without drawing the boxes were confused by conflicting notions of "working backwards" and reading an equation from right to left. The researchers warn teachers about the dangers of promoting the use of informal and non-standard notations, for backtracking or any other processes. They also question the use of backtracking as a written procedure beyond the introductory stages of equation solving.

Misunderstanding the purpose and use of brackets is a major obstacle in algebra learning. MacGregor and Stacey (1994a) showed that a single lesson on brackets, based on the distributive law and its parallel in ordinary language, was highly successful with Year 9 classes. When tested some months later, most of the students involved had retained their skills in using brackets correctly. A more thorough study by Hodge (1995) compared the effectiveness of two methods of teaching the use of brackets in one school. One group of students was exposed to a "conflict-discussion" method, and the other to a traditional method involving explanation, examples, and exercises. Both groups improved equally well.

Barcham (1994) developed a short and well-focused teaching program designed to reduce difficulties in interpreting graphs. It was found to be effective at all levels 8-

11, with gains being well maintained. Barcham concluded that many misconceptions and difficulties, once recognised by teachers, can be dealt with successfully in the classroom.

The benefits of a real-life context for understanding functional relationships were studied by Afamasaga-Fuata'i (1992, 1993). Using the technique of "teaching-interview" and a problem about a real-life situation, the researcher was able to follow students' ideas from the iterative conception of a quadratic relationship to its representation as a product of two linear variations.

Learning with Technology

The use of computing technology and multi-representational software, and the increasing availability of graphics calculators, offer the possibility of new beginnings, new routes and new outcomes in algebra learning. Research on teaching and learning with technology is in its early stages, as is the development of teaching materials.

The Technology-Enriched Algebra [TEA] Project (Asp, Dowsey, & Stacey, 1992, 1993) was carried out to investigate the ways in which spreadsheets and function graphers might be used to enhance students' understanding of secondary school algebra. The use of spreadsheets as an algebra learning environment was investigated by Asp, Dowsey, and Stacey (1992) with six classes of Year 9 students. Another study of the effect of spreadsheets for algebra was carried out by Horne (1994) with one class of Year 7 students. Horne found evidence of increased understanding and confidence by the students involved, who were all low achievers. Asp and co-workers found that students did not learn as well as had been expected since they tended to ignore relevant information provided in the spreadsheet display. They concluded that students would have benefited from more input by the teachers to promote reflection and discussion about the tasks. Horne's (1995) work with senior students found that they used spreadsheets successfully to explore a fictitious problem about the spread of a disease.

Another trial carried out by Asp, Dowsey, and Stacey (1993), this time with Year 10 classes, made use of the ANUgraph software package for interpreting linear and quadratic graphs. A control group drew graphs for themselves on paper and used previously prepared graphs. Testing showed that both groups improved on the topic. However the group who drew their own graphs, plotting points by hand, seemed to be advantaged. Although these students were at a level where it is expected that they can read graphs and plot points, they benefited from practice at interpreting coordinates. The computer group worked without interruption at the machines, missing opportunities for reflection and discussion. In the trial of a spreadsheet for equation-solving (Asp et al., 1992) the researchers had also noticed this behaviour. As they point out, when classes have severely limited access to computers, both students and teachers feel that time working with the machines should not be sacrificed for class discussion.

In a three-year study involving secondary students and preservice teachers, Arnold (1995) found evidence that beliefs about algebra and algebra learning obstructed the use of computer software for certain purposes. Representational tools, especially graph plotters, were accepted by the students and teachers concerned since they fit comfortably alongside existing instructional patterns, but tools for the manipulation of algebraic symbols were not. Arnold points out that this impediment to the use of mathematical software tools will be difficult to overcome, since it arises from the perceived nature of algebra as taught and learned in schools for 150 years. According to Arnoid, it is not yet clear whether computer algebra tools can be incorporated into existing patterns of instruction and assessment. However algebra software offers exciting opportunities for versatile thinking, improved understanding, and exploration of mathematical ideas.

Studies by Quinlan (1994c) with two classes using computer software packages (Excel and ANUgraph) indicated that the software helped students to relate graphs with tables of values and to understand that algebraic letters could represent a range of values that were not necessarily integers. Quinlan concluded that many of the students, even at Year 8, were working at the SOLO relational level (Biggs & Collis, 1982) in the computer software environment.

A study of eight classes of Year 11 students by Steele (1995) showed that students' understanding of graphs was enhanced by frequent personal use of the graphics calculator, especially when teachers emphasised aspects of scaling and the viewing window. Steele concluded that when graphics calculators become more affordable they will become essential tools for senior students.

Boers and Jones (1992a, 1992b) investigated the use of the graphics calculator by first year tertiary students and the positive aspects of its use as perceived by the students. Students valued the calculator first as a device for helping them do mathematics and to a lesser extent as an aid to understanding and learning. The researchers found that during an examination students did not use its full potential. They point out that students may have difficulty integrating use of the graphics calculator with previously learned knowledge.

An interesting outcome of the TEA project was the finding that different function graphers promoted different concepts and solving procedures (Tynan, Stacey, Asp & Dowsey, 1995). Minor variations in the capabilities of function graphers led their users to use significantly different approaches to solving a problem. Although it is accepted that there are usually several pencil-and-paper methods to solve a problem, it has not been recognised that with technological tools the number of options is greatly increased. Tynan et al. point out that the decisions made by software developers about what notations to use and what capabilities to emphasise will influence the patterns and directions of students' thinking. Differences in technological tools will promote the development of different concepts and procedures.

Conclusion

Researchers in Australasia are concerned with many questions of importance in algebra teaching and learning. What is the best way to begin? What prior knowledge needs to be developed? What cognitive factors need to be taken into account? How can concrete analogs be used most effectively? In what ways can new technology be used to enhance learning, and how will its presence affect what is learned? Some research has focused on ways to improve students' learning in the traditional curriculum. Other research provides an impetus for rethinking teaching methods and goals in the light of the increasing availability of software for algebra. One overall message is clear: good teaching is necessary, no matter what approaches and materials are used. Whether students are looking for patterns, modelling expressions with concrete analogs, learning about variables from spreadsheets, trying to formulate equations from tables of data, or graphing functions, the role of the classroom teacher is essential for promoting the reflection and discussion that lead to the growth of understanding.

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Robert Peard works at Queensland University of Technology in the School of Mathematics, Science and Technology Education. He teaches units in mathematics curriculum at under graduate and graduate levels and courses in assessment, diagnosis and remediation at the graduate level. He has a particular interest in probability in the school syllabus and is presently researching students' misconceptions in probability. His previous research examined the influence of social background on the development of probabilistic ideas.

John Pegg is an Associate Professor in the Department of Curriculum Studies at the University of New England, Armidale, NSW. His research interests include the exploration and analysis of students' understanding in various subject and topic areas. Related to this work is the application and development of theoretical frameworks from the post-Piagetian paradigm, such as that offered by the SOLO Taxonomy. Currently, John holds the position of Director of research in a newly created centre on Cognition in Mathematics, Science and Technology Education. He is also President of the New England Mathematical Association.

Ian Putt is a senior lecturer in mathematics education in the School of Education at James Cook University, Townsville campus, in Queensland. He has been involved in teaching mathematics education courses at a distance via printed text to inservice teachers and through Interactive Multimedia (IMM) courseware to Aboriginal and Torres Strait Islander teacher education students in their home communities. His current research interests focus on tertiary students' thinking while studying from IMM materials and their usage patterns of IMM courseware. He also supervises post graduate students in the area of teaching problem solving in upper primary and lower secondary school.

Cyril Quinlan taught school mathematics (and quite a few other subjects) from 1951 to 1978 in classes covering Year 4 to Year 12. Since 1979 he has been lecturing in mathematics and mathematics education at what has become the Castle Hill campus of Australian Catholic University where he is currently Program Coordinator for Preservice Secondary Education. He founded the NSW Algebra Research Group which produced the popular series *A Concrete Approach to Algebra* and is working with teachers on school mathematics textbooks for Years 7 and 8. Continuing research interest focuses on the teaching/learning of early algebra. A participant presenter at ICME in 1984 and regular presenter at MERGA and MANSW Conferences in Australia.

Rita Stewart is a qualified and experienced primary teacher and holds a BA with a major in psychology from the University of Newcastle and a BTeach from Southern Cross University. In the last three years Rita has worked as a research officer on ARC funded projects in early mathematics and her interests include research and programs which target children with special needs in mathematics, and programs which may enhance the learning of those children.

Margaret Taplin is a lecturer in the Department of Early Childhood and Primary Education at the University of Tasmania. During 1995/96 she is employed as a lecturer in the Department of Curriculum Studies at the University of Hong Kong. Her main research interest is concerned with students' management of problem-solving strategies. Her current work is focussing on improving the problem-solving skills of pre-service teachers.

Jan Thomas is a senior lecturer in mathematics education at Victoria University of Technology at Footscray. Her research interests are in linguistic and cultural effects on mathematics learning and the politics of mathematics education. She is President of the Australian Mathematical Sciences Council.

Kath Truran had many years as a primary school teacher with a special interest in teaching mathematics and in teaching both gifted and very weak children. She currently teaches mathematics education at the University of South Australia. Her research work is concerned with young children's understanding of the behaviour of random generators. She convenes the Probability and Statistics Special Interest Group for MERGA, assists with a similar group at PME. Since her research interests are very close to those of John Truran, she works with him on some papers, and on creating support materials for use by classroom teachers.

John Truran worked as a teacher of secondary school mathematics specialising in linking mathematics to environmental studies He has spent more than 20 years as a freelance mathematics educator working in a very wide variety of situations and occasionally writing textbooks. His doctoral research interest is on the teaching and learning of probability in secondary schools. He is on the editorial board of the Australian Mathematics Teacher, on the executive committee of the Adelaide Consortium for Mathematics Education, and several other more general education committees.

Paul White is currently a Senior Lecturer at Australian Catholic University and Vice President (Membership) of MERGA. His teaching involvement include mathematics, (number theory and non-Euclidean geometry) and undergraduate and postgraduate mathematics education. Research interests include concept development in calculus, functions and deductive geometry. He is also involved in researching younger children's developing concept of angle.

Bob Wright has had an extensive career in teacher education and research and is a qualified and experienced primary teacher. His academic qualifications include bachelor's and master's degrees in mathematics from the University of Queensland and a doctoral degree in education from the University of Georgia. Bob's interests include early mathematics learning and teaching, and recovery education for at-risk students at the junior primary level. Bob has published widely in research and professional journals and has undertaken several research projects in early mathematics funded by the ARC.

Robyn Zevenbergen. Having completed her doctoral studies at Deakin University, she now lectures in mathematics education and education theory at Griffith University - Gold Coast campus. She teaches critical mathematics at the undergraduate level. Her research interests include the social context of mathematics education, social justice and the construction of difference, critical theory, and workplace numeracy. She is currently involved in a number of action research projects with teachers and a workplace numeracy project which investigates the situatedness of numeracy.

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Adult Numeracy Teaching: making meaning in mathematics

Tout, David and Johnston, Betty, National Staff Development Committee for Vocational Education and Training, Melbourne, 1995

474 pp. $80.00 plus postage

Professional development program designed to train and support teachers of adult numeracy and mathematics in adult literacy and basic education

Breaking the Maths Barrier : A Kit for Building Staff Development Skills in Adult Numeracy

Marr, Beth and Helme, Sue with Tout, Dave, Department of Employment, Education and Training, Canberra, 1991 302 pp. Cost: $35.00

A professional development kit in adult numeracy and includes background notes, training activities and sample student activities.

Numeracy in Focus

Adult Literacy Information Office (ALIO) and ARIS, Ashfield, NSW, 1995

76 pp. $10.00

Three sections: Articles, Reviews and Readings

Numeracy on the Line: Language Based Numeracy Activities for Adults

Marr, Beth; Anderson, Chris; Tout, Dave, National Automotive Industry Training Board, Doncaster, Vic., 1994, 260 pp. $35.00 plus postage and handling

Meets the needs identified by industry trainers in the automotive sector for materials, activities and trainer's notes in the area of basic mathematical skills.

Strength in Numbers : A resource book for teaching adult numeracy

Goddard, Ruth; Marr, B.; Martin, J., Holmesglen College of TAFE, Holmesglen, Vic., 1991 350 pp. $30.00

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Mathematics : A New Beginning: A Resource Book for Teachers of Adults Returning to Study

Marr, B. and Helme, S. (eds.), State Training Board, Victoria, Melbourne, 1987 330pp. $33.00 plus postage and handling. (contact ARIS for information about availability):

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